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Thesis

Joint Pricing and Inventory Control Policies for a Multi-period Dual-channel System

By

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ABSTRACT

As an important research issue, joint pricing and inventory management with uncertain demand has received considerable attention. The literature on the joint pricing and inventory management of a multi-period system with uncertain demand has mostly focused on the setting with a single distribution channel. In recent years, however, dual-channel or multi-channel distribution has become a common business mode due to the rapid advances in e-commerce and the increasingly fierce market competition. The addition of a distribution channel would bring price competition and complicate the formulation of the optimal joint pricing and inventory control policy. Hence, we aim at studying the joint pricing and inventory management problems of a multi-period dual-channel system with uncertain demand in this thesis.

First, a periodic review, joint dynamic pricing and inventory control problem of a dual-channel supply chain with one manufacturer and one retailer, where demand is stochastic and price sensitive, is studied. Stochastic dynamic programming models are built to determine how to adjust the pricing and inventory decisions in every period so that each member's total expected discounted profit over the planning horizon is maximized. Considering the manufacturer is the dominator in the dual-channel supply chain, the structural properties of the optimal joint dynamic pricing and inventory control policy under Manufacturer Stackelberg are analyzed. Moreover, the effects of the dual-channel setting on the optimal policy are clarified by comparing the optimal joint dynamic pricing and inventory control policy of the manufacturer-retailer dual-channel supply chain with that of the dual-parallel-channel supply chain. Our main findings include: (i) the optimal joint dynamic pricing and inventory control policy of a dual-channel supply chain under Manufacturer Stackelberg is an inventory-dependent base-stock-list-price policy; (ii) base stock levels and reduced prices are affected by members' starting inventory levels; (iii) the structural properties of the optimal policy are not affected by the dual-channel setting, while the influence rules of starting inventory levels on the optimal policy vary in different dual-channel settings.

Second, considering the channel power held by the retailer may be greater than or equal to that held by the manufacturer in a practical dual-channel supply chain, the joint dynamic pricing and inventory control problems of a dual-channel supply chain under Retailer Stackelberg and Vertical Nash are further studied. The effects of the channel power structure are analyzed by comparing the optimal joint dynamic pricing and inventory control policies under different channel power structures. Our main findings include: (i) the influence rules on reduced prices under different channel power structures are the same, while the influence rules on base stock levels vary according to the channel power structure; (ii) optimal pricing and inventory decisions are affected by the channel power structure, although the structural properties of the optimal policies under different channel power structures are the same. Results from numerical examples show that, for the two-period dual-channel supply chain, manufacturer and retailer prefer Vertical Nash when the wholesale price is low, while they prefer Manufacturer Stackelberg when the wholesale price is high.

Finally, the dynamic versus static pricing problem of a manufacturer-retailer dual-channel supply chain where demand is stochastic and price sensitive, and inventory can be replenished periodically is studied. Four different pricing strategies, i.e., both members take dynamic pricing (DD strategy), retailer takes dynamic pricing while manufacturer takes static pricing (DS strategy), retailer takes static pricing while manufacturer takes dynamic pricing (SD strategy), and both members take static pricing (SS strategy), are considered. Under each of the pricing strategies, stochastic dynamic programming models are developed to determine the optimal joint pricing and inventory control policy so that each member's total expected discounted profit over the planning horizon is maximized. Numerical studies are conducted to compare the performance of different pricing strategies. Results show that: (i) the optimal inventory control policies under different pricing strategies belong to a base-stock type; (ii) under DD strategy, both members should reduce prices as long as one member's initial inventory level is above its base stock level; (iii) under DS and SD strategies, the member adopting dynamic pricing should reduce price if its initial inventory level is above its base stock level; (iv) there exists a Nash equilibrium of the pricing strategy which is affected by market parameters including demand variety, market size, channel preference, and price sensitivities.

Contents

Chapter 1

Introduction

In an imperfect competitive market, it is widely acknowledged that the selling price of a product and the quantity to produce or order are two fundamental operational decisions of a firm, which play an important role in the firm's performance. Traditionally, the pricing and inventory decisions are made separately, i.e., the selling price is determined by the sales department to maximize the net revenue without consideration of the inventory-related costs, and the inventory decision is made by the manufacturing or purchasing department to minimize the cost by seeing the selling price as an exogenous variable. However, from the end of last century, with the increased emphasis on the integration of different departments decisions, which was attributed to a certain degree to the success of Japanese firms that commonly possess a highly integrated organizational structure, academic and industrial circles began to pay attention to how to make the pricing and inventory decisions jointly with the goal of maximizing the total profit of a firm (Eliashberg and Steinberg, 1993). Many studies like Damon and Schramm (1972) and Welam (1977) have theoretically demonstrated the importance of making the pricing and inventory decisions jointly. In addition, using the data from a local store of a large US retail chain, Mantrala and Rao (2001) suggested that making the pricing and inventory decisions jointly can help a firm obtain the highest profit. As a result, the issue of joint pricing and inventory management has attracted more and more consideration and has become an important research topic.

In practice, owing to the incompleteness of demand forecast information and the existence of uncertain factors or events such as irregular purchase or natural disasters, the demand for a firm's product is inherently uncertain. It is worth noting that, the inherent demand uncertainty is becoming more nonnegligible in the recent period, because of the increasingly fierce market competition, the shorter product life cycles, the COVID-19 pandemic, and geopolitical events. Since a firm typically needs to make pricing and inventory decisions before the demand is realized, the inherent uncertainty over demand usually induces a mismatch between the supply and demand, which has an adverse impact on a firm's profitability and efficiency. Specifically, the understocking, in which the supply falls short of the realized demand, incurs the cost of lost sales or backorder and may result in customer dissatisfaction even if the product can be backordered. And the overstocking where the supply exceeds the actual demand brings the inventory holding cost or disposal cost. A real-world example which vividly demonstrates the adverse impact of demand uncertainty on a firm's performance is IBM. In 1993 and 1994, due to demand uncertainty, the ValuePoint product line of IBM was overstocked which incurred 700 million dollars in unsold personal computers, and meanwhile, the understocking of its new Aptiva home computer line caused tens of millions of dollars to revenue lost (Fisher et al., 1997). Therefore, considering the growing nonnegligible demand uncertainty, and its negative, sometimes even deadly, effects on a firm, it is important to take demand uncertainty into account in the joint pricing and inventory management.

In this dissertation, with the wish of providing managerial insights and guidance for the long-term operation of a system, we focus on studying the joint pricing and inventory management with uncertain demand over multiple periods. There is a vast literature on the multi-period joint pricing and inventory management with uncertain demand, which has mostly focused on analyzing the joint pricing and inventory control policy of a system where the firm distributes products via a single channel. For the details, the reader is referred to the literature review in the next chapter. With the boom in e-commerce and the rapid development of third-party logistics, however, a growing number of manufacturers in various industries, who traditionally sell products through retailers, have engaged in direct sales via the Internet in order to address a wider range of customers with low operational costs, enhance competitiveness, and increase profits. For example, many leading manufacturers in the computer industry (like Apple, Hewlett-Packard, Dell, Lenovo, and Xiaomi) are distributing products through their own online channels alongside the existing traditional retail channels (Xu et al., 2014; Ding et al., 2016). Adding an online channel introduces horizontal price competition between the manufacturer and retailer, which affects the decision-making of supply chain members and complicates the coordination of pricing and inventory decisions. Hence, an important and interesting issue is to address the joint pricing and inventory control problem of a multi-period dual-channel supply chain with uncertain demand.

In Chapter 2, considering that dynamic pricing has recently been a focused issue due to its merits and implementability in reality, we aim to study the joint dynamic pricing and inventory control problem of a multi-period dual-channel supply chain with one dominant manufacturer and one weak retailer. In each period, the manufacturer and retailer are faced with uncertain demand which is assumed to be stochastic and price sensitive. With the goal of maximizing members' respective total expected profits over the multiple periods, the problem is formulated as stochastic dynamic programming models. Then, structural properties of the optimal joint dynamic pricing and inventory control policy are analyzed with a transformation technique and game theory. Moreover, the effects of the dual-channel setting on the structural properties of the optimal joint dynamic pricing and inventory control policy are investigated. Part of this work is included in Li and Mizuno (2022a).

In Chapter 3, considering that different dual-channel supply chains may have different channel power structures, and the channel power structure of a dual-channel supply chain may change over time because of some internal and external factors, we aim to study the optimal joint dynamic pricing and inventory control policies of a multi-period dual-channel supply chain under different channel power structures as well as explore the effects of the channel power structure on the optimal joint dynamic pricing and inventory control policy. Moreover, as an extension of Chapter 2, the effects of the dual-channel setting on the structural properties of the optimal joint dynamic pricing and inventory control policies under different channel power structures are also investigated. This chapter is related to Li and Mizuno (2022a).

In Chapter 4, considering the fact that many firms are still taking a wait-and-see attitude to dynamic pricing, and one of the critical reasons for this fact is the concern about the performance of dynamic pricing in improving the profit, we aim to compare the performance of dynamic pricing strategy with static pricing strategy in the multi-period dual-channel supply chain where inventory control policy is implemented to deal with demand uncertainty. As either of the channels can adopt dynamic or static pricing, the optimal joint pricing and inventory control policies under four different pricing strategies are analyzed and compared. Then, the performance of dynamic pricing and static pricing in a dual-channel supply chain is compared, and the effects of market parameters including demand variety, market size, channel preference, and price sensitivities on the comparison results are studied with numerical examples. This chapter is related to Li and Mizuno (2022b).

The originalities of this thesis mainly lie in exploring the joint pricing and inventory control problem over multiple periods in a dual-channel supply chain with uncertain demand. Specifically, the original points are as follows. First, compared with existing literature on the joint dynamic pricing and inventory management with uncertain demand, this thesis focuses on analyzing the optimal joint dynamic pricing and inventory control policy of a dual-channel supply chain with horizontal price competition and supply-demand relationship between members. Second, compared with existing literature on decision-making of a dual-channel supply chain under different channel power structures, this thesis focuses on analyzing the effects of channel power structure on the optimal joint dynamic pricing and inventory control policy. Last, compared with existing literature on comparison of static pricing and dynamic pricing with inventory control, this thesis focuses on studying the comparison results of static pricing and dynamic pricing with inventory control in a dual-channel supply chain with horizontal price competition and supply-demand relationship between members.

The main contributions of this thesis are as follows. First, it can provide a framework for the joint dynamic pricing and inventory management problem of a multi-period dual-channel system with uncertain demand. Second, it reveals the structural properties of the optimal joint dynamic pricing and inventory control policy of a dual-channel supply chain in Chapter 2, and clarifies the effects of dual-channel setting and channel power structure on the optimal policy in Chapter 3. The results of Chapters 2 and 3 can provide managerial implications for the joint dynamic pricing and inventory management of dual-channel supply chain members, and can, to some extent, help the dual-channel supply chain members understand the effects of the dual-channel setting and channel power structure in the joint dynamic pricing and inventory management. Third, it shows that dynamic pricing may underperform static pricing in coping with additive demand uncertainty in a dual-channel supply chain, and the performance of dynamic pricing is affected by market parameters in Chapter 4. The results of Chapter 4 can, to a certain extent, provide dual-channel supply chain members with the guidance in deciding whether to take dynamic pricing strategy or not.

Chapter 2

Dynamic Pricing and Inventory Management of a Dual-channel Supply Chain

2.1 Introduction

Dynamic pricing is a business strategy that dynamically adjusts prices based on the factors such as time of sales, inventory availability, and demand conditions to maximize the total profit. Owing to the cost and effort of changing prices, the early application of dynamic pricing is centered on the industries where the capacity over a finite planning horizon is relatively fixed and perishable, such as airline, hotel, and car rental industries (Elmaghraby and Keskinocak, 2003; Chen and Chen, 2015). With numerous successful application stories including American Airlines, Marriott Hotels and National Car Rental, dynamic pricing has become one of the most fundamental management tools in the above industries (Netessine and Shumsky, 2002; Zhuang et al., 2017; Geraghty and Johnson, 1997). In recent years, with the boom in e-commerce and the rapid development of information technology like digital price tags, firms in the retail and manufacturing industries where the capacity is commonly flexible through inventory replenishment are able to implement dynamic pricing easily and cheaply (Bitran and Caldentey, 2003). The success of dynamic pricing in the industries with fixed and perishable capacity and the possession of capability to adjust prices at low cost and effort have stimulated considerable interests in dynamic pricing among firms in the retail and manufacturing industries. As a result, joint dynamic pricing and inventory control has received considerable attention.

Existing research in the area of joint dynamic pricing and inventory management has mostly focused on analyzing the joint dynamic pricing and inventory control policy of a firm or a supply chain with a single distribution channel. Driven by the popularity of dual-channel supply chain, in this chapter, we aim to investigate the optimal joint dynamic pricing and inventory control policy of a dual-channel supply chain. We consider a dual-channel supply chain where the manufacturer produces a single type of products and sells them to customers through the traditional retail channel and its direct online channel over a planning horizon with a finite number of periods. In every period, demand in each of the two channels is assumed to be stochastic and price sensitive, the unmet demand is assumed to be backordered, and the leftover inventory is assumed to be carried over to the next period. With the goal of maximizing their respective total expected discounted profit over the entire planning horizon, the manufacturer decides the selling price in the online channel and production quantity at the beginning of every period, and the retailer decides the selling price in the retail channel and order quantity at the beginning of every period. The manufacturer and retailer's profit-maximizing problems are formulated with stochastic dynamic programming, respectively. The interaction between the manufacturer's problem and the retailer's problem is formulated as a Stackelberg game where one player named as the leader moves first, and then the other players named as the followers move sequentially. In this chapter, we focus on the case, called as Manufacturer Stackelberg, where manufacturer is the leader and retailer is the follower. Moreover, we further discuss the effects of the dual-channel setting by investigating the optimal policy of a dual-channel system where the two channels are parallel to each other and compete on selling prices.

Results show that, an inventory-dependent base-stock-list-price policy in which base stock levels and reduced prices are influenced by starting inventory levels, is optimal for a dual-channel supply chain in the face of demand uncertainty and horizontal price competition. Specifically, base stock level is the minimum amount of inventory that should be maintained in operation, and list price is the basic price before any adjustment is taken. The manufacturer is optimal to produce when its starting inventory level is below its base stock level, and the retailer is optimal to order when its starting inventory level is below its base stock level. List prices should be charged only when both members' starting inventory levels are below their respective base stock levels, and as long as one member's starting inventory level is above its base stock level, both members should reduce prices. Moreover, the higher the initial inventory level exceeds the base stock level, the lower the reduced prices. Results also reveal that the structural properties of the optimal policy of the manufacturer-retailer dual-channel supply chain are also suitable for the dual-channel system with two competing retailers. However, the influence rules of starting inventory levels on the optimal policy are simpler in the dual-parallel-channel setting.

The reminder of this chapter is organized as follows. In Section 2.2, we review the relevant literature. In Section 2.3, we make assumptions and notations for the joint dynamic pricing and inventory control problem in a dual-channel supply chain and establish the basic decision model for this problem with stochastic dynamic programming. In Section 2.4, the optimal joint dynamic pricing and inventory control policy of a dual-channel supply chain is explored and analyzed. Section 2.5 provides the analysis of the optimal joint dynamic pricing and inventory control policy of a dual-parallel-channel supply chain. Finally, some conclusions are summarized in Section 2.6.

2.2 Literature review

The work in this chapter is related to the following streams of research literature: (i) joint dynamic pricing and inventory management with uncertain demand; (ii) joint pricing and inventory decisions of a dual-channel supply chain.

2.2.1 Joint dynamic pricing and inventory management with uncertain demand

The literature on joint dynamic pricing and inventory management with uncertain demand can be categorized based on whether the decision-making framework is discrete or continuous. In the discrete-time decision-making framework, pricing and inventory decisions are made periodically at discrete time points, while in the continuous-time decision-making framework, a firm is allowed to continuously adjust the selling price and production or replenishment rate over the planning horizon. Considering it is impractical for a firm to continuously adjust the selling price and production or replenishment rate at any time due to the operational cost and the risk of bringing adverse impacts on consumer psychology, this work focuses on studying the periodic-review joint dynamic pricing and inventory management with uncertain demand.

Zabel (1972) is the first to consider a monopoly firm makes the pricing and output decisions periodically with the goal of maximizing its discounted expected profit over a finite planning horizon. By assuming the expected demand follows a concave function, the unsatisfied demand in each period is lost, and the lead time is zero, Zabel investigates the questions of existence and characteristics of the joint dynamic pricing and inventory control policies in the cases of multiplicative and additive demand, respectively. Since the seminal work of Zabel (1972), a number of papers have studied the periodic-review joint dynamic pricing and inventory management problem with uncertain demand under different settings.

Specifically, from the aspect of supply, different settings include: with a positive lead time (see for example, Bernstein et al., 2016; Feng et al., 2021), considering supply uncertainty (e.g., Amihud and Mendelson, 1983; Li and Zheng, 2006), constrained by supply capacity (e.g., Feng, 2010; Feng and Shi, 2012), etc. From the aspect of demand, different settings include: considering reference price effects (e.g., Taudes and Rudloff, 2012; Chen et al., 2016), in the presence of online reviews (Yang and Zhang, 2022; Vahdani and Sazvar, 2022), general demand function (e.g., Federgruen and Heching, 1999; Feng et al., 2020), demand learning (e.g., Katehakis et al., 2020; Perakis et al., 2022), etc.

Besides the above different settings on the aspects of supply and demand, existing literature has also extended the work of Zabel (1972) from other directions including with fixed ordering cost (e.g., Chen and Simchi-Levi, 2004; Gurkan et al., 2022), in the presence of price adjustment cost (e.g., Aguirregabiria, 1999; Chen et al., 2011), perishable products (e.g., Chen et al., 2014), considering multiple products are sold in the system (e.g., Ceryan et al., 2018; Bhatia et al., 2020), oligopolistic competition (Kirman and Sobel, 1974; Adida and Perakis, 2010), etc. For a detailed and comprehensive review of research on joint dynamic pricing and inventory management with uncertain demand, interested readers are referred to Elmaghraby and Keskinocak (2003), Chen and Simchi-Levi (2012) and Yang and Zhang (2022).

To the best of the author's knowledge, the setting where a manufacturer uses its own and third-party channels to distribute products has not been considered. Driven by the popularity of the dual-channel supply chain in the modern business world, this chapter tries to analyze the optimal joint dynamic pricing and inventory control policy of a dual-channel supply chain with uncertain demand.

2.2.2 Joint pricing and inventory decisions of a dual-channel supply chain

Joint pricing and inventory decisions of a dual-channel supply chain have obtained considerable attention in the marketing and supply chain management literature in recent years. Most of the studies in this area focus on deciding the optimal prices and production/order quantities for a dual-channel supply chain in a single-period setting (see, e.g., Dumrongsiri et al., 2008; Ryan et al., 2013; Modak and Kelle, 2019; Huang et al., 2021; Sun et al., 2022a) or economic order quantity (EOQ) models with pricing consideration (e.g., Panda et al., 2015; Batarfi et al., 2016; Karthick and Uthayakumar, 2022) where the pricing decisions are generally considered to be fixed during the planning horizon.

To the best of our knowledge, only a little literature on the joint pricing and inventory management of a dual-channel supply chain considers the selling prices can be adjusted during the planning horizon. Moon et al. (2010) present a continuous-time optimization model to investigate the joint dynamic pricing and inventory control problem of a dual-channel supply chain where the pricing and inventory decisions are assumed to be able to change at any time. Huang et al. (2012, 2013) develop two-period pricing and production decision models for the joint pricing and inventory problem of a dual-channel supply chain which experiences demand disruption or production cost disruption in the planning horizon. In the above works, demand at each channel is assumed to be deterministic. Considering supply chain members commonly face demand uncertainty which strongly affects their decisions and profits, Li and Wang (2023) study the joint dynamic pricing and ordering problem of a dual-channel retailer which charges identical price in its online and offline channels under different shipping policies. Differing from Li and Wang (2023), our work considers a dual-channel supply chain consisting of a manufacturer and a retailer, and the selling prices of products sold in the online and offline channels are not restricted to be identical.

To summarize, contributions of this study to the literature are that: (i) joint dynamic pricing and inventory management problem with uncertain demand in a setting with dual distribution channels is studied; (ii) considering pricing and inventory decisions can be adjusted over time and the demand is stochastic, the optimal pricing and inventory decisions of a dual-channel supply chain over a finite planning horizon are studied; (iii) the structural properties of the optimal joint dynamic pricing and inventory control policy of a dual-channel supply chain are revealed; (iv) the impact of the dual-channel setting on the optimal joint dynamic pricing and inventory control policy is investigated.

2.3 Model description and assumptions

We consider a dual-channel supply chain where a manufacturer produces a single type of products and distributes the products to customers through an online channel opened by himself and a traditional retail channel over a finite planning horizon with T periods, indexed by $t = 1, ..., T$.

We now outline the sequence of events in each period t and define the notations and assumptions.

(1) At the beginning of period t, manufacturer reviews its inventory level I_t^m, and retailer reviews its inventory level I_t^r. Manufacturer decides the quantity q_t^m to produce with the unit production cost c and the price of products sold in online channel p_t^m, and retailer determines the quantity q_t^r to order with the wholesale price w, $w > c$, and the retail price p_t^r.

It is assumed that, in each period t, the manufacturer and retailer choose to share their initial inventory levels of this period with each other. Theoretical studies have shown that sharing of inventory information can enhance collaborative relationships between supply chain members, increase competitiveness and improve performance (Cachon and Fisher, 2000; Yao and Dresner, 2008). In practice, some firms have implemented information technologies to share inventory information. For example, Apple and its partners have the visibility of pipeline inventory in different stages of the entire supply chain with Apple-Fritz's supplier hub (Lee and Whang, 2000). It is common that manufacturers in the high-tech, automotive, chemical, and home appliance industries access their retailers' inventory levels (Armony and Plambeck, 2005).

Assume w is determined through negotiation between manufacturer and retailer in advance of the planning horizon and fixed during a planning horizon with T periods. While the assumption of the exogenous wholesale price is mainly for analytical tractability, it is realistic in certain settings. For example, when the manufacturer is operating in a highly competitive environment, the manufacturer is a price taker and the wholesale price is determined by the market competition (Dong and Rudi, 2004; Dumrongsiri et al., 2008). According to Zhang et al. (2019), under the circumstances where holding the power of wholesale price-setting may diminish the incentive to share information and cause information leakage, the manufacturer and retailer may commit to the exogenous wholesale price. Production time and lead time are assumed to be negligible.

(2) During period t, customers observe the prices (p_t^m, p_t^r) and may choose the online channel (online store) or retail channel (physical store) to obtain the product. The orders placed through the online store D_t^m are shipped directly to customers with the on-hand inventory at the manufacturer warehouse y_t^m, while the demand at the physical store D_t^r is met with the retailer's on-hand inventory y_t^r. Specifically, $y_t^m = I_t^m + q_t^m - q_t^r$ and $y_t^r = I_t^r + q_t^r$.

D_t^m and D_t^r are assumed to be stochastic and only dependent on the prices of the current period. Similar to Chen et al.(2014), stochastic demand D_t^m and D_t^r are assumed to take the following additive form $D_t^m = d_t^m + \epsilon_t^m,\ D_t^r = d_t^r + \epsilon_t^r$, where d_t^m and d_t^r are the mean demand in the online channel and retail channel respectively, and ϵ_t^m and ϵ_t^r are random terms with $\mathbf{E}(\epsilon_t^m) = \mathbf{E}(\epsilon_t^r) = 0$. It is assumed that the firm has no control over the random term, that is ϵ_t^m and ϵ_t^r are independent of decision variables. Moreover, without loss of generality, the means of the random terms are assumed to be zero.

Following the demand model which are commonly adopted in the dual-channel literature such as Huang et al. (2012) and Ding et al. (2016), we assume the mean demand d_t^m and d_t^r are given by: $d_t^m = \theta a - \alpha_1 p_t^m + \beta_1 p_t^r$, $d_t^r = (1-\theta)a - \alpha_2 p_t^r + \beta_2 p_t^m$, where a represents the forecasted potential demand if the products are free of charge. Moreover, the share of the demand goes to the direct channel is θ, and the rest $1-\theta$ goes to the retail channel. θ captures customers' preference for the direct channel when the products are free of charge. α_1 and α_2 are the coefficients of self-price elasticity of d_t^m and d_t^r respectively. β_1 and β_2 are the coefficients of cross-price sensitivity which reflect the degree to which the products sold via the two channels are substitutes. Assume $\alpha_i > \beta_i$, $i = 1, 2$, i.e., the coefficients of self-price sensitivity are greater than that of cross-price sensitivity, which is widely used in the literature such as Huang et al. (2012), Ryan et al. (2013) and Huang et al. (2021).

(3) At the end of period t, the unmet demand is assumed to be backordered, and the leftover inventory is assumed to be carried over to the next period. Like Feng (2010) and Bernstein et al. (2016), the backlogged cost function or the inventory holding cost function of the manufacturer and retailer, denoted as $h^m(x)$ and $h^r(x)$ respectively, are assumed to have the following common form $h^m(x) = h^r(x) = c_1 x^+ + c_2 x^-$ where $x^+ = max\{0, x\}, x^- = max\{0, -x\}$, c_1 is the unit inventory holding cost with $c_1 > 0$ and c_2 is the unit backorder cost with $c_2 > 0$.

The problem faced by the manufacturer and retailer is to determine their dynamic pricing and inventory control policy to maximize their respective total expected profits over T periods, respectively. This profit maximization problem can be formulated through stochastic dynamic programming models.

Let $V_t^{m\prime}(I_t^m)$ denote the manufacturer's maximum expected discount profit from period t until the end of the planning horizon with I_t^m (the manufacturer's beginning inventory level in period t), $V_t^{r\prime}(I_t^r)$ denote the retailer's maximum expected discount profit from period t until the end of the planning horizon with I_t^r (the retailer's initial inventory level in period t). I_t^m and I_t^r are state variables, in the sense that their values depend on the decisions made from period 1 up to period $t-1$ $(t = 2, ..., T)$.

$V_t^{m\prime}(I_t^m)$ satisfies the following dynamic recursion denoted as model (2.1).

$$V_t^{m\prime}(I_t^m) = \begin{cases} \max\limits_{p_t^m, y_t^m} & d_t^m p_t^m + (w-c)(y_t^r - I_t^r) - c(y_t^m - I_t^m) - \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] \\ & + \rho \mathbf{E}[V_{t+1}^{m\prime}(y_t^m - d_t^m - \epsilon_t^m)] \\ s.\ t. & y_t^m - I_t^m + y_t^r - I_t^r \geq 0 \end{cases} \tag{2.1}$$

where $d_t^m p_t^m$ is manufacturer's expected revenue from selling product online in period t; $(w-c)(y_t^r - I_t^r)$ is manufacturer's revenue from selling product to retailer in period t; $c(y_t^m - I_t^m)$ is the production cost in period t ; $\mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)]$ is manufacturer's expected backlogged cost or inventory holding cost in period t; ρ is the one-period discount factor of money, with $0 < \rho \leq 1$.

$V_t^{r\prime}(I_t^r)$ satisfies the following dynamic recursion denoted as model (2.2).

$$V_t^{r\prime}(I_t^r) = \begin{cases} \max\limits_{p_t^r, y_t^r} & d_t^r p_t^r - w(y_t^r - I_t^r) - \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho \mathbf{E}[V_{t+1}^{r\prime}(y_t^r - d_t^r - \epsilon_t^r)] \\ s.\ t. & y_t^r - I_t^r \geq 0 \end{cases} \tag{2.2}$$

where $d_t^r p_t^r$ is retailer's expected revenue in period t; $w(y_t^r - I_t^r)$ is the ordering cost in period t; $\mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)]$ is retailer's expected backlogged cost or inventory holding cost in period t.

In addition, we assume the terminal value is given by $V_{T+1}^{m\prime} = c_m I_{T+1}^m$, $V_{T+1}^{r\prime} = c_r I_{T+1}^r$, which means any backlogged demand incurs with per unit cost c_m or c_r, or any leftover inventory incurs with per unit value c_m or c_r at the end of the planning horizon.

Considering the fact that the appearance of state variables in the objective functions greatly increases the difficulty of solving the models, we use a transformation technique by letting $V_t^m(I_t^m) = V_t^{m\prime}(I_t^m) - cI_t^m$, $V_t^r(I_t^r) = V_t^{r\prime}(I_t^r) - wI_t^r$ to facilitate the analysis.

With the transformation $V_t^m(I_t^m) = V_t^{m\prime}(I_t^m) - cI_t^m$ and the dynamic recursion of the manufacturer's inventory $I_{t+1}^m = y_t^m - d_t^m - \epsilon_t^m$, we have

$$V_t^m(I_t^m) = \begin{cases} \max\limits_{p_t^m, y_t^m} & J_t^m(p_t^m, y_t^m) \\ s.\ t. & y_t^m - I_t^m + y_t^r - I_t^r \geq 0 \end{cases} \tag{2.3}$$

where $J_t^m(p_t^m, y_t^m) = d_t^m(p_t^m - \rho c) + (w-c)(y_t^r - I_t^r) - c y_t^m(1-\rho) - \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] + \rho \mathbf{E}[V_{t+1}^m(y_t^m - d_t^m - \epsilon_t^m)]$.

With the transformation $V_t^r(I_t^r) = V_t^{r\prime}(I_t^r) - wI_t^r$ and the dynamic recursion of the retailer's inventory $I_{t+1}^r = y_t^r - d_t^r - \epsilon_t^r$, we have

$$V_t^r(I_t^r) = \begin{cases} \max\limits_{p_t^r, y_t^r} \ J_t^r(p_t^r, y_t^r) \\ s.\ t. \quad y_t^r - I_t^r \geq 0 \end{cases} \tag{2.4}$$

where $J_t^r(p_t^r, y_t^r) = d_t^r(p_t^r - \rho w) - y_t^r w(1-\rho) - \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho \mathbf{E}[V_{t+1}^r(y_t^r - d_t^r - \epsilon_t^r)]$.

Obviously, with the assumption of exogenous wholesale price w, this transformation will not alternate the structure of the optimal dynamic pricing and inventory control policies. Therefore, we will focus on (2.3) and (2.4) instead of (2.1) and (2.2) to analyze the manufacturer and retailer's problems.

2.4 Analysis of the optimal joint dynamic pricing and inventory control policy

Like a lot of studies on supply chain management (e.g., Sajadieh and Jokar, 2009; Li et al., 2018), we consider manufacturer and retailer play a Stackelberg game under perfect information with manufacturer as the leader and retailer as the follower in this chapter.

Acting as the leader, manufacturer can perfectly anticipate the retailer's optimal response to its decisions and make decisions by taking the retailer's optimal response into account. After knowing manufacturer's decisions, retailer acting as the follower determines its optimal decisions. We use backward induction method to pursue manufacturer and retailer's optimal decisions. That is, retailer's problem is first focused to get its optimal response functions for any given manufacturer's decisions. Then, with retailer's optimal response functions, manufacturer's problem will be solved to get its optimal decisions. Last, retailer's optimal decisions can be obtained with its optimal response functions and manufacturer's optimal decisions.

According to the backward induction method, (2.4) is solved first to get the retailer's optimal response functions with given (p_t^m, y_t^m). When p_t^m is given, the mean demand d_t^r has an inverse function $p_t^r(d_t^r) = \frac{(1-\theta)a + \beta_2 p_t^m - d_t^r}{\alpha_2}$. Therefore, the price decision variable p_t^r can be replaced by the mean demand variable d_t^r in (2.4). Following the change of variable, (2.4) can be rewritten as follows.

$$V_t^r(I_t^r) = \begin{cases} \max\limits_{d_t^r, y_t^r} \; J_t^r(d_t^r, y_t^r) \\ s.\ t. \quad y_t^r - I_t^r \geq 0 \end{cases} \tag{2.5}$$

where $J_t^r(d_t^r, y_t^r) = d_t^r(p_t^r(d_t^r) - \rho w) - y_t^r w(1-\rho) - \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho\mathbf{E}[V_{t+1}^r(y_t^r - d_t^r - \epsilon_t^r)]$.

Lemma 2.1. In each period t, with given (p_t^m, y_t^m)

(a) $V_t^r(I_t^r)$ is concave and nonincreasing in I_t^r;

(b) $J_t^r(d_t^r, y_t^r)$ is jointly concave in (d_t^r, y_t^r).

Based on Lemma 2.1, the optimal joint response functions of (d_t^r, y_t^r) can be derived with the following steps:

Step 1: With any y_t^r, define $d_t^{r\prime}(y_t^r) = \arg\max J_t^r(d_t^r, y_t^r)$. Then, substituting $d_t^{r\prime}(y_t^r)$ into $p_t^r(d_t^r)$ to get the optimal price function $p_t^{r\prime}(y_t^r)$.

Step 2: Substitute $d_t^{r\prime}(y_t^r)$ into (2.5) with relaxing the constraint $y_t^r \geq I_t^r$, we can get $y_t^{r\prime} = \arg\max J_t^r(d_t^{r\prime}(y_t^r), y_t^r)$.

Step 3: Get the optimal price $p_t^{r\prime}$ by substituting $y_t^{r\prime}$ into the function $p_t^{r\prime}(y_t^r)$.

Theorem 2.1. In each period t, with given (p_t^m, y_t^m)

(a) if $I_t^r < y_t^{r\prime}$, the optimal response decision of (p_t^r, y_t^r) is $(p_t^{r\prime}, y_t^{r\prime})$;

(b) if $I_t^r \geq y_t^{r\prime}$, the optimal response decision of (p_t^r, y_t^r) is $(p_t^{r\prime}(I_t^r), I_t^r)$, where $p_t^{r\prime}(I_t^r)$ declines with the rise in I_t^r.

Theorem 2.1 states that, when manufacturer's pricing and inventory decisions are given, the optimal policy of retailer is a base-stock-list-price policy. In detail, retailer should place an order to reach the base stock level and charge a list price if its initial inventory level is below the base stock level, otherwise it should place no order and mark down the retail price. Furthermore, in the case of no order should be placed, the higher the retailer's initial inventory level, the lower the retail price.

Proposition 2.1. With the retailer's optimal response decisions, the mean demand d_t^m has an inverse function $p_t^m(d_t^m) = \frac{\theta a + \beta_1 p_t^r - d_t^m}{\alpha_1}$ which is strictly decreasing, where $p_t^r = p_t^{r\prime}$ when $I_t^r < y_t^{r\prime}$ and $p_t^r = p_t^{r\prime}(I_t^r)$ when $I_t^r \geq y_t^{r\prime}$.

With Proposition 2.1, we can get that optimizing the price p_t^m in period t is equivalent to optimizing the mean demand d_t^m. Then, the manufacturer's problem can be represented as

$$V_t^m(I_t^m) = \begin{cases} \max\limits_{d_t^m, y_t^m} \ J_t^m(d_t^m, y_t^m) \\ s.\ t. \quad y_t^m - I_t^m + y_t^r - I_t^r \geq 0 \end{cases} \tag{2.6}$$

where $J_t^m(d_t^m, y_t^m) = d_t^m(p_t^m(d_t^m) - \rho c) + (w - c)(y_t^r - I_t^r) - cy_t^m(1 - \rho) - \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] + \rho\mathbf{E}[V_{t+1}^m(y_t^m - d_t^m - \epsilon_t^m)]$, p_t^r and y_t^r are retailer's optimal response decisions shown in Theorem 2.1.

Lemma 2.2. In each period t, with retailer's optimal response decisions,

(a) $V_t^m(I_t^m)$ is concave and nonincreasing in I_t^m;

(b) $J_t^m(d_t^m, y_t^m)$ is jointly concave in (d_t^m, y_t^m) .

Based on Lemma 2.2, the optimal policy of the manufacturer exists and can be obtained with the same steps as the retailer's problem. Define $d_t^{m\star}(y_t^m) = \arg\max J_t^m(d_t^m, y_t^m)$. Then, define $y_t^{m\star} = \arg\max J_t^m(d_t^{m\star}(y_t^m), y_t^m)$ where (p_t^r, y_t^r) is $(p_t^{r\prime}, y_t^{r\prime})$, and $y_t^{m\star}(I_t^r) = \arg\max J_t^m(d_t^{m\star}(y_t^m), y_t^m)$ where (p_t^r, y_t^r) is $(p_t^{r\prime}(I_t^r), I_t^r)$.

Theorem 2.2. In each period t, manufacturer's optimal decisions are:

(a) Under the scenario where $I_t^r < y_t^{r\prime}$, if $I_t^m < y_t^{m\star} + y_t^{r\prime} - I_t^r$, the optimal solution of (p_t^m, y_t^m) is $(p_t^{m\star}, y_t^{m\star})$; otherwise, the optimal solution of (p_t^m, y_t^m) is $(p_t^{m(1)\star}(I_t^r, I_t^m), I_t^m + I_t^r - y_t^{r\prime})$. Moreover, $y_t^{m\star}$ and $p_t^{m\star}$ are independent of I_t^m, and $p_t^{m(1)\star}(I_t^r, I_t^m)$ is decreasing with I_t^m or I_t^r.

(b) Under the scenario where $I_t^r \geq y_t^{r\prime}$, if $I_t^m < y_t^{m\star}(I_t^r)$, the optimal solution of (p_t^m, y_t^m) is $(p_t^{m\star}(I_t^r), y_t^{m\star}(I_t^r))$; otherwise, the optimal solution of (p_t^m, y_t^m) is $(p_t^{m(2)\star}(I_t^r, I_t^m), I_t^m)$. Moreover, $p_t^{m\star}(I_t^r)$ and $y_t^{m\star}(I_t^r)$ are decreasing with I_t^r, and $p_t^{m(2)\star}(I_t^r, I_t^m)$ is decreasing with I_t^r or I_t^m.

Theorem 2.2 indicates that, whether the retailer places an order or not, the manufacturer's optimal policy is a base-stock-list-price policy. Specifically, under the situation where retailer decides to order products, manufacturer should increase the inventory level to the base stock level $y_t^{m\star} + y_t^{r\prime} - I_t^r$ by producing and charge a list price $p_t^{m\star}$ if its initial inventory level is below the base stock level, otherwise it should not produce and mark down the online price. Under the situation where retailer decides to place no order, manufacturer should produce-up to the base stock level $y_t^{m\star}(I_t^r)$ and charge a list price $p_t^{m\star}(I_t^r)$ if its initial inventory level is below the base stock level, otherwise it should not produce and mark down the online price.

Besides, in the case of not producing goods, the higher the manufacturer's initial inventory level, the lower the online price.

We need to substitute the manufacturer's optimal decisions into the retailer's optimal response decisions to get the retailer's final optimal policy. The retailer's final optimal decisions are denoted with asterisk superscript. The following theorem is about the structural analysis of the dual-channel supply chain's optimal joint dynamic pricing and inventory control policy when the manufacturer is the Stackelberg leader.

Theorem 2.3. The optimal joint dynamic pricing and inventory control policy of a dual-channel supply chain where the manufacturer is the Stackelberg leader is:

(a) if $I_t^r < y_t^{r\star}$ and $I_t^m < y_t^{m\star} + y_t^{r\star} - I_t^r$, the optimal decisions are $(p_t^{r\star}, y_t^{r\star})$ and $(p_t^{m\star}, y_t^{m\star})$, where $y_t^{r\star}$, $p_t^{r\star}$, $y_t^{m\star}$ and $p_t^{m\star}$ are independent of I_t^m and I_t^r .

(b) if $I_t^r < y_t^{r(1)\star}(I_t^r, I_t^m)$ and $I_t^m \geq y_t^{m\star} + y_t^{r\star} - I_t^r$, the optimal decisions are $(p_t^{r(1)\star}(I_t^r, I_t^m), y_t^{r(1)\star}(I_t^r, I_t^m))$ and $(p_t^{m(1)\star}(I_t^r, I_t^m), I_t^m + I_t^r - y_t^{r(1)\star}(I_t^r, I_t^m))$. Additionally, $p_t^{r(1)\star}(I_t^r, I_t^m)$, $y_t^{r(1)\star}(I_t^r, I_t^m)$ and $p_t^{m(1)\star}(I_t^r, I_t^m)$ decrease as either I_t^r or I_t^m increases.

(c) if $I_t^r \geq y_t^{r\star}(I_t^r)$ and $I_t^m < y_t^{m\star}(I_t^r)$, the optimal decisions are $(p_t^{r\star}(I_t^r), I_t^r)$ and $(p_t^{m\star}(I_t^r), y_t^{m\star}(I_t^r))$, where $y_t^{r\star}(I_t^r)$, $p_t^{r\star}(I_t^r)$, $y_t^{m\star}(I_t^r)$ and $p_t^{m\star}(I_t^r)$ decrease as I_t^r increases.

(d) if $I_t^r \geq y_t^{r(2)\star}(I_t^r, I_t^m)$ and $I_t^m \geq y_t^{m\star}(I_t^r)$, the optimal decisions are $(p_t^{r(2)\star}(I_t^r, I_t^m), I_t^r)$ and $(p_t^{m(2)\star}(I_t^r, I_t^m), I_t^m)$, where $y_t^{r(2)\star}(I_t^r, I_t^m)$, $p_t^{r(2)\star}(I_t^r, I_t^m)$ and $p_t^{m(2)\star}(I_t^r, I_t^m)$ decrease as either I_t^r or I_t^m increases.

Theorem 2.3 indicates that, for the dual-channel supply chain where manufacturer acts as the leader and retailer acts as the follower, manufacturer and retailer's optimal joint dynamic pricing and inventory control policies are inventory-dependent base-stock-list-price policies. Specifically, one member should increase its inventory to its base stock level through producing or ordering if its initial inventory level is below its base stock level, otherwise it should keep its inventory at the initial level. Moreover, one member's base stock level may be affected by its initial inventory level or the other's initial inventory level. Manufacturer and retailer should charge list prices if their initial inventory levels are both below their respective base stock levels, otherwise they should reduce online price and retail price.

2.5 Modeling and analysis of the optimal policy for a dual-parallel-channel system

The dual-channel supply chain where the two channels have supply-demand relationship besides competing on selling prices is studied in Section 2.4. In this section, we consider the two channels are parallel to each other and compete on selling prices, and discuss the effect of the dual-channel setting.

Consider two retailers, referred to as retailer i, $i = 1, 2$, purchase products of a single type from a manufacturer with the same wholesale price w and sell to customers over T periods, indexed by $t = 1, ..., T$.

At the beginning of period t, retailer i, reviews its inventory level I_t^i, and decides the order quantity q_t^i and selling price p_t^i. During period t, retailer i's customer demand D_t^i is met with its on-hand inventory level y_t^i where $y_t^i = I_t^i + q_t^i$. At the end of period t, unmet demand is backordered, and leftover inventory is carried over to the next period. Retailer i incurs the backlogged cost or inventory holding cost $h^i(\cdot)$. Same as the manufacturer-retailer dual-channel setting, we assume $D_t^i = d_t^i + \epsilon_t^i$ where $\mathbf{E}(\epsilon_t^i) = 0$, d_t^i is given by $d_t^i = \theta_i a - \alpha_i p_t^i + \beta_i p_t^{3-i}$ where $0 < \theta_i < 1$, $\theta_i + \theta_{3-i} = 1$ and $\alpha_i > \beta_i$, and $h^i(\cdot)$ is convex and continuous derivable.

The problems faced by the two retailers under different channel power structures are to decide the optimal joint dynamic pricing and inventory policies to maximize their respective total expected profits. Retailer i's problem is built as (2.7) by letting $V_t^{i\prime}(I_t^i)$ be retailer i's maximum expected discount profit from period t until the end of period T and doing transformation to $V_t^{i\prime}(I_t^i)$ with $V_t^i(I_t^i) = V_t^{i\prime}(I_t^i) - wI_t^i$.

$$V_t^i(I_t^i) = \begin{cases} \max\limits_{p_t^i, y_t^i} \ J_t^i(p_t^i, y_t^i) \\ \\ s.\ t. \quad y_t^i - I_t^i \geq 0 \end{cases} \tag{2.7}$$

where $J_t^i(p_t^i, y_t^i) = d_t^i(p_t^i - \rho w) - (1-\rho)wy_t^i - \mathbf{E}[h^i(y_t^i - d_t^i - \epsilon_t^i)] + \rho\mathbf{E}[V_{t+1}^i(y_t^i - d_t^i - \epsilon_t^i)]$.

Referring to the steps to solve the problems of the manufacturer-retailer dual-channel supply chain in Section 2.4, the two retailers' problems under Retailer i Stackelberg can be solved. Theorems 2.4 describes the structural properties of the two retailers' optimal policies.

Theorem 2.4. The two retailers' optimal joint dynamic pricing and inventory decisions under Retailer i Stackelberg are:

(a) if $I_t^{3-i} < y_t^{3-i\star}$ and $I_t^i < y_t^{i\star}$, the optimal decisions are $(p_t^{3-i\star}, y_t^{3-i\star})$ and $(p_t^{i\star}, y_t^{i\star})$, where $y_t^{3-i\star}$, $p_t^{3-i\star}$, $y_t^{i\star}$ and $p_t^{i\star}$ are independent of I_t^i and I_t^{3-i} .

(b) if $I_t^{3-i} < y_t^{3-i\star}(I_t^i)$ and $I_t^i \geq y_t^{i\star}$, the optimal decisions are $(p_t^{3-i\star}(I_t^i), y_t^{3-i\star}(I_t^i))$ and $(p_t^{i\star}(I_t^i), I_t^i)$, where $p_t^{3-i\star}(I_t^i)$, $y_t^{3-i\star}(I_t^i)$ and $p_t^{i\star}(I_t^i)$ decrease as either I_t^i increases.

(c) if $I_t^{3-i} \geq y_t^{3-i\star}(I_t^{3-i})$ and $I_t^i < y_t^{i\star}(I_t^{3-i})$, the optimal decisions are $(p_t^{3-i\star}(I_t^{3-i}), I_t^{3-i})$ and $(p_t^{i\star}(I_t^{3-i}), y_t^{i\star}(I_t^{3-i}))$, where $y_t^{3-i\star}(I_t^{3-i})$, $p_t^{3-i\star}(I_t^{3-i})$, $y_t^{i\star}(I_t^{3-i})$ and $p_t^{i\star}(I_t^{3-i})$ decrease as I_t^{3-i} increases.

(d) if $I_t^{3-i} \geq y_t^{3-i\star}(I_t^{3-i}, I_t^i)$ and $I_t^i \geq y_t^{i\star}(I_t^{3-i})$, the optimal decisions are $(p_t^{3-i\star}(I_t^{3-i}, I_t^i), I_t^{3-i})$ and $(p_t^{i\star}(I_t^{3-i}, I_t^i), I_t^i)$, where $p_t^{3-i\star}(I_t^{3-i}, I_t^i)$, $y_t^{3-i\star}(I_t^{3-i}, I_t^i)$ and $p_t^{i\star}(I_t^{3-i}, I_t^i)$ decrease as I_t^{3-i} or I_t^i increases.

Theorem 2.4 indicates that, under Retailer i Stackelberg power structure, the member should order up to its base stock level if its initial inventory level is below its base stock level, otherwise it places no order and both members should reduce their selling prices. This means that the structural properties of the optimal policies under the two different dual-channel settings are the same. Optimal selling prices are negatively correlated with the initial inventory level of the member whose initial inventory level is above its base stock level. However, in the manufacturer-retailer setting, because of the supply-demand relationship between manufacturer and retailer, optimal prices may be negatively affected by retailer's initial inventory level though its initial inventory level is below its base stock level.

Theorems 2.3 and 2.4 show that the influence rules of initial inventory levels on base stock levels are affected by the dual-channel setting. Manufacturer's base stock level is affected by retailer's initial inventory level if retailer's initial inventory level is below its base stock level, while retailer i's base stock level is not affected by retailer $3 - i$'s base stock level if retailer $3 - i$'s initial inventory level is below its base stock level. In addition, retailer's base stock level is negatively correlated with its initial inventory level when manufacturer's initial inventory level is above its base stock level and retailer's initial inventory level is below its base stock level, while retailer i's base stock level is not affected by retailer $3 - i$'s base stock level if retailer $3 - i$'s initial inventory level is above its base stock level and retailer i's initial inventory level is below its base stock level.

2.6 Conclusions

This chapter investigates the joint dynamic pricing and inventory control problem of a dual-channel supply chain faced by stochastic and price-sensitive demand.

We find that, the optimal joint dynamic pricing and inventory control policy of a dual-channel supply chain is a base-stock-list-price type, that is, manufacturer should produce goods only if its starting inventory level is below its base stock level, retailer should place an order only if its starting inventory level is below its base stock level, and list prices should be charged only when both members' starting inventory levels are below their respective base stock levels. Moreover, as long as one member's starting inventory level is above its base stock level, both members should reduce prices. The higher the initial inventory level exceeds the base stock level, the lower the reduced prices. The base stock levels of manufacturer and retailer may be affected by their initial inventory levels.

We also find that the above structural properties are also suitable for the optimal joint dynamic pricing and inventory control policy of a dual-channel supply chain with two competing retailers. However, due to the supply-demand relationship between manufacturer and retailer, the influence rules of starting inventory levels on the optimal joint dynamic pricing and inventory control policy in the manufacturer-retailer dual-channel setting are more complex.

2.7 Appendices

Proof of Lemma 2.1

Lemma 2.1 can be proved by induction. Since $V^r_{T+1} = (c_r - w)I^r_{T+1}$, it is obviously concave in I^r_{T+1}. We assume inductively that $V^r_{t+1}(I^r_{t+1})$ is concave in I^r_{t+1}. In what follows, we should prove the result also holds for period t.

We first prove that $J^r_t(d^r_t, y^r_t)$ is jointly concave in (d^r_t, y^r_t). $J^r_t(d^r_t, y^r_t)$ consists of three parts: (1) $d^r_t(p^r_t(d^r_t) - \rho w) - y^r_t w(1-\rho)$, (2) $-\mathbf{E}[h^r(y^r_t - d^r_t - \epsilon^r_t)]$, and (3) $\rho\mathbf{E}[V^r_{t+1}(y^r_t - d^r_t - \epsilon^r_t)]$. Since $p^r_t(d^r_t) = \frac{(1-\theta)a+\beta p^m_t - d^r_t}{\alpha_2}$, the Hessian matrix of part

(1), denoted as $\mathbf{H}$, is $\begin{bmatrix} \frac{-2}{\alpha_2} & 0 \\ 0 & 0 \end{bmatrix}$. For any vector $\mathbf{x}$, we have $\mathbf{x^T H x} \leq 0$. Hence, we can get that $d_t^r(p_t^r - \rho w) - y_t^r w(1-\rho)$ is jointly concave in (d_t^r, y_t^r). Note that $h^r(y_t^r - d_t^r - \epsilon_t^r) = c_1(y_t^r - d_t^r - \epsilon_t^r)^+ + c_2(y_t^r - d_t^r - \epsilon_t^r)^-$ is a convex function and $y_t^r - d_t^r - \epsilon_t^r$ is a linear combination of (d_t^r, y_t^r) for any ϵ_t^r. Then, we can easily get that $h^r(y_t^r - d_t^r - \epsilon_t^r)$ is jointly convex in (d_t^r, y_t^r) for any ϵ_t^r. After taking expectation over ϵ_t^r, $-\mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)]$ is also jointly concave in (d_t^r, y_t^r). Similarly, since $V_{t+1}^r(I_{t+1}^r)$ is concave in $I_{t+1}^r = y_t^r - d_t^r - \epsilon_t^r$, we can get that $\mathbf{E}[V_{t+1}^r(y_t^r - d_t^r - \epsilon_t^r)]$ is also jointly concave in (d_t^r, y_t^r).

Then, we will prove $V_t^r(I_t^r)$ is concave in I_t^r. The constraint $y_t^r - I_t^r \geq 0$ implies that the optimal value of y_t^r is greater than I_t^r or equal to I_t^r. If the optimal value of y_t^r is greater than I_t^r, with $J_t^r(d_t^r, y_t^r) = d_t^r(p_t^r(d_t^r) - \rho w) - y_t^r w(1-\rho) - \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho\mathbf{E}[V_{t+1}^r(y_t^r - d_t^r - \epsilon_t^r)]$, we get that $V_t^r(I_t^r) = \max_{(d_t^r, y_t^r)} J_t^r(d_t^r, y_t^r)$ is unrelated to I_t^r. Since $J_t^r(d_t^r, y_t^r)$ is jointly concave in (d_t^r, y_t^r) and concavity is preserved after maximization, we can get $\max_{d_t^r} J_t^r(d_t^r, y_t^r)$ is concave in y_t^r. Therefore, if the optimal value of y_t^r is I_t^r, $V_t^r(I_t^r) = \max_{d_t^r} J_t^r(d_t^r, I_t^r)$ is concave in I_t^r.

$V_t^r(I_t^r)$ is nonincreasing in I_t^r, because $J_t^r(d_t^r, y_t^r)$ is independent of I_t^r and a larger I_t^r leads to a more restrictive feasible domain of y_t^r and so a smaller maximum objective function value. Hence, Lemma 2.1 is completely proved.

Proof of Theorem 2.1

If $I_t^r < y_t^{r\prime}$, the constraint $y_t^{r\prime} - I_t^r \geq 0$ is satisfied, it is obvious that the optimal solution of (p_t^r, y_t^r) is $(p_t^{r\prime}, y_t^{r\prime})$. If $I_t^r \geq y_t^{r\prime}$, the optimal y_t^r must be in the boundary line $y_t^{r\prime} - I_t^r = 0$. Therefore, if $I_t^r \geq y_t^{r\prime}$, the optimal solution of (p_t^r, y_t^r) is $(p_t^{r\prime}(I_t^r), I_t^r)$. Next, we will prove $p_t^{r\prime}(I_t^r)$ is decreasing in I_t^r.

For any y_t^r, $d_t^{r\prime}(y_t^r)$ is obtained with $\frac{\partial J_t^r(d_t^r, y_t^r)}{\partial d_t^r} = 0$, where $\frac{\partial J_t^r(d_t^r, y_t^r)}{\partial d_t^r} = \frac{(1-\theta)a + \beta p_t^m - 2d_t^r}{\alpha_2} - \rho w + \frac{\partial \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)]}{\partial(y_t^r - d_t^r)} - \rho\frac{\partial \mathbf{E}[V_{t+1}^r(y_t^r - d_t^r - \epsilon_t^r)]}{\partial(y_t^r - d_t^r)} = 0$. Since $-\mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho\mathbf{E}[V_{t+1}^r(y_t^r - d_t^r - \epsilon_t^r)]$ is concave in $y_t^r - d_t^r$, which is proved in Lemma 2.1, the coefficient of $y_t^r - d_t^r$ in $\frac{\partial \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)]}{\partial(y_t^r - d_t^r)} - \rho\frac{\partial \mathbf{E}[V_{t+1}^r(y_t^r - d_t^r - \epsilon_t^r)]}{\partial(y_t^r - d_t^r)}$ is greater than 0. Then, we can get $0 < \frac{\partial d_t^{r\prime}(y_t^r)}{\partial y_t^r} < 1$.

When $I_t^r \geq y_t^{r\prime}$, the optimal solution of y_t^r is I_t^r. With $0 < \frac{\partial d_t^{r\prime}(y_t^r)}{\partial y_t^r} < 1$, we can get $d_t^{r\prime}(I_t^r)$ increases as I_t^r increases. With $p_t^r(d_t^r) = \frac{(1-\theta)a + \beta p_t^m - d_t^r}{\alpha_2}$, we can further get $p_t^{r\prime}(I_t^r)$ decreases with the increase in I_t^r. Hence, $p_t^{r\prime}(I_t^r)$ decreases as I_t^r increases is proved. Theorem 2.1 is completely proved.

Proof of Proposition 2.1

For any y_t^r, $d_t^{r\prime}(y_t^r)$ is obtained with $\frac{\partial J_t^r(d_t^r,y_t^r)}{\partial d_t^r}=0$ where $\frac{\partial J_t^r(d_t^r,y_t^r)}{\partial d_t^r}=\frac{(1-\theta)a+\beta p_t^m-2d_t^r}{\alpha_2}-\rho w+\frac{\partial \mathbf{E}[h^r(y_t^r-d_t^r-\epsilon_t^r)]}{\partial (y_t^r-d_t^r)}-\rho\frac{\partial \mathbf{E}[V_{t+1}^r(y_t^r-d_t^r-\epsilon_t^r)]}{\partial (y_t^r-d_t^r)}$. With $d_t^{r\prime}(y_t^r)$, we can get $\frac{\partial J_t^r(d_t^{r\prime}(y_t^r),y_t^r)}{\partial y_t^r}=-w(1-\rho)-\frac{\partial \mathbf{E}[h^r(y_t^r-d_t^{r\prime}(y_t^r)-\epsilon_t^r)]}{\partial (y_t^r-d_t^{r\prime}(y_t^r))}+\rho\frac{\partial \mathbf{E}[V_{t+1}^r(y_t^r-d_t^{r\prime}(y_t^r)-\epsilon_t^r)]}{\partial (y_t^r-d_t^{r\prime}(y_t^r))}$. Then, $y_t^{r\prime}$ is derived with $\frac{\partial J_t^r(d_t^{r\prime}(y_t^r),y_t^r)}{\partial y_t^r}=0$.

If $I_t^r<y_t^{r\prime}$, we get $\frac{\partial d_t^{r\prime}}{\partial p_t^m}=\frac{\beta}{2}$ by combining $\frac{\partial J_t^r(d_t^r,y_t^r)}{\partial d_t^r}=0$ and $\frac{\partial J_t^r(d_t^{r\prime}(y_t^r),y_t^r)}{\partial y_t^r}=0$. If $I_t^r\geq y_t^{r\prime}$, the optimal value of y_t^r is I_t^r. Since the coefficient of $y_t^r-d_t^r$ in $\frac{\partial \mathbf{E}[h^r(y_t^r-d_t^r-\epsilon_t^r)]}{\partial (y_t^r-d_t^r)}-\rho\frac{\partial \mathbf{E}[V_{t+1}^r(y_t^r-d_t^r-\epsilon_t^r)]}{\partial (y_t^r-d_t^r)}$ is greater than 0, $0<\frac{\partial d_t^{r\prime}(I_t^r)}{\partial I_t^r}<1$ and $0<\frac{\partial d_t^{r\prime}(I_t^r)}{\partial p_t^m}<\frac{\beta}{2}$.

With $d_t^m=\theta a-\alpha_1 p_t^m+\beta p_t^r$ and $d_t^r=(1-\theta)a-\alpha_2 p_t^r+\beta p_t^m$, we get $p_t^m(d_t^m)=\frac{\theta\alpha_2 a+(1-\theta)\beta a-\beta d_t^r-\alpha_2 d_t^m}{\alpha_1\alpha_2-\beta^2}$, where $d_t^r=d_t^{r\prime}$ when $I_t^r<y_t^{r\prime}$ and $d_t^r=d_t^{r\prime}(I_t^r)$ when $I_t^r\geq y_t^{r\prime}$. Then, with $\frac{\partial d_t^{r\prime}}{\partial p_t^m}=\frac{\beta}{2}$ and $0<\frac{\partial d_t^{r\prime}(I_t^r)}{\partial p_t^m}<\frac{\beta}{2}$, we can get that $\frac{\partial p_t^m(d_t^m)}{\partial d_t^m}\leq\frac{-2\alpha_2}{2\alpha_1\alpha_2-\beta^2}$. With the assumption $\alpha_1>\beta>0$ and $\alpha_2>\beta>0$, we get $\frac{-2\alpha_2}{2\alpha_1\alpha_2-\beta^2}<0$. Therefore, $p_t^m(d_t^m)$ is strictly decreasing d_t^m. Proposition 2.1 is completely proved.

Proof of Lemma 2.2

Similar to Lemma 2.1, Lemma 2.2 can be proved by induction. Since $V_{T+1}^m=(c_m-c)I_{T+1}^m$, it is obviously concave in I_{T+1}^m. Now, we assume inductively that $V_{t+1}^m(I_{t+1}^m)$ is concave in I_{t+1}^m. In what follows, we should prove the result also holds for period t.

We first prove that $J_t^m(d_t^m,y_t^m)$ is jointly concave in (d_t^m,y_t^m). $J_t^m(d_t^m,y_t^m)$ is divided into three parts: (1) $d_t^m(p_t^m(d_t^m)-\rho c)+(w-c)(y_t^r-I_t^r)-cy_t^m(1-\rho)$; (2) $-\mathbf{E}[h^m(y_t^m-d_t^m-\epsilon_t^m)]$; and (3) $\rho\mathbf{E}[V_{t+1}^m(y_t^m-d_t^m-\epsilon_t^m)]$. The Hessian matrix of part (1), denoted as $\mathbf{H}'$, is $\begin{bmatrix} 2\frac{\partial p_t^m(d_t^m)}{\partial d_t^m} & 0 \\ 0 & 0 \end{bmatrix}$. Since $\frac{\partial p_t^m(d_t^m)}{\partial d_t^m}<0$ which is proved in Proposition 2.1, we can get that, for any vector $\mathbf{x}$, $\mathbf{x^T H' x}\leq 0$. Hence, part (1) is jointly concave in (d_t^m,y_t^m). Parts (2) and (3) are jointly concave in (d_t^m,y_t^m) can be proved in the same way as Lemma 2.1.

Then, we will prove $V_t^m(I_t^m)$ is concave in I_t^m. Under the situation where $I_t^r\geq y_t^{r\prime}$, the optimal solution of y_t^r is I_t^r, then $J_t^m(d_t^m,y_t^m)=d_t^m(p_t^m(d_t^m)-\rho c)-cy_t^m(1-\rho)-\mathbf{E}[h^m(y_t^m-d_t^m-\epsilon_t^m)]+\rho\mathbf{E}[V_{t+1}^m(y_t^m-d_t^m-\epsilon_t^m)]$, and the constraint $y_t^m-I_t^m+y_t^r-I_t^r\geq 0$ can be represented as $y_t^m\geq I_t^m$. If the optimal value of y_t^m is greater than I_t^m, with $J_t^m(d_t^m, y_t^m)$, we get that $V_t^m(I_t^m) = \max\limits_{(d_t^m, y_t^m)} J_t^m(d_t^m, y_t^m)$ is unrelated to I_t^m. If the optimal value of y_t^m is I_t^m, similar to Lemma 2.1, we can get that $V_t^m(I_t^m) = \max\limits_{d_t^m} J_t^m(d_t^m, I_t^m)$ is concave in I_t^m.

Under the situation where $I_t^r < y_t^{r\prime}$, the optimal solution of y_t^r is $y_t^{r\prime}$, then $J_t^m(d_t^m, y_t^m) = d_t^m(p_t^m(d_t^m) - \rho c) + (w-c)(y_t^{r\prime} - I_t^r) - c y_t^m(1-\rho) - \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] + \rho \mathbf{E}[V_{t+1}^m(y_t^m - d_t^m - \epsilon_t^m)]$, and the constraint $y_t^m - I_t^m + y_t^r - I_t^r \geq 0$ can be represented as $y_t^m \geq I_t^m + I_t^r - y_t^{r\prime}$. In view of $\frac{\partial J_t^r(d_t^{r\prime}(y_t^r), y_t^r)}{\partial y_t^r}$ which is displayed in Proposition 2.1, we get $\frac{\partial y_t^{r\prime}}{\partial p_t^m} = \frac{\partial d_t^{r\prime}}{\partial p_t^m} = \frac{\beta}{2}$. This indicates that $y_t^{r\prime}$ is related to d_t^m and $\frac{\partial y_t^{r\prime}}{\partial d_t^m} = \frac{\partial y_t^{r\prime}}{\partial p_t^m}\frac{\partial p_t^m(d_t^m)}{\partial d_t^m} = \frac{-\alpha_2\beta}{2\alpha_1\alpha_2 - \beta^2}$. If the optimal solution of (d_t^m, y_t^m) satisfies $y_t^m > I_t^m + I_t^r - y_t^{r\prime}$, then $\max\limits_{(d_t^m, y_t^m)} J_t^m(d_t^m, y_t^m)$ is unrelated to I_t^m. If the optimal solution of (d_t^m, y_t^m) satisfies $y_t^m = I_t^m + I_t^r - y_t^{r\prime}$, then $\max\limits_{(d_t^m, y_t^m)} J_t^m(d_t^m, y_t^m)$ equals to $\max\limits_{d_t^m} J_t^m(d_t^m, I_t^m)$, where $J_t^m(d_t^m, I_t^m) = d_t^m(p_t^m(d_t^m) - \rho c) + (w-c)(y_t^{r\prime} - I_t^r) - c(1-\rho)(I_t^m + I_t^r - y_t^{r\prime}) - \mathbf{E}[h^m(I_t^m + I_t^r - y_t^{r\prime} - d_t^m - \epsilon_t^m)] + \rho\mathbf{E}[V_{t+1}^m(I_t^m + I_t^r - y_t^{r\prime} - d_t^m - \epsilon_t^m)]$. Since $J_t^m(d_t^m, y_t^m)$ is jointly concave in (d_t^m, y_t^m) and $\frac{\partial y_t^{r\prime}}{\partial d_t^m} = \frac{-\alpha_2\beta}{2\alpha_1\alpha_2-\beta^2}$, we get that $J_t^m(d_t^m, I_t^m)$ is jointly concave in (d_t^m, I_t^m). Since concavity is preserved after maximization, $V_t^m(I_t^m) = \max\limits_{d_t^m} J_t^m(d_t^m, I_t^m)$ is concave in I_t^m.

$V_t^m(I_t^m)$ is nonincreasing in I_t^m, because $J_t^m(d_t^m, y_t^m)$ is independent of I_t^m and a larger I_t^m leads to a more restrictive feasible domain of (d_t^m, y_t^m) and so a smaller maximum objective function value. Hence, Lemma 2.2 is completely proved.

Proof of Theorem 2.2

(a) Under the scenario where $I_t^r < y_t^{r\prime}$, $J_t^m(d_t^m, y_t^m) = d_t^m(p_t^m(d_t^m) - \rho c) + (w - c)(y_t^{r\prime} - I_t^r) - c y_t^m(1-\rho) - \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] + \rho\mathbf{E}[V_{t+1}^m(y_t^m - d_t^m - \epsilon_t^m)]$, and the constraint is $y_t^m - I_t^m + y_t^{r\prime} - I_t^r \geq 0$, where $\frac{\partial y_t^{r\prime}}{\partial d_t^m} = \frac{-\alpha_2\beta}{2\alpha_1\alpha_2-\beta^2}$ which is displayed in the proof of Lemma 2.2. Since $(d_t^{m(1)\star}, y_t^{m(1)\star}) = \arg\max J_t^m(d_t^m, y_t^m)$, it is obvious that if $I_t^m < y_t^{m(1)\star} + y_t^{r(1)\star} - I_t^r$, the optimal solution is $(p_t^{m(1)\star}, y_t^{m(1)\star})$ which is independent of I_t^r and I_t^m. If $I_t^m \geq y_t^{m(1)\star} + y_t^{r(1)\star} - I_t^r$, $d_t^{m(1)\star}(I_t^r, I_t^m) = \arg\max J_t^m(d_t^m, I_t^m + I_t^r - y_t^r)$, which is derived with $\frac{\partial J_t^m(d_t^m)}{\partial d_t^m} = 0$, where $\frac{\partial J_t^m(d_t^m)}{\partial d_t^m} = p_t^m(d_t^m) + d_t^m\frac{\partial p_t^m(d_t^m)}{\partial d_t^m} - \rho c + (w-c)\frac{\partial y_t^{r\prime}}{\partial d_t^m} + \frac{\partial \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)]}{\partial(y_t^m - d_t^m)}(1 - \frac{\partial y_t^{r\prime}}{\partial d_t^m}) - \rho\frac{\partial \mathbf{E}[V_{t+1}^m(y_t^m - d_t^m - \epsilon_t^m)]}{\partial(y_t^m - d_t^m)}(1 - \frac{\partial y_t^{r\prime}}{\partial d_t^m})$. Since $\frac{\partial p_t^m(d_t^m)}{\partial d_t^m} = \frac{-2\alpha_2}{2\alpha_1\alpha_2-\beta^2} < 0$, $-1 < \frac{\partial y_t^{r\prime}}{\partial d_t^m} = \frac{-\alpha_2\beta}{2\alpha_1\alpha_2-\beta^2} < 0$, and $-\mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] + \rho\mathbf{E}[V_{t+1}^m(y_t^m - d_t^m - \epsilon_t^m)]$ is concave in $y_t^m - d_t^m$, we get that $\frac{\partial d_t^{m(1)\star}(I_t^r, I_t^m)}{\partial I_t^r} > 0$ and $\frac{\partial d_t^{m(1)\star}(I_t^r, I_t^m)}{\partial I_t^m} > 0$. Then, with $\frac{\partial p_t^m(d_t^m)}{\partial d_t^m} < 0$ and $\frac{\partial y_t^{r\prime}}{\partial d_t^m} < 0$, we get that $p_t^{m(1)\star}(I_t^r, I_t^m)$ and $y_t^{r(1)\star}(I_t^r, I_t^m)$ decrease with increase in I_t^r or I_t^m.

(b) Under the scenario where $I_t^r \geq y_t^{r\prime}$, $J_t^m(d_t^m, y_t^m) = d_t^m(p_t^m(d_t^m) - \rho c) - c y_t^m(1-\rho) - \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] + \rho \mathbf{E}[V_{t+1}^m(y_t^m - d_t^m - \epsilon_t^m)]$, and the constraint is $y_t^m - I_t^m \geq 0$. Since $(d_t^{m(2)\star}(I_t^r), y_t^{m(2)\star}(I_t^r)) = \arg\max J_t^m(d_t^m, y_t^m)$, it is obvious that if $I_t^m < y_t^{m(2)\star}(I_t^r)$, the optimal solution is $(p_t^{m(2)\star}(I_t^r), y_t^{m(2)\star}(I_t^r))$. $d_t^{m(2)\star}(I_t^r)$ and $y_t^{m(2)\star}(I_t^r)$ are derived with $\frac{\partial J_t^m(d_t^m, y_t^m)}{\partial d_t^m} = 0$ and $\frac{\partial J_t^m(d_t^m, y_t^m)}{\partial y_t^m} = 0$, where $\frac{\partial J_t^m(d_t^m, y_t^m)}{\partial d_t^m} = p_t^m(d_t^m) + d_t^m \frac{\partial p_t^m(d_t^m)}{\partial d_t^m} - \rho c + \frac{\partial \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)]}{\partial (y_t^m - d_t^m)} - \rho \frac{\partial \mathbf{E}[V_{t+1}^m(y_t^m - d_t^m - \epsilon_t^m)]}{\partial (y_t^m - d_t^m)}$ and $\frac{\partial J_t^m(d_t^m, y_t^m)}{\partial y_t^m} = -(1-\rho)c - \frac{\partial \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)]}{\partial (y_t^m - d_t^m)} + \rho \frac{\partial \mathbf{E}[V_{t+1}^m(y_t^m - d_t^m - \epsilon_t^m)]}{\partial (y_t^m - d_t^m)}$. With $p_t^m(d_t^m) = \frac{\theta \alpha_2 a + (1-\theta)\beta a - \beta d_t^{r\prime}(I_t^r) - \alpha_2 d_t^m}{\alpha_1 \alpha_2 - \beta^2}$, $0 < \frac{\partial d_t^{r\prime}(I_t^r)}{\partial I_t^r}$ and $0 < \frac{\partial d_t^{r\prime}(I_t^r)}{\partial p_t^m} < \frac{\beta}{2}$ which are proved in Proposition 2.1, we get that $\frac{\partial d_t^{m(2)\star}(I_t^r)}{\partial I_t^r} = \frac{\partial y_t^{m(2)\star}(I_t^r)}{\partial I_t^r} = \frac{-\beta}{2\alpha_2} \frac{\partial d_t^{r\prime}(I_t^r)}{\partial I_t^r} < 0$. Then, with $p_t^m(d_t^m)$, we can further get that $\frac{\partial p_t^{m(2)\star}(I_t^r)}{\partial I_t^r} < 0$. If $I_t^m \geq y_t^{m(2)\star}(I_t^r)$, $d_t^{m(2)\star}(I_t^r, I_t^m) = \arg\max J_t^m(d_t^m, I_t^m)$, which is derived with $\frac{\partial J_t^m(d_t^m)}{\partial d_t^m} = 0$, where $\frac{\partial J_t^m(d_t^m)}{\partial d_t^m} = p_t^m(d_t^m) + d_t^m \frac{\partial p_t^m(d_t^m)}{\partial d_t^m} - \rho c + \frac{\partial \mathbf{E}[h^m(I_t^m - d_t^m - \epsilon_t^m)]}{\partial (I_t^m - d_t^m)} - \rho \frac{\partial \mathbf{E}[V_{t+1}^m(I_t^m - d_t^m - \epsilon_t^m)]}{\partial (I_t^m - d_t^m)}$. Since $-\mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] + \rho \mathbf{E}[V_{t+1}^m(y_t^m - d_t^m - \epsilon_t^m)]$ is concave in $y_t^m - d_t^m$, we can get that $\frac{-\beta}{2\alpha_2} \frac{\partial d_t^{r\prime}(I_t^r)}{\partial I_t^r} < \frac{\partial d_t^{m(2)\star}(I_t^r, I_t^m)}{\partial I_t^r} < 0$ and $0 < \frac{\partial d_t^{m(2)\star}(I_t^r, I_t^m)}{\partial I_t^m}$. Then, with $p_t^m(d_t^m)$, we can further get that $\frac{\partial p_t^{m(2)\star}(I_t^r, I_t^m)}{\partial I_t^r} < 0$ and $\frac{\partial p_t^{m(2)\star}(I_t^r, I_t^m)}{\partial I_t^m} < 0$. Hence, Theorem 2.2 is completely proved.

Proof of Theorem 2.3

Since the influence rules of (I_t^r, I_t^m) on the optimal solutions of (p_t^m, y_t^m) have been proved in Theorem 2.2, we only need to prove the influence rules of (I_t^r, I_t^m) on the optimal solutions of (p_t^r, y_t^r) which are shown in Theorem 2.3 is true.

(a) It is obvious that, if $I_t^r < y_t^{r(1)\star}$ and $I_t^m < y_t^{m(1)\star} + y_t^{r(1)\star} - I_t^r$, $p_t^{r(1)\star}$ and $y_t^{r(1)\star}$ are independent of I_t^m and I_t^r.

(b) if $I_t^r < y_t^{r(1)\star}(I_t^r, I_t^m)$ and $I_t^m \geq y_t^{m(1)\star} + y_t^{r(1)\star} - I_t^r$, $p_t^{r(1)\star}(I_t^r, I_t^m)$ and $y_t^{r(1)\star}(I_t^r, I_t^m)$ are obtained by substituting $p_t^{m(1)\star}(I_t^r, I_t^m)$ into $p_t^{r\prime}$ and $y_t^{r\prime}$. With $\frac{\partial y_t^{r\prime}}{\partial p_t^m} = \frac{\partial d_t^{r\prime}}{\partial p_t^m} = \frac{\beta}{2}$ which is proved in Lemma 2.2, $p_t^r(d_t^r) = \frac{(1-\theta)a + \beta p_t^m - d_t^r}{\alpha_2}$, and $p_t^{m(1)\star}(I_t^r, I_t^m)$ decreases with increase in I_t^r or I_t^m which is proved in Theorem 2.2, we get that $p_t^{r(1)\star}(I_t^r, I_t^m)$ and $y_t^{r(1)\star}(I_t^r, I_t^m)$ decrease with increase in I_t^r or I_t^m.

(c) if $I_t^r \geq y_t^{r(2)\star}(I_t^r)$ and $I_t^m < y_t^{m(2)\star}(I_t^r)$, $p_t^{r(2)\star}(I_t^r)$ and $y_t^{r(2)\star}(I_t^r)$ are obtained by substituting $p_t^{m(2)\star}(I_t^r)$ into $p_t^{r\prime}(I_t^r)$ and $y_t^{r\prime}$. With $0 < \frac{\partial d_t^{r\prime}(I_t^r)}{\partial I_t^r}$ which is proved in Proposition 2.1, $p_t^r(d_t^r) = \frac{(1-\theta)a + \beta p_t^m - d_t^r}{\alpha_2}$, $\frac{\partial y_t^{r\prime}}{\partial p_t^m} = \frac{\beta}{2}$, and $p_t^{m(2)\star}(I_t^r)$ decreases with increase in I_t^r which is proved in Theorem 2.2, we get that $p_t^{r(2)\star}(I_t^r)$ and $y_t^{r(2)\star}(I_t^r)$ decrease as I_t^r increases.

(d) if $I_t^r \geq y_t^{r(2)\star}(I_t^r, I_t^m)$ and $I_t^m \geq y_t^{m(2)\star}(I_t^r)$, $p_t^{r(2)\star}(I_t^r, I_t^m)$ and $y_t^{r(2)\star}(I_t^r, I_t^m)$ are obtained by substituting $p_t^{m(2)\star}(I_t^r, I_t^m)$ into $p_t^{r\prime}(I_t^r)$ and $y_t^{r\prime}$. Similar to (c), we can prove that $p_t^{r(2)\star}(I_t^r, I_t^m)$ and $y_t^{r(2)\star}(I_t^r, I_t^m)$ decrease as I_t^r or I_t^m increases.

Hence, with (a)-(d), Theorem 2.3 is proved.

Proof of Theorem 2.4

Theorem 2.4 can be proved in a similar way to Theorem 2.3.

Chapter 3

Effects of Channel Power Structure on Dynamic Pricing and Inventory Management of a Dual-channel Supply Chain

3.1 Introduction

Channel power refers to the ability of a particular member to control or affect the decision making and behavior of another member in a channel. In practical supply chains, there are three possible channel power structures including Manufacturer Stackelberg (MS) where the manufacturer has dominated channel power over the retailer (e.g., Apple is often much powerful than its retailers), Retailer Stackelberg (RS) where the retailer is the dominant member (retail giants such as Tesco), and Vertical Nash (VN) in which the manufacturer and the retailer have balanced power (like P&G and Walmart) (Wu et al., 2012; Huang et al., 2016). Furthermore, in a practical supply chain, especially the long-running supply chain, the dominant role in channel power may shift from one member to another member at different periods according to the developments of members and changes in the environment. An example is a supply chain consisting of Zhuhai Gree Corporation (an air conditioning manufacturer) and GOME Appliance Co., Ltd. (a home appliances retail giant in China), which is illustrated in detail in the work of Wang et al. (2011).

As the member who is dominant in channel power moves first in decision making, supply chain members' dynamic pricing and inventory decisions may be affected by the channel power structure.

Motivated by the above observations, we pursue the following research questions in this chapter: (1) What are the structural properties of the optimal joint dynamic pricing and inventory control policies of a dual-channel supply chain under RS and VN power structures? (2) What impacts the channel power structure might have on the optimal joint dynamic pricing and inventory control policy of a dual-channel supply chain? To address these questions, same as Chapter 2 where the optimal joint dynamic pricing and inventory control policy under MS is analyzed, we consider a dual-channel supply chain where the manufacturer produces a single type of products and sells them to customers through the traditional retail channel and its direct online channel over multiple periods. The optimal joint dynamic pricing and inventory control policies under RS and VN are investigated with stochastic dynamic programming and game theory. To derive the effects of the channel power structure, the optimal joint dynamic and inventory control policies under different channel power structures are compared and numerical experiments are conducted. Moreover, considering that there are also different channel power structures in a dual-parallel-channel system, we further explore the effects of the dual-channel setting on the structural properties of the optimal joint dynamic pricing and inventory control policies under different channel power structures.

Results show that, an inventory-dependent base-stock-list-price policy in which base stock levels and reduced prices are influenced by starting inventory levels, is also optimal for a dual-channel supply chain under RS or VN power structure. The influence rules of starting inventory levels on reduced prices under different channel power structures are the same, while the influence rules of starting inventory levels on base stock levels vary with the channel power structure. Results also reveal that, for a dual-parallel-channel system consisting of two competing retailers, the structural properties of the optimal policy are also not affected by the channel power structure, while the influence rules of initial inventory levels on base stock levels under different channel power structures are partially different. Numerical results show that, for the two-period dual-channel supply chain, VN is the most profitable channel power structure when the wholesale price is low, and MS is the most profitable channel power structure when the wholesale price is high.

The reminder of this chapter is organized as follows. In Section 3.2, we review the relevant literature. In Section 3.3, the optimal pricing and inventory control policies under RS and VN power structures are explored and analyzed by developing decentralized game models, and the optimal joint dynamic pricing and inventory control policies under three different channel power structures are compared. Section 3.4 investigates the impacts of the dual-channel setting on the optimal dynamic pricing and inventory control policies under differernt power structures. Section 3.5 provides numerical experiments and discusses the management insights that arise. Finally, some conclusions are summarized in Section 3.6.

3.2 Literature review

This work is closely related to decisions in a dual-channel supply chain under different channel power structures. Under this stream of work, some literature (e.g., Wang et al., 2011; Wu et al., 2012; Huang et al., 2016; Shi et al., 2020; Liu et al., 2022) studies a dual-channel supply chain in which a manufacturer sells products through two competing retailers. Chen et al., (2013), Zhao et al. (2017), and Wang et al. (2017) studies a dual-channel supply chain with two manufacturers and one common retailer where one of the two manufacturers uses dual channels including an online channel and a traditional retail channel to sell its product. Yan et al. (2020) and Chen et al. (2021) consider a dual-channel supply chain in which the supplier sold products through its own direct offline channel and an online retailer, while Chai et al. (2021) consider online and offline channels are owned by a single retailer.

Different from the above literature, this work focuses on a dual-channel supply chain where a manufacturer distributes products through traditional retail and his own online channels. For such type of dual-channel supply chains, Yao and Liu (2005) study the pricing decisions under manufacturer Stackelberg and Nash power structures. Cai et al. (2009) examine price discount contracts and pricing schemes under three different channel power structures. Xiao et al. (2014) develop the retailer-Stackelberg model and the manufacturer-Stackelberg model to study pricing decisions and manufacturer's distribution channel strategy when the indirect channel sells standard products whereas the direct channel offers custom products. Chen and Wang (2015) examine the impact of channel power structure on the pricing decision and channel selection strategy for a smartphone supply chain. Rodríguez and Aydın (2015) study pricing and assortment decisions in the presence of inventory costs. Zheng et al. (2019) investigate the manufacturer's distribution channel strategy and its effects on the performance of a closed-loop supply chain system. Zhou et al. (2018) and Liu et al. (2020) analyze the impacts of supply disruption and market fluctuations on the pricing decisions and profits, respectively. Matsui (2022) studies whether bargaining to determine the wholesale price with a manufacturer really benefits a retailer. Sun et al. (2022b) study the timing of pricing decisions by considering the selling price in the direct channel affects the consumers' reservation price in both channels. In this paper, we focus on investigating the joint dynamic pricing and inventory control policies under different channel power structures and analyzing the impact of the channel power structure on the joint dynamic pricing and inventory control policy.

To summarize, contributions of this study to the literature are that: (i) the structural properties of the optimal dynamic pricing and inventory control policy of a dual-channel supply chain under different channel power structures are revealed; (ii) the impact of the channel power structure on the dynamic pricing and inventory control policy of a dual-channel supply chain is investigated; (iii) the effects of the dual-channel setting on the optimal joint dynamic pricing and inventory control policies under different channel power structure are investigated.

3.3 Analysis of the optimal joint dynamic pricing and inventory control policies under different channel power structures

In this section, the structure of the optimal dynamic pricing and inventory control policies for the manufacturer and retailer under RS and VN power structures is characterized, and a comparison and analysis of the optimal dynamic pricing and inventory control policies for different channel power structures is performed.

The model description and assumptions under RS and VN power structures are the same as that under MS power structure. Therefore, we will also focus on the following models which have already been established in Section 2.3 to analyze the manufacturer and retailer's problems under RS or VN power structure.

$$V_t^m(I_t^m) = \begin{cases} \max\limits_{p_t^m, y_t^m} & J_t^m(p_t^m, y_t^m) \\ s.\ t. & y_t^m - I_t^m + y_t^r - I_t^r \geq 0 \end{cases} \tag{3.1}$$

where $J_t^m(p_t^m, y_t^m) = d_t^m(p_t^m - \rho c) + (w - c)(y_t^r - I_t^r) - c y_t^m(1-\rho) - \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] + \rho\mathbf{E}[V_{t+1}^m(y_t^m - d_t^m - \epsilon_t^m)]$.

$$V_t^r(I_t^r) = \begin{cases} \max\limits_{p_t^r, y_t^r} & J_t^r(p_t^r, y_t^r) \\ s.\ t. & y_t^r - I_t^r \geq 0 \end{cases} \tag{3.2}$$

where $J_t^r(p_t^r, y_t^r) = d_t^r(p_t^r - \rho w) - y_t^r w(1-\rho) - \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho\mathbf{E}[V_{t+1}^r(y_t^r - d_t^r - \epsilon_t^r)]$.

3.3.1 Analysis of the optimal joint dynamic pricing and inventory control policy under RS

In the RS dual-channel supply chain, retailer plays the dominant role and decides its optimal policy $(p_t^{r-r\star}, y_t^{r-r\star})$ first by taking manufacturer's response functions into account. Then, manufacturer decides its optimal policy $(p_t^{m-r\star}, y_t^{m-r\star})$ with the consideration of retailer's optimal policy.

With given (p_t^r, y_t^r), manufacturer's problem (3.1) will be solved first. The mean demand d_t^m has an inverse function $p_t^m(d_t^m) = \frac{\theta a + \beta_1 p_t^r - d_t^m}{\alpha_1}$ when given p_t^r is given. Therefore, optimizing p_t^m is equivalent to optimizing d_t^m. After replacing the decision variable p_t^m by d_t^m, (3.1) under RS is rewritten as (3.3).

$$V_t^{m-r}(I_t^m) = \begin{cases} \max\limits_{d_t^m, y_t^m} & J_t^{m-r}(d_t^m, y_t^m) \\ s.\ t. & y_t^m - I_t^m + y_t^r - I_t^r \geq 0 \end{cases} \tag{3.3}$$

where $J_t^{m-r}(d_t^m, y_t^m) = d_t^m(p_t^m(d_t^m) - \rho c) + (w - c)(y_t^r - I_t^r) - c y_t^m(1-\rho) - \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] + \rho\mathbf{E}[V_{t+1}^{m-r}(y_t^m - d_t^m - \epsilon_t^m)]$.

Lemma 3.1. In each period t, with retailer's given decisions (p_t^r, y_t^r),

(a) $V_t^{m-r}(I_t^m)$ is concave and nonincreasing in I_t^m;

(b) $J_t^{m-r}(d_t^m, y_t^m)$ is jointly concave in (d_t^m, y_t^m).

With Lemma 3.1, manufacturer's optimal response functions in period t exist and can be obtained with the steps used in Section 2.4. Denote $(d_t^{m-r\prime}, y_t^{m-r\prime}) = \arg\max J_t^{m-r}(d_t^m, y_t^m)$ and $p_t^{m-r\prime} = p_t^m(d_t^{m-r\prime})$.

Theorem 3.1. In each period t, manufacturer's optimal decisions are:

(a) if $I_t^m < y_t^{m-r\prime} + y_t^r - I_t^r$, the optimal solution of (p_t^m, y_t^m) is $(p_t^{m-r\prime}, y_t^{m-r\prime})$;

(b) if $I_t^m \geq y_t^{m-r\prime} + y_t^r - I_t^r$, the optimal solution of (p_t^m, y_t^m) is $(p_t^{m-r\prime}(I_t^m + I_t^r - y_t^r), I_t^m + I_t^r - y_t^r)$, where $p_t^{m-r\prime}(I_t^m + I_t^r - y_t^r)$ is decreasing with $I_t^m + I_t^r - y_t^r$.

From Theorem 3.1, we can find that, for any given retailer's decisions, the manufacturer's optimal policy is a base-stock-list-price policy. If the manufacturer's initial inventory is less than the base stock level, it should produce up to this base stock level and charge a list price, otherwise it should not produce and reduce the online price. Moreover, the more the initial inventory level exceeds the base stock level, the more the online price should be reduced.

Proposition 3.1. With manufacturer's optimal response decisions, the mean demand d_t^r has an inverse function $p_t^r(d_t^r) = \frac{(1-\theta)a+\beta_2 p_t^m - d_t^r}{\alpha_2}$ which is strictly decreasing, where $p_t^m = p_t^{m-r\prime}$ when $I_t^m < y_t^{m-r\prime} + y_t^r - I_t^r$ and $p_t^m = p_t^{m-r\prime}(I_t^m + I_t^r - y_t^r)$ when $I_t^m \geq y_t^{m-r\prime} + y_t^r - I_t^r$.

With Proposition 3.1, optimizing price p_t^r in period t is equivalent to optimizing d_t^r. Hence, the retailer's problem (3.2) can be rewritten as

$$V_t^{r-r}(I_t^r) = \begin{cases} \max\limits_{d_t^r, y_t^r} \; J_t^{r-r}(d_t^r, y_t^r) \\ s.\ t. \quad y_t^r - I_t^r \geq 0 \end{cases} \tag{3.4}$$

where $J_t^{r-r}(d_t^r, y_t^r) = d_t^r(p_t^r(d_t^r) - \rho w) - y_t^r w(1-\rho) - \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho\mathbf{E}[V_{t+1}^{r-r}(y_t^r - d_t^r - \epsilon_t^r)]$.

Lemma 3.2. In each period t, with the manufacturer's optimal response decisions,

(a) $V_t^{r-r}(I_t^r)$ is concave and nonincreasing in I_t^r;

(b) $J_t^{r-r}(d_t^r, y_t^r)$ is jointly concave in (d_t^r, y_t^r).

Lemma 3.2 indicates that retailer's optimal decisions exist. Define $d_t^{r-r\star}(y_t^r) =$

$\arg\max J_t^{r-r}(d_t^r, y_t^r)$. Define $y_t^{r-r\star} = \arg\max J_t^{r-r}(d_t^{r-r\star}(y_t^r), y_t^r)$ where (p_t^m, y_t^m) is $(p_t^{m-r\prime}, y_t^{m-r\prime})$, and $y_t^{r-r\star}(I_t^r, I_t^m) = \arg\max J_t^{r-r}(d_t^{r-r\star}(y_t^r), y_t^r)$ where (p_t^m, y_t^m) is $(p_t^{m-r\prime}(I_t^m + I_t^r - y_t^r), I_t^m + I_t^r - y_t^r)$.

Theorem 3.2. In each period t, retailer's optimal decisions are:

(a) Under the scenario where $I_t^m < y_t^{m-r\prime} + y_t^r - I_t^r$, if $I_t^r < y_t^{r-r\star}$, the optimal solution of (p_t^r, y_t^r) is $(p_t^{r-r\star}, y_t^{r-r\star})$; otherwise, the optimal solution of (p_t^r, y_t^r) is $(p_t^{r-r\star}(I_t^r), I_t^r)$. Moreover, $y_t^{r-r\star}$ and $p_t^{r-r\star}$ are independent of I_t^r, and $p_t^{r-r\star}(I_t^r)$ is decreasing with I_t^r.

(b) Under the scenario where $I_t^m \geq y_t^{m-r\prime} + y_t^r - I_t^r$, if $I_t^r < y_t^{r-r\star}(I_t^r, I_t^m)$, the optimal solution of (p_t^r, y_t^r) is $(p_t^{r-r(1)\star}(I_t^r, I_t^m), y_t^{r-r\star}(I_t^r, I_t^m))$; otherwise, the optimal solution of (p_t^r, y_t^r) is $(p_t^{r-r(2)\star}(I_t^r, I_t^m), I_t^r)$. Moreover, $y_t^{r-r\star}(I_t^r, I_t^m)$, $p_t^{r-r(1)\star}(I_t^r, I_t^m)$ and $p_t^{r-r(2)\star}(I_t^r, I_t^m)$ are decreasing with I_t^r or I_t^m.

To get the manufacturer's final optimal policy, we need to substitute retailer's optimal decisions into the manufacturer's optimal response decisions which are shown in Theorem 3.1. Theorem 3.3 describes the structural properties of the dual-channel supply chain's optimal dynamic pricing and inventory control policy under RS.

Theorem 3.3. The optimal pricing and inventory control policies under RS are:

(a) if $I_t^r < y_t^{r-r\star}$ and $I_t^m < y_t^{m-r\star} + y_t^{r-r\star} - I_t^r$, the optimal decisions are $(p_t^{r-r\star}, y_t^{r-r\star})$ and $(p_t^{m-r\star}, y_t^{m-r\star})$, where $y_t^{r-r\star}$, $p_t^{r-r\star}$, $y_t^{m-r\star}$ and $p_t^{m-r\star}$ are independent of I_t^m and I_t^r .

(b) if $I_t^r < y_t^{r-r\star}(I_t^r, I_t^m)$ and $I_t^m \geq y_t^{m-r(1)\star}(I_t^r, I_t^m) + y_t^{r-r\star}(I_t^r, I_t^m) - I_t^r$, the optimal decisions are $(p_t^{r-r(1)\star}(I_t^r, I_t^m), y_t^{r-r\star}(I_t^r, I_t^m))$ and $(p_t^{m-r(1)\star}(I_t^r, I_t^m), I_t^m + I_t^r - y_t^{r-r\star}(I_t^r, I_t^m))$. In addition, $y_t^{r-r\star}(I_t^r, I_t^m)$, $p_t^{r-r(1)\star}(I_t^r, I_t^m)$, $y_t^{m-r(1)\star}(I_t^r, I_t^m)$ and $p_t^{m-r(1)\star}(I_t^r, I_t^m)$ decrease as either I_t^r or I_t^m increases.

(c) if $I_t^r \geq y_t^{r-r\star}$ and $I_t^m < y_t^{m-r\star}(I_t^r)$, the optimal decisions are $(p_t^{r-r\star}(I_t^r), I_t^r)$ and $(p_t^{m-r\star}(I_t^r), y_t^{m-r\star}(I_t^r))$, where $p_t^{r-r\star}(I_t^r)$, $y_t^{m-r\star}(I_t^r)$ and $p_t^{m-r\star}(I_t^r)$ are independent of I_t^m but decrease as I_t^r increases.

(d) if $I_t^r \geq y_t^{r-r\star}(I_t^r, I_t^m)$ and $I_t^m \geq y_t^{m-r(2)\star}(I_t^r, I_t^m)$, the optimal decisions are $(p_t^{r-r(2)\star}(I_t^r, I_t^m), I_t^r)$ and $(p_t^{m-r(2)\star}(I_t^r, I_t^m), I_t^m)$, where $p_t^{r-r(2)\star}(I_t^r, I_t^m)$, $y_t^{m-r(2)\star}(I_t^r, I_t^m)$

and $p_t^{m-r(2)\star}(I_t^r, I_t^m)$ decrease as either I_t^r or I_t^m increases.

3.3.2 Analysis of the optimal joint dynamic pricing and inventory control policy under VN

In the VN dual-channel supply chain, manufacturer and retailer make their decisions simultaneously and independently. The decision sequence in each period t is: with the initial inventory level information, manufacturer decides online price and production quantity to maximize its expected discount profit from period t to the end of period T given retail price and order quantity, and retailer decides retail price and order quantity to maximize its expected discount profit from period t to the end of period T given online price and production quantity.

Similar to the problems under MS and RS, price variables (p_t^m, p_t^r) are also replaced with mean demand variables (d_t^m, d_t^r) in the VN case. But, in the VN case, the change of variables is achieved by simultaneously solving the mean demand functions instead of supposing p_t^m or p_t^r is given. With $p_t^m(d_t^m, d_t^r) = \frac{\theta\alpha_2 a+(1-\theta)\beta_1 a-\alpha_2 d_t^m-\beta_1 d_t^r}{\alpha_1\alpha_2-\beta_1\beta_2}$ and $p_t^r(d_t^m, d_t^r) = \frac{(1-\theta)\alpha_1 a+\theta\beta_2 a-\alpha_1 d_t^r-\beta_2 d_t^m}{\alpha_1\alpha_2-\beta_1\beta_2}$, retailer and manufacturer's problems are represented as below.

$$V_t^{m-v}(I_t^m) = \begin{cases} \max\limits_{d_t^m, y_t^m} & J_t^{m-v}(d_t^m, y_t^m) \\ s.\ t. & y_t^m - I_t^m + y_t^r - I_t^r \geq 0 \end{cases} \tag{3.5}$$

where $J_t^{m-v}(d_t^m, y_t^m) = d_t^m(p_t^m(d_t^m, d_t^r)-\rho c)+(w-c)(y_t^r-I_t^r)-cy_t^m(1-\rho)-\mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] + \rho\mathbf{E}[V_{t+1}^{m-v}(y_t^m - d_t^m - \epsilon_t^m)]$.

$$V_t^{r-v}(I_t^r) = \begin{cases} \max\limits_{d_t^r, y_t^r} & J_t^{r-v}(d_t^r, y_t^r) \\ s.\ t. & y_t^r - I_t^r \geq 0 \end{cases} \tag{3.6}$$

where $J_t^{r-v}(d_t^r, y_t^r) = d_t^r(p_t^r(d_t^m, d_t^r) - \rho w) - y_t^r w(1-\rho) - \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho\mathbf{E}[V_{t+1}^{r-v}(y_t^r - d_t^r - \epsilon_t^r)]$.

Since manufacturer and retailer make decisions independently under VN, we can easily get that Lemma 2.1 and Lemma 3.1 are also true for retailer's problem and manufacturer's problem under VN. That is, $J_t^{r-v}(d_t^r, y_t^r)$ is jointly concave in (d_t^r, y_t^r) and $J_t^{m-v}(d_t^m, y_t^m)$ is jointly concave in (d_t^m, y_t^m). Therefore, manufacturer and retailer's optimal decisions in period t exist. Manufacturer and retailer's optimal decisions can be obtained with the following steps.

Step 1: For any (d_t^r, y_t^r) and y_t^m, define $d_t^{m-v}(y_t^m) = \arg\max J_t^{m-v}(d_t^m, y_t^m)$. Get $J_t^{m-v}(y_t^m)$ by substituting $d_t^{m-v}(y_t^m)$ for d_t^m in $J_t^{m-v}(d_t^m, y_t^m)$. For any (d_t^m, y_t^m) and y_t^r, define $d_t^{r-v} = \arg\max J_t^{r-v}(d_t^r, y_t^r)$. Get $J_t^{r-v}(y_t^r)$ by substituting $d_t^{r-v}(y_t^r)$ for d_t^r in $J_t^{r-v}(d_t^r, y_t^r)$. Obtain optimal response functions of d_t^m and d_t^r, denoted as $d_t^{m-v\star}(y_t^m, y_t^r)$ and $d_t^{r-v\star}(y_t^m, y_t^r)$, by simultaneously solving d_t^{m-v} and d_t^{r-v}.

Step 2: Solve (3.7) to derive the optimal decisions of (y_t^m, y_t^r).

$$\begin{aligned} &\max_{y_t^m} \quad J_t^{m-v}(y_t^m) && \max_{y_t^r} \quad J_t^{r-v}(y_t^r) \\ &s.\ t. \quad y_t^m + y_t^r \geq I_t^r + I_t^m && s.\ t. \quad y_t^r \geq I_t^r \end{aligned} \tag{3.7}$$

Since the feasible set of y_t^m depends on y_t^r, (3.7) is a Generalized Nash game where each player's feasible strategy set can depend on the other players' strategies. To ensure the domain of (y_t^r, y_t^m) is compact, let $M_1 \geq y_t^r$ and $M_2 \geq y_t^m + y_t^r$ where M_1 and M_2 are large numbers. Referring to Harker (1991) and Facchinei et al. (2007), (3.7) has a generalized Nash equilibrium and can be solved as follows: find (y_t^{m-v}, y_t^{r-v}) such that $-\frac{\partial J_t^{r-v}(y_t^{r-v})}{\partial y_t^{r-v}}(y_t^r - y_t^{r-v}) \geq 0$ and $-\frac{\partial J_t^{m-v}(y_t^{m-v})}{\partial y_t^{m-v}}(y_t^m - y_t^{m-v}) \geq 0$ for any $y_t^r \in K^r$ and any $y_t^m \in K^m(y_t^{r-v})$, where $K^r = \{y_t^r : M_1 \geq y_t^r \geq I_t^r\}$ and $K^m(y_t^{r-v}) = \{y_t^m : M_2 \geq y_t^m \geq I_t^r + I_t^m - y_t^{r-v}\}$.

(a) Consider there exist $y_t^r \in K^r$ such that $\frac{\partial J_t^{r-v}(y_t^r)}{\partial y_t^r} = 0$ and $y_t^m \in K^m(y_t^{r-v})$ such that $\frac{\partial J_t^{m-v}(y_t^m)}{\partial y_t^m} = 0$. It is obvious that (y_t^{m-v}, y_t^{r-v}) can be derived by solving $\frac{\partial J_t^{m-v}(y_t^m)}{\partial y_t^m} = 0$ and $\frac{\partial J_t^{r-v}(y_t^r)}{\partial y_t^r} = 0$ simultaneously. With (y_t^{m-v}, y_t^{r-v}), $d_t^{m-v\star}(y_t^m, y_t^r)$ and $d_t^{r-v\star}(y_t^m, y_t^r)$, obtain the optimal solutions of (y_t^m, y_t^r), denoted as $(y_t^{m-v\star}, y_t^{r-v\star})$. Therefore, if $I_t^r < y_t^{r-v\star}$ and $I_t^m < y_t^{m-v\star} + y_t^{r-v\star} - I_t^r$, the optimal decisions are $(y_t^{m-v\star}, y_t^{r-v\star})$.

(b) Consider there exists $y_t^r \in K^r$ such that $\frac{\partial J_t^{r-v}(y_t^r)}{\partial y_t^r} = 0$, and $\frac{\partial J_t^{m-v}(y_t^m)}{\partial y_t^m} \neq 0$ for any $y_t^m \in K^m(y_t^{r-v})$. Since $\frac{\partial J_t^{m-v}(y_t^m)}{\partial y_t^m} \neq 0$ for any $y_t^m \in K^m(y_t^{r-v})$ and $J_t^{m-v}(y_t^m)$ is concave in y_t^m, $\frac{\partial J_t^{m-v}(y_t^m)}{\partial y_t^m} < 0$ for any $y_t^m \in K^m(y_t^{r-v})$. Therefore, y_t^{m-v} which ensures $-\frac{\partial J_t^{m-v}(y_t^{m-v})}{\partial y_t^{m-v}}(y_t^m - y_t^{m-v}) \geq 0$ for any $y_t^m \in K^m(y_t^{r-v})$ should be $I_t^m + I_t^r - y_t^{r-v}$. With $y_t^{m-v} = I_t^m + I_t^r - y_t^{r-v}$ and $\frac{\partial J_t^{r-v}(y_t^r)}{\partial y_t^r} = 0$, we can obtain y_t^{r-v} which is a function of (I_t^r, I_t^m). With (y_t^{m-v}, y_t^{r-v}), $d_t^{m-v\star}(y_t^m, y_t^r)$ and $d_t^{r-v\star}(y_t^m, y_t^r)$, obtain the optimal solutions of y_t^r, denoted as $y_t^{r-v(1)\star}(I_t^r, I_t^m)$. With $y_t^{r-v\star}(I_t^r, I_t^m)$, obtain the solution of $\frac{\partial J_t^{m-v}(y_t^m)}{\partial y_t^m}=0$, denoted as $y_t^{m-v(1)\star}(I_t^r,I_t^m)$. Therefore, if $I_t^r<y_t^{r-v(1)\star}(I_t^r,I_t^m)$ and $I_t^m\geq y_t^{m-v(1)\star}(I_t^r,I_t^m)+y_t^{r-v(1)\star}(I_t^r,I_t^m)-I_t^r$, the optimal decisions are $(I_t^m+I_t^r-y_t^{r-v(1)\star}(I_t^r,I_t^m),y_t^{r-v(1)\star}(I_t^r,I_t^m))$.

(c) Consider $\frac{\partial J_t^{r-v}(y_t^r)}{\partial y_t^r}\neq 0$ for any $y_t^r\in K^r$, and there exists $y_t^m\in K^m(y_t^{r-v})$ such that $\frac{\partial J_t^{m-v}(y_t^m)}{\partial y_t^m}=0$. Similar to (b), we get that y_t^{r-v} which should be I_t^r and y_t^{m-v} which is a function of I_t^r. With (y_t^{m-v},y_t^{r-v}), $d_t^{m-v\star}(y_t^m,y_t^r)$ and $d_t^{r-v\star}(y_t^m,y_t^r)$, obtain the optimal solution of y_t^m, denoted as $y_t^{m-v\star}(I_t^r)$. Then, obtain the solution of $\frac{\partial J_t^{r-v}(y_t^r)}{\partial y_t^r}=0$, denoted as $y_t^{r-v\star}(I_t^r)$. Hence, if $I_t^r\geq y_t^{r-v\star}(I_t^r)$ and $I_t^m<y_t^{m-v\star}(I_t^r)$, the optimal decisions are $(y_t^{m-v\star}(I_t^r),I_t^r)$.

(d) Consider $\frac{\partial J_t^{r-v}(y_t^r)}{\partial y_t^r}\neq 0$ for any $y_t^r\in K^r$ and $\frac{\partial J_t^{m-v}(y_t^m)}{\partial y_t^m}\neq 0$ for any $y_t^m\in K^m(y_t^{r-v})$. Similar to (b), we get that (y_t^{m-v},y_t^{r-v}) should be (I_t^m,I_t^r). Then, with $d_t^{m-v\star}(I_t^m,I_t^r)$ and $d_t^{r-v\star}(I_t^m,I_t^r)$, obtain the solution of $\frac{\partial J_t^{r-v}(y_t^r)}{\partial y_t^r}=0$, denoted as $y_t^{r-v(2)\star}(I_t^r,I_t^m)$, and the solution of $\frac{\partial J_t^{m-v}(y_t^m)}{\partial y_t^m}=0$, denoted as $y_t^{m-v(2)\star}(I_t^r,I_t^m)$. Therefore, if $I_t^r\geq y_t^{r-v(2)\star}(I_t^r,I_t^m)$ and $I_t^m\geq y_t^{m-v(2)\star}(I_t^r,I_t^m)$, the optimal decisions are (I_t^r,I_t^m).

Step 3: With results of Step 2, $p_t^m(d_t^m,d_t^r)$ and $p_t^r(d_t^m,d_t^r)$, obtain the optimal decisions of (p_t^m,p_t^r).

Theorem 3.4. The optimal pricing and inventory control policies under VN are:

(a) if $I_t^r<y_t^{r-v\star}$ and $I_t^m<y_t^{m-v\star}+y_t^{r-v\star}-I_t^r$, the optimal decisions are $(p_t^{r-v\star},y_t^{r-v\star})$ and $(p_t^{m-v\star},y_t^{m-v\star})$, where $y_t^{r-v\star}$, $p_t^{r-v\star}$, $y_t^{m-v\star}$ and $p_t^{m-v\star}$ are unrelated to the initial inventory levels I_t^r and I_t^m.

(b) if $I_t^r<y_t^{r-v(1)\star}(I_t^r,I_t^m)$ and $I_t^m\geq y_t^{m-v(1)\star}(I_t^r,I_t^m)+y_t^{r-v(1)\star}(I_t^r,I_t^m)-I_t^r$, the optimal decisions are $(p_t^{r-v(1)\star}(I_t^r,I_t^m),y_t^{r-v(1)\star}(I_t^r,I_t^m))$ and $(p_t^{m-v(1)\star}(I_t^r,I_t^m),I_t^m+I_t^r-y_t^{r-v(1)\star}(I_t^r,I_t^m))$, where $y_t^{m-v(1)\star}(I_t^r,I_t^m)$ increases as either I_t^r or I_t^m increases, while $y_t^{r-v(1)\star}(I_t^r,I_t^m)$, $p_t^{r-v(1)\star}(I_t^r,I_t^m)$ and $p_t^{m-v(1)\star}(I_t^r,I_t^m)$ decrease as either I_t^r or I_t^m increases.

(c) if $I_t^r\geq y_t^{r-v\star}(I_t^r)$ and $I_t^m<y_t^{m-v\star}(I_t^r)$, the optimal decisions are $(p_t^{r-v\star}(I_t^r),I_t^r)$ and $(p_t^{m-v\star}(I_t^r),y_t^{m-v\star}(I_t^r))$, where $y_t^{r-v\star}(I_t^r)$ increases as I_t^r increases, while $y_t^{m-v\star}(I_t^r)$, $p_t^{r-v\star}(I_t^r)$ and $p_t^{m-v\star}(I_t^r)$ decrease as I_t^r increases.

(d) if $I_t^r\geq y_t^{r-v(2)\star}(I_t^r,I_t^m)$ and $I_t^m\geq y_t^{m-v(2)\star}(I_t^r,I_t^m)$, the optimal decisions are $(p_t^{r-v(2)\star}(I_t^r,I_t^m),I_t^r)$ and $(p_t^{m-v(2)\star}(I_t^r,I_t^m),I_t^m)$, where $y_t^{m-v(2)\star}(I_t^r,I_t^m)$ decreases as

I_t^r increases but increases as I_t^m increases, and $y_t^{r-v(2)\star}(I_t^r, I_t^m)$ decreases as I_t^m increases but increases as I_t^r increases. Furthermore, $p_t^{r-v(2)\star}(I_t^r, I_t^m)$ and $p_t^{m-v(2)\star}(I_t^r, I_t^m)$ decrease as either I_t^r or I_t^m increases.

3.3.3 Comparison study

There are four possible situations of the relationships between members' initial inventory levels and their base stock levels. As shown in Fig. 3.1, the situation, where retailer's initial inventory level I_t^r and manufacturer's initial inventory level I_t^m are both below their respective base stock levels, is represented by case I, and other situations are represented by cases II, III and IV.

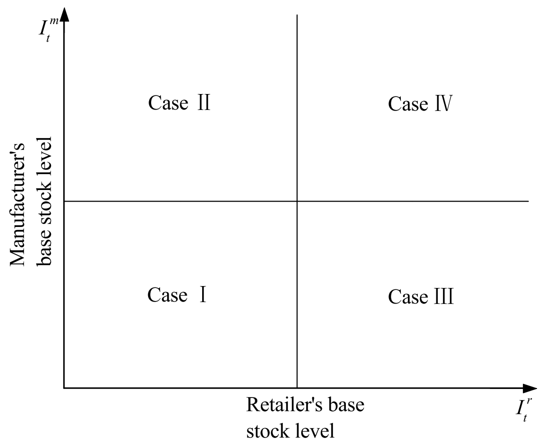


Fig. 3.1. The division of cases I, II, III and IV

Corollary 3.1. The structural properties of the optimal dynamic pricing and inventory control policies of a dual-channel supply chain under different channel power structures are the same.

(a) In Case I, retailer and manufacturer are optimal to order or produce up to their respective base stock levels and charge list prices.

(b) In Case II, retailer should reduce the retail price and order up to its base stock level, while the manufacturer should reduce the online price and not produce.

(c) In Case III, retailer should reduce the retail price and place no order, and the manufacturer should mark down the online price and produce up to its base stock level.

(d) In Case IV, retailer should reduce the retail price and place no order, and the manufacturer should reduce the online price and not produce.

Corollary 3.1 shows that, no matter what channel power structure, retailer's order behavior or manufacturer's produce behavior only depends on whether its starting inventory level is below its base stock level. This kind of order or produce behavior is the same as the results of the study on the joint dynamic pricing and inventory management in a single distribution channel setting (Federgruen and Heching, 1999; Elmaghraby and Keskinocak, 2003; Chen and Simchi-Levi, 2012). However, in a dual-channel supply chain, as long as one member's initial inventory level is above its base stock level, both manufacturer and retailer should reduce the prices. This is caused by the horizontal price competition between the manufacturer and retailer.

Corollary 3.2. In every period, the optimal pricing and inventory decisions of a dual-channel supply chain vary in different channel power structures.

From Corollary 3.1 and Corollary 3.2, we can find that the optimal pricing and inventory decisions in every period are affected by the channel power structure, although the structural properties of the optimal dynamic pricing and inventory control policy are the same under different channel power structure.

Corollary 3.3. The influences of members' starting inventory levels on pricing decisions under different channel power structures are as follows.

(a) In Case I, retail price and online price are not affected by retailer's starting inventory level and manufacturer's starting inventory level under different channel power structures.

(b) In Case II and Case IV, retail price and online price are negatively affected by retailer's starting inventory level as well as manufacturer's starting inventory level under different channel power structures.

(c) In Case III, retail price and online price are negatively affected by retailer's starting inventory level but independent of manufacturer's starting inventory level under different channel power structures.

Corollary 3.3 indicates that the influence rules of members' starting inventory levels on prices are the same under different channel power structures. Moreover, online and retail prices are fixed and not affected by starting inventory levels if manufacturer and retailer's starting inventory levels are below their respective base stock levels. Online and retail prices decrease as retailer's starting inventory level increases if manufacturer's starting inventory level is below its base stock level and retailer's starting inventory level is above its base stock level. Online and retail prices decrease as each member's starting inventory level increase if manufacturer's starting inventory level is above its base stock level.

Corollary 3.4. The influences of members' starting inventory levels on base stock levels under different channel power structures are shown in Tables 3.1 and 3.2.

Table 3.1
The influences of initial inventory levels on manufacturer's base stock level

	Case I	Case II	Case III	Case IV
MS	$I_t^m(\circ), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$
RS	$I_t^m(\circ), I_t^r(-)$	$I_t^m(-), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$	$I_t^m(-), I_t^r(-)$
VN	$I_t^m(\circ), I_t^r(-)$	$I_t^m(-), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$	$I_t^m(+), I_t^r(-)$

[1] $I_t^m(\circ)$ or $I_t^r(\circ)$ denotes base stock level is independent of I_t^m or I_t^r, $I_t^m(-)$ or $I_t^r(-)$ denotes base stock level is negatively correlated with I_t^m or I_t^r, and $I_t^m(+)$ or $I_t^r(+)$ denotes base stock level is positively correlated with I_t^m or I_t^r.

Table 3.2
The influences of initial inventory levels on retailer's base stock level

	Case I	Case II	Case III	Case IV
MS	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(-), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$	$I_t^m(-), I_t^r(-)$
RS	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(-), I_t^r(-)$	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(-), I_t^r(-)$
VN	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(-), I_t^r(-)$	$I_t^m(\circ), I_t^r(+)$	$I_t^m(-), I_t^r(+)$

Corollary 3.4 shows that one member's base stock level may be affected by not only its starting inventory level but also by the other member's starting inventory level. This is caused by the interaction between manufacturer and retailer in decision-making.

Corollary 3.4 also indicates that the influence rules of starting inventory levels on base stock levels are partially different under different channel power structures. Different aspects lie in: (i) the influence of members' initial inventory levels on manufacturer's base stock level when manufacturer's initial inventory level is no less than its base stock level, (ii) the influence of retailer's initial inventory level on its base stock level when retailer's initial inventory level is no less than its base stock level. Common aspects are: (i) when manufacturer's initial inventory level is below its base stock level, manufacturer's base stock level is independent of its initial inventory level but decreases as retailer's initial inventory level increases, (ii) retailer's base stock level is independent of manufacturer's initial inventory level if manufacturer's initial inventory level is below its base stock level, otherwise it decreases as manufacturer's initial inventory level increases, and (iii) when retailer's initial inventory level is below its base stock level, retailer's base stock level is independent of its initial inventory level if manufacturer's initial inventory level is below its base stock level, otherwise retailer's base stock level decreases as its initial inventory level increases.

3.4 Analysis of the optimal policies for a dual-parallel-channel system under different channel power structures

In this section, we analyze the optimal joint dynamic pricing and inventory control policy of dual-parallel-channel system under Bertrand Nash, and then discuss the effect of the dual-channel setting under different channel power structures.

The model description and assumptions under Bertrand Nash are the same as that under Retailer i Stackelberg in Section 2.5. Therefore, we will also focus on the following model which has already been built in Section 2.5 to analyze the optimal policy under Bertrand Nash where the two retailers have balanced power and act simultaneously.

$$V_t^i(I_t^i) = \begin{cases} \max\limits_{p_t^i, y_t^i} \ J_t^i(p_t^i, y_t^i) \\ s.\ t. \quad y_t^i - I_t^i \geq 0 \end{cases} \tag{3.8}$$

where $J_t^i(p_t^i, y_t^i) = d_t^i(p_t^i - \rho w) - (1-\rho) w y_t^i - \mathbf{E}[h^i(y_t^i - d_t^i - \epsilon_t^i)] + \rho \mathbf{E}[V_{t+1}^i(y_t^i - d_t^i - \epsilon_t^i)]$.

Referring to the steps to solve the problems of the manufacturer-retailer dual-channel supply chain in Section 3.3, the two retailers' problems under Retailer i Stackelberg and Bertrand Nash can be solved. Theorems 3.5 describes the structural properties of the two retailers' optimal policies.

Theorem 3.5. The two retailers' optimal dynamic pricing and inventory decisions under Bertrand Nash are:

(a) if $I_t^1 < y_t^{1\star}$ and $I_t^2 < y_t^{2\star}$, the optimal decisions are $(p_t^{1\star}, y_t^{1\star})$ and $(p_t^{2\star}, y_t^{2\star})$, where $y_t^{1\star}$, $p_t^{1\star}$, $y_t^{2\star}$ and $p_t^{2\star}$ are independent of I_t^1 and I_t^2 .

(b) if $I_t^1 < y_t^{1\star}(I_t^2)$ and $I_t^2 \geq y_t^{2\star}(I_t^2)$, the optimal decisions are $(p_t^{1\star}(I_t^2), y_t^{1\star}(I_t^2))$ and $(p_t^{2\star}(I_t^2), I_t^2)$, where $y_t^{2\star}(I_t^2)$ increases as I_t^2 increases, while $p_t^{1\star}(I_t^2)$, $y_t^{1\star}(I_t^2)$, $p_t^{2\star}(I_t^2)$ decrease as I_t^2 increases.

(c) if $I_t^1 \geq y_t^{1\star}(I_t^1)$ and $I_t^2 < y_t^{2\star}(I_t^1)$, the optimal decisions are $(p_t^{1\star}(I_t^1), I_t^1)$ and $(p_t^{2\star}(I_t^1), y_t^{2\star}(I_t^1))$, where $y_t^{1\star}(I_t^1)$ increases as I_t^1 increases, $p_t^{1\star}(I_t^1)$, $p_t^{2\star}(I_t^1)$ and $y_t^{2\star}(I_t^1)$ decrease as I_t^1 increases.

(d) if $I_t^1 \geq y_t^{1\star}(I_t^1, I_t^2)$ and $I_t^2 \geq y_t^{2\star}(I_t^1, I_t^2)$, the optimal decision are $(p_t^{1\star}(I_t^1, I_t^2), I_t^1)$ and $(p_t^{2\star}(I_t^1, I_t^2), I_t^2)$, where, $i = 1, 2$, $y_t^{i\star}(I_t^1, I_t^2)$ increases as I_t^i increases but decreases as I_t^{3-i} increases, and $p_t^{i\star}(I_t^1, I_t^2)$ decrease as either I_t^1 or I_t^2 increases.

Theorem 3.5 indicates that, the member should order up to its base stock level if its initial inventory level is below its base stock level, otherwise it places no order and both members should reduce their selling prices. This implies that the sutructural properties of the optimal policies for a dual-parallel-channel system are also not affected by the channel power structure. Moreover, the same and different aspects between the optimal policies of the two different dual-channel settings are not affected by the channel power structure.

Theorems 2.4 and 3.5 also shows that, the influence rules of initial inventory levels on base stock levels under different channel power structures are partially different. Specifically, (i) when members' initial inventory levels are above their respective base stock levels, retailer i's base stock level is negatively affected by the other's initial inventory level under Retailer i Stackelberg, negatively affected by both members' inventory levels under Retailer $3 - i$ Stackelberg, and positively affected by its initial inventory level while negatively affected by the other's initial inventory level under Bertrand Nash, and (ii) when retailer i's initial inventory level is above its base stock level and retailer $3-i$'s initial inventory level is below its base stock level, retailer i's base stock level is independent of members' initial inventory levels under Retailer i Stackelberg, negatively affected by its initial inventory level under Retailer $3-i$ Stackelberg, positively affected by its initial inventory level under Bertrand Nash. Common aspects are: (i) when members' initial inventory levels are below their respective base stock levels, retailer i's base stock level is independent of members' initial inventory levels, and (ii) when retailer i's initial inventory level is below its base stock level and retailer $3-i$'s initial inventory level is above its base stock level, retailer i's base stock level is negatively correlated with the other's initial inventory level.

3.5 Numerical studies

In this section, we provide numerical examples to demonstrate the proposed theoretical results and explore the impact of channel power structure on the two-period dual-channel supply chain by comparing the equilibrium results under different channel power structures. Referring to the existing numerical studies of dual-channel supply chain (Li et al., 2016; Huang et al., 2021; Modak and Kelle, 2019) and taking account of assumptions made in this paper, the initial values of parameters for the dual-channel supply chain are set as follows: $a = 200$, $\theta = 0.4$, $c = 10$, $w = 15$, $c_m = c_r = 10$, $\alpha_1 = 4$, $\alpha_2 = 6$, $\beta_1 = \beta_2 = 2$, $\rho = 0.9$, $h^m(x) = h^r(x) = x^+ + 23x^-$, $\epsilon_t^m \in [-20, 20]$, $\epsilon_t^r \in [-20, 20]$.

Based on the proposed structural properties of the optimal pricing and inventory control policies, we first obtain the optimal pricing and inventory control policies in period 2 given initial inventory levels (I_2^m, I_2^r). Optimal policies in period 2 given (I_2^m, I_2^r) under different channel power structures are shown in Tables 3.3 to 3.5.

Table 3.3
Optimal pricing and inventory control policies in period 2 under MS

	Case I [1]	Case II	Case III	Case IV
p_2^m	21.36	$37.7 - 0.17(I_2^m + I_2^r)$	$22.38 - 0.03I_2^r$	$31.13 - 0.15I_2^m - 0.05I_2^r$
y_2^m	53.33	$1.17(I_2^m + I_2^r) - 61.04$	$59.4 - 0.11I_2^r$	I_2^m
p_2^r	21.06	$23.78 - 0.03(I_2^m + I_2^r)$	$26.13 - 0.12I_2^r$	$28.52 - 0.04I_2^m - 0.12I_2^r$
y_2^r	44.7	$61.04 - 0.17(I_2^m + I_2^r)$	I_2^r	I_2^r

[1] Case I: $I_2^r < 44.7$, $I_2^m < 98.03 - I_2^r$; Case II: $I_2^r < 61.04 - 0.17(I_2^m + I_2^r)$, $I_2^m \geq 98.03 - I_2^r$; Case III: $I_2^r \geq 45.71 - 0.03I_2^r$, $I_2^m < 59.4 - 0.11I_2^r$; Case IV: $I_2^r \geq 54.47 - 0.15I_2^m - 0.05I_2^r$, $I_2^m \geq 59.4 - 0.11I_2^r$.

From Tables 3.3 to 3.5, we can see that, under different channel power structures, as long as one member's initial inventory level is above its base stock level, both manufacturer and retailer should reduce their selling prices, and the impacts of members' initial inventory levels on base stock levels which are displayed in corollary 4 are further identified. With the optimal policies in period 2, we then get the

Table 3.4
Optimal pricing and inventory control policies in period 2 under RS

	Case I [1]	Case II	Case III	Case IV
p_2^m	20.28	$38.77 - 0.17(I_2^m + I_2^r)$	$21.48 - 0.03I_2^r$	$31.03 - 0.15I_2^m - 0.05I_2^r$
y_2^m	57.8	$1.21(I_2^m + I_2^r) - 72.04$	$62.57 - 0.11I_2^r$	I_2^m
p_2^r	21.14	$23.15 - 0.03(I_2^m + I_2^r)$	$25.9 - 0.11I_2^r$	$28.54 - 0.04I_2^m - 0.12I_2^r$
y_2^r	42.08	$72.04 - 0.21(I_2^m + I_2^r)$	I_2^r	I_2^r

[1] Case I: $I_2^r < 42.08$, $I_2^m < 99.89 - I_2^r$; Case II: $I_2^r < 72.04 - 0.21(I_2^m + I_2^r)$, $I_2^m \geq 131.86 - 1.23I_2^r - 0.23I_2^m$; Case III: $I_2^r \geq 42.08$, $I_2^m < 62.57 - 0.11I_2^r$; Case IV: $I_2^r \geq 72.04 - 0.21(I_2^m + I_2^r)$, $I_2^m \geq 65.21 - 0.04I_2^m - 0.12I_2^r$.

Table 3.5
Optimal pricing and inventory control policies in period 2 under VN

	Case I [1]	Case II	Case III	Case IV
p_2^m	21.35	$36.78 - 0.16(I_2^m + I_2^r)$	$22.61 - 0.03I_2^r$	$31.19 - 0.15I_2^m - 0.05I_2^r$
y_2^m	54.49	$1.15(I_2^m + I_2^r) - 55.41$	$58.7 - 0.1I_2^r$	I_2^m
p_2^r	21.61	$24.41 - 0.03(I_2^m + I_2^r)$	$26.24 - 0.11I_2^r$	$28.61 - 0.04I_2^m - 0.12I_2^r$
y_2^r	41.38	$55.41 - 0.15(I_2^m + I_2^r)$	I_2^r	I_2^r

[1] Case I: $I_2^r < 41.38$, $I_2^m < 95.87 - I_2^r$; Case II: $I_2^r < 55.41 - 0.15(I_2^m + I_2^r)$, $I_2^m \geq 107.56 - 0.12I_2^m - 1.12I_2^r$; Case III: $I_2^r \geq 40.32 + 0.03I_2^r$, $I_2^m < 58.7 - 0.1I_2^r$; Case IV: $I_2^r \geq 47.72 - 0.13I_2^m + 0.01I_2^r$, $I_2^m \geq 58.21 - 0.1I_2^r + 0.0084I_2^m$.

optimal pricing and inventory decisions in period 1 and period 2 and the total expected profit of each member under different channel power structures which are presented in Table 3.6.

Table 3.6
Optimal pricing and inventory decisions and total expected profits under different channel power structures

	p_2^m	y_2^m	p_2^r	y_2^r	p_1^m	y_1^m	p_1^r	y_1^r	$V_1^{m\prime}$	$V_1^{r\prime}$
MS	21.36	53.33	21.06	44.7	21.36	53.33	21.06	52.2	1112.87	284.69
RS	20.28	57.8	21.14	42.08	21.46	53.51	21.34	50.71	1104.95	273.49
VN	21.35	54.49	21.61	41.38	21.35	54.49	21.61	48.88	1105.23	280.87

As shown in Table 3.6, manufacturer and retailer are optimal to charge list prices and increase their inventory levels to the base stock levels in each period. Manufacturer's base stock levels under RS are higher than those under MS, and retailer's base stock levels under MS are higher than those under RS, which means the follower of this two-period dual-channel supply chain is optimal to have a higher on-hand inventory level. As compared with MS and RS, under VN, retailer's base stock levels are the lowest, and manufacturer's base stock level in the first period is the highest. The relationship of list prices under different channel power structures is opposite to the relationship of base stock levels, except manufacturer's list price under MS is lower than that under RS in the first period.

Comparing the profits under different channel power structures in Table 3.6, we can find that both manufacturer and retailer get the most profits under MS than under other channel power structures. Manufacturer gets the most profit under MS can be attributed to its lowest base stock levels and retailer's highest base stock levels under MS. Manufacturer's lowest base stock levels induce the lowest production cost of selling products to customers, and retailer's highest base stock levels induce the highest profit for manufacturer by selling products to retailer. Although retailer's base stock levels are the highest under MS, it gets the most profit under MS can be attributed to the fact that it charges the lowest selling prices under MS which induces the highest demand and thus the highest revenue.

Considering the wholesale price decided by the negotiation between manufacturer and retailer in advance is affected by the channel power structure, we calculate optimal decisions and total expected discounted profits of the two-period dual-channel supply chain at different wholesale prices, and then explore the effect of the wholesale price on the most profitable channel power structure. The computed optimal decisions and demand are shown in Table 3.7, and the total expected discounted profits are shown in Table 3.8.

From Table 3.7 we find that, optimal decisions of manufacturer and retailer in each period are to charge list prices and increase their inventory levels to the base stock levels when the wholesale price is low. However, when the wholesale price is high, manufacturer and retailer may reduce their selling prices, and retailer may not place an order. Retailer's on-hand inventory levels are the highest under MS and the lowest under VN. Manufacturer's on-hand inventory levels are the lowest under VN when the wholesale price is low. However, when the wholesale price is high, manufacturer's on-hand inventory levels under MS are lower than that under VN.

Table 3.8 shows that MS remains the most profitable channel power structure for the two-period dual-channel supply chain when the wholesale price is high. However, when the wholesale price is low, the most profitable channel power structure is VN. For manufacturer, the change of the most profitable channel power structure is caused by the fact that the contribution of retailer's higher on-hand inventory levels to the increase of manufacturer's profit is less obvious when the wholesale price is low.

Table 3.7
Optimal prices, demand and inventory decisions under different channel power structures when the wholesale price changes

w	Power structure	Demand uncertainty	p_2^m	d_2^m	y_2^m	p_2^r	d_2^r	y_2^r	p_1^m	d_1^m	y_1^m	p_1^r	d_1^r	y_1^r
	MS		20	36.67	53.33	18.33	50	66.67	20	36.67	53.33	18.33	50	66.67
10	RS	$\epsilon_1^r \in [-20, 20]$	19.66	38.64	55.3	18.64	47.5	64.17	9.66	38.64	55.3	18.64	47.5	64.17
	VN		20.7	35.65	52.32	19.22	46.09	62.75	20.7	35.65	52.32	19.22	46.09	62.75
	MS		20.55	36.67	53.33	19.42	44.55	57.88	20.55	36.67	53.33	19.42	44.55	60.88
12	RS	$\epsilon_1^r \in [-20, 20]$	19.91	39.64	56.3	19.64	42	55.33	20.38	37.92	54.58	19.72	42.45	58.78
	VN		20.95	36.52	53.19	20.17	40.87	54.2	20.96	36.52	53.19	20.17	40.87	57.2
	MS		21.09	36.67	53.33	20.52	39.09	49.09	21.09	36.67	53.33	20.52	39.09	55.09
14	RS	$\epsilon_1^r \in [-20, 20]$	20.1591	40.64	57.3	20.64	36.5	46.5	21.1	37.2	53.87	20.8	37.4	53.4
	VN		21.2174	37.39	54.06	21.13	35.65	45.65	1.22	37.39	54.06	21.1304	35.65	51.65
	MS		21.64	36.67	53.33	21.6061	33.64	40.3	21.64	36.67	53.33	21.61	33.64	49.3
16	RS	$\epsilon_1^r \in [-20, 20]$	20.41	41.64	58.3	21.64	31	37.67	21.82	36.48	53.15	21.88	32.35	48.02
	VN		21.48	38.26	54.93	22.09	30.43	37.1	21.48	38.26	54.93	22.09	30.43	46.1
	MS	$\epsilon_1^r \in [-20, -16.56]$	$21.92+0.03\epsilon_1^r$	$41.14+0.11\epsilon_1^r$	$57.8+0.11\epsilon_1^r$	$24.4+0.12\epsilon_1^r$	$17.44-0.63\epsilon_1^r$	$14.95-\epsilon_1^r$	22.18	36.67	53.33	22.7	28.18	43.13
		$\epsilon_1^r \in [-16.56, 20]$	22.18	36.67	53.33	22.7	28.18	31.52						
18	RS	$\epsilon_1^r \in [-20, -13.67]$	$21.05+0.03\epsilon_1^r$	$44.18+0.11\epsilon_1^r$	$60.85+0.11\epsilon_1^r$	$24.18+0.11\epsilon_1^r$	$16.99-0.62\epsilon_1^r$	$15.16-\epsilon_1^r$	24.94	42.64	59.3	22.64	25.5	40.66
		$\epsilon_1^r \in [-13.67, 20]$	20.66	42.64	59.3	22.64	25.5	28.83						
	VN	$\epsilon_1^r \in [-20, -13.34]$	$22.15+0.03\epsilon_1^r$	$40.49+0.1\epsilon_1^r$	$57.15+0.1\epsilon_1^r$	$24.54+0.11\epsilon_1^r$	$17.08-0.61\epsilon_1^r$	$15.21-\epsilon_1^r$	21.74	39.13	55.8	23.04	25.22	40.43
		$\epsilon_1^r \in [-13.34, 20]$	21.74	39.13	55.8	23.04	25.22	28.55						

Table 3.8
Total expected discounted profits under different channel power structures when the wholesale price changes

		$V_1^{m'}$			$V_1^{r'}$	
w	MS	RS	VN	MS	RS	VN
10	627	639.4	654.85	722	709.77	737.46
12	861.54	865.16	873.19	528.39	516.32	534.76
14	1042.49	1038.2	1040.61	359.7	348.12	358.81
16	1169.85	1158.52	1157.11	215.91	205.16	209.62
18	1244.23	1220.07	1197.18	96.33	69.94	87.05

Moreover, manufacturer's on-hand inventory levels are the lowest under VN instead of MS when the wholesale price is low. As for retailer, this change can be attributed to the fact that the contribution of higher demand to the increase of retailer's profit is less obvious because the selling price is low at a low wholesale price. Table 3.8 also indicates that, under different channel power structures, manufacturer's profits increase while retailer's profits decrease as the wholesale price increases.

To further analyze the profit potential or loss to manufacturer and retailer if the channel power structure changes, let $\eta^m = \frac{V_1^{m'-n} - V_1^{m'-o}}{V_1^{m'-o}} \times 100\%$ and $\eta^r = \frac{V_1^{r'-n} - V_1^{r'-o}}{V_1^{r'-o}} \times 100\%$, where $V_1^{m'-o}$ and $V_1^{r'-o}$ are the profits of manufacturer and retailer under the original channel power structure o, and $V_1^{m'-n}$ and $V_1^{r'-n}$ are the profits of manufacturer and retailer under the new channel power structure n. Let $o \to n$ denote the change of the channel power structure, where $o,\ n \in \{MS, RS, VN\}$ and $n \neq o$. η^m and η^r at different wholesale prices for all possibilities of $o \to n$ are shown in Table 3.9.

Table 3.9
The magnitude of the profit impact of the channel power structure at different wholesale prices

		$(\eta^m,\ \eta^r)$ (%)	
w	MS $\to$ RS	MS $\to$ VN	RS $\to$ VN
10	(1.98, -1.69)	(4.44, 2.14)	(2.42, 3.9)
12	(0.42, -2.28)	(1.35, 1.21)	(0.93, 3.57)
14	(-0.43, -3.22)	(-0.2, -0.25)	(0.23, 3.07)
15	(-0.71, -3.93)	(-0.69, -1.34)	(0.03, 2.7)
16	(-0.97, -4.98)	(-1.09, -2.91)	(-0.12, 2.17)
18	(-1.94, -27.4)	(-3.78, -9.63)	(-1.88, 24.46)

Table 3.9 shows that: (i) when the most profitable channel power structure is VN, the profit potential or loss to retailer is lower than that to manufacturer if the

channel power structure changes from MS to VN or from VN to MS, while the profit potential or loss to retailer is greater than that to manufacturer if the channel power structure changes from RS to VN or from VN to RS, (ii) when the most profitable channel power structure is MS, the profit potential or loss to retailer is higher than that to manufacturer for any change of the channel power structure, (iii) the impact degree of the change of the channel power structure on manufacturer's profit first declines and then increases with the increase in the wholesale price, and (iv) the impact degree of the change of the channel power structure on retailer's profit first declines and then increases as the wholesale price increases if VN is involved in the change, otherwise the impact degree of the change of the channel power structure on retailer's profit increases as the wholesale price increases.

From the numerical analysis we find that, in a two-period dual-channel supply chain, if the wholesale price is mainly decided by the highly competitive market and can be hardly influenced by manufacturer and retailer, the Vertical Nash power structure is the best for manufacturer and retailer when the wholesale price is low, while the Manufacturer Stackelberg power structure is the best for manufacturer and retailer when the wholesale price is high. If the exogenous wholesale price is not completely determined by the market and can be decided by the negotiation between manufacturer and retailer in advance, although retailer cannot benefit from its dominant power with a given wholesale price, it can lower the wholesale price with its dominant power to get more profit. As for manufacturer, it should consider using its dominant power to achieve a higher wholesale price and then get the highest profit under its dominant channel power structure.

3.6 Conclusions

This chapter investigates the effects of the channel power structure on the joint dynamic pricing and inventory management of a dual-channel supply chain under demand uncertainty.

We find that, the structural properties of the optimal joint dynamic pricing and inventory control policy of a manufacturer-retailer dual-channel supply chain are not affected by the channel power structure. This means, under the Retailer Stackelberg or Vertical Nash power structure, the optimal policy is also a base-stock-list-price type. Moreover, the structural properties of the optimal joint dynamic pricing and inventory control policy of a dual-channel supply chain with two competing retailers are also not affected by the channel power structure.

We also find that when manufacturer's initial inventory level is below its base stock level, there is no need for retailer to know the exact value of manufacturer's initial inventory level to make its inventory decision. However, manufacturer should always pay attention to retailer's initial inventory level and take it into account to make its inventory decision. By clarifying the influence rules of starting inventory levels on reduced prices and base stock levels, we find that the influence rules on reduced prices under different channel power structures are the same, while the influence rules on base stock levels under different channel power structures are partially different.

Through numerical analysis, we can obtain some valuable managerial insights for the dual-channel supply chain where decisions are made dynamically in two periods. For the dual-channel supply chain where the products' wholesale prices are not dominated by the market, such as the dual-channel supply chain consisting of Apple and its retailers, it is important for both manufacturer and retailer to consider enhancing their bargaining power on the wholesale price and then get more profits. For the dual-channel supply chain where the products' wholesale prices are dominated by the competitive market, such as the dual-channel supply chain consisting of Walmart and some of its suppliers, members benefit from the balanced channel power structure when the exogenous wholesale price is low. And when the exogenous wholesale price is high, the most benefitable channel power structure for them is the manufacturer dominant channel power structure.

3.7 Appendices

Proof of Lemma 3.1.

Lemma 3 can be proved in a similar way to Lemma 2.1.

Proof of Theorem 3.1.

Theorem 3.1 can be proved in a similar way to Theorem 2.1.

Proof of Proposition 3.1.

We first obtain $d_t^{m-r\prime}(y_t^m)$ with $\frac{\partial J_t^{m-r}(d_t^m,y_t^m)}{\partial d_t^m}=0$, where $\frac{\partial J_t^{m-r}(d_t^m,y_t^m)}{\partial d_t^m}=\frac{\theta a+\beta_1 p_t^r-2d_t^m}{\alpha_1}-\rho c+\frac{\partial \mathbf{E}[h^m(y_t^m-d_t^m-\epsilon_t^m)]}{\partial(y_t^m-d_t^m)}-\rho\frac{\partial \mathbf{E}[V_{t+1}^{m-r}(y_t^m-d_t^m-\epsilon_t^m)]}{\partial(y_t^m-d_t^m)}$. Then, with $d_t^{m-r\prime}(y_t^m)$, we can get that $\frac{\partial J_t^{m-r}(d_t^{m-r\prime}(y_t^m),y_t^m)}{\partial y_t^m}=-(1-\rho)c-\frac{\partial \mathbf{E}[h^m(y_t^m-d_t^{m-r\prime}(y_t^m)-\epsilon_t^m)]}{\partial(y_t^m-d_t^{m-r\prime}(y_t^m))}+\rho\frac{\partial \mathbf{E}[V_{t+1}^{m-r}(y_t^m-d_t^{m-r\prime}(y_t^m)-\epsilon_t^m)]}{\partial(y_t^m-d_t^{m-r\prime}(y_t^m))}$ and $\frac{\partial^2 J_t^{m-r}(d_t^{m-r\prime}(y_t^m),y_t^m)}{\partial^2 y_t^m}=[-\frac{\partial^2 \mathbf{E}[h^m(y_t^m-d_t^{m-r\prime}(y_t^m)-\epsilon_t^m)]}{\partial^2(y_t^m-d_t^{m-r\prime}(y_t^m))}+\rho\frac{\partial^2 \mathbf{E}[V_{t+1}^{m-r}(y_t^m-d_t^{m-r\prime}(y_t^m)-\epsilon_t^m)]}{\partial^2(y_t^m-d_t^{m-r\prime}(y_t^m))}](1-\frac{\partial d_t^{m-r\prime}(y_t^m)}{\partial y_t^m})$. Since $\mathbf{E}[h^m(y_t^m-d_t^m-\epsilon_t^m)]$ is strictly convex and $\mathbf{E}[V_{t+1}^{m-r}(y_t^m-d_t^m-\epsilon_t^m)]$is concave, the coefficient before $y_t^m-d_t^m$ in $\frac{\partial \mathbf{E}[h^m(y_t^m-d_t^m-\epsilon_t^m)]}{\partial(y_t^m-d_t^m)}-\rho\frac{\partial \mathbf{E}[V_{t+1}^{m-r}(y_t^m-d_t^m-\epsilon_t^m)]}{\partial(y_t^m-d_t^m)}$ is greater than zero. Then, we get $0<\frac{\partial d_t^{m-r\prime}(y_t^m)}{\partial y_t^m}<1$ and $\frac{\partial^2 J_t^{m-r}(d_t^{m-r\prime}(y_t^m),y_t^m)}{\partial^2 y_t^m}<0$. Therefore, $y_t^{m-r\prime}$ can be derived with $\frac{\partial J_t^{m-r}(d_t^{m-r\prime}(y_t^m),y_t^m)}{\partial y_t^m}=0$.

If $I_t^m<y_t^{m-r\prime}+y_t^r-I_t^r$, we can get $\frac{\partial d_t^{m-r\prime}}{\partial p_t^r}=\frac{\beta_1}{2}$ by combining $\frac{\partial J_t^{m-r}(d_t^m,y_t^m)}{\partial d_t^m}$ and $\frac{\partial J_t^{m-r}(d_t^{m-r\prime}(y_t^m),y_t^m)}{\partial y_t^m}$. Moreover, it is obvious that $\frac{\partial y_t^{m-r\prime}}{\partial p_t^r}=\frac{\partial d_t^{m-r\prime}}{\partial p_t^r}=\frac{\beta_1}{2}$ in view of $\frac{\partial J_t^{m-r}(d_t^{m-r\prime}(y_t^m),y_t^m)}{\partial y_t^m}$. If $I_t^m\geq y_t^{m-r\prime}+y_t^r-I_t^r$, the optimal value of y_t^m is $I_t^m+I_t^r-y_t^r$. Since the coefficient before d_t^m in $\frac{\partial \mathbf{E}[h^m(y_t^m-d_t^m-\epsilon_t^m)]}{\partial(y_t^m-d_t^m)}-\rho\frac{\partial \mathbf{E}[V_{t+1}^{m-r}(y_t^m-d_t^m-\epsilon_t^m)]}{\partial(y_t^m-d_t^m)}$ is less than zero, we can get $0<\frac{\partial d_t^{m-r\prime}(I_t^m+I_t^r-y_t^r)}{\partial p_t^r}<\frac{\beta_1}{2}$ with $\frac{\partial J_t^{m-r}(d_t^m,y_t^m)}{\partial d_t^m}=0$. With $p_t^m(d_t^m)=\frac{\theta a+\beta_1 p_t^r-d_t^m}{\alpha_1}$, we can further get $\frac{\partial p_t^{m-r\prime}}{\partial p_t^r}=\frac{\beta_1}{2\alpha_1}$ and $\frac{\beta_1}{2\alpha_1}<\frac{\partial p_t^{m-r\prime}(I_t^m+I_t^r-y_t^r)}{\partial p_t^r}<\frac{\beta_1}{\alpha_1}$.

With $d_t^r=(1-\theta)a-\alpha_2 p_t^r+\beta_2 p_t^m$, we can get that $p_t^r(d_t^r)=\frac{(1-\theta)a+\beta_2 p_t^m-d_t^r}{\alpha_2}$, where $p_t^m=p_t^{m-r\prime}$ when $I_t^m<y_t^{m-r\prime}+y_t^r-I_t^r$ and $p_t^m=p_t^{m-r\prime}(I_t^m+I_t^r-y_t^r)$ when $I_t^m\geq y_t^{m-r\prime}+y_t^r-I_t^r$. Then, with $\frac{\partial p_t^{m-r\prime}}{\partial p_t^r}=\frac{\beta_1}{2\alpha_1}$ and $\frac{\beta_1}{2\alpha_1}<\frac{\partial p_t^{m-r\prime}(I_t^m+I_t^r-y_t^r)}{\partial p_t^r}<\frac{\beta_1}{\alpha_1}$, it is obvious that $\frac{\partial p_t^r(d_t^r)}{\partial d_t^r}\leq\frac{-2\alpha_1}{2\alpha_1\alpha_2-\beta_1\beta_2}$. With the assumption $\alpha_1>\beta_1$ and $\alpha_2>\beta_2$, we can get that $\frac{-2\alpha_1}{2\alpha_1\alpha_2-\beta_1\beta_2}<0$. Therefore, $p_t^r(d_t^r)$ is strictly decreasing d_t^r.

Proof of Lemma 3.2.

Lemma 3.2 can be proved by induction. It is obvious that V_{T+1}^{r-r} is concave in I_{T+1}^r. Now, we assume inductively that $V_{t+1}^{r-r}(I_{t+1}^r)$ is concave in I_{t+1}^r. In what follows, we should prove the result also holds for period t.

We first prove that $J_t^{r-r}(d_t^r,y_t^r)$ is jointly concave in (d_t^r,y_t^r). $J_t^{r-r}(d_t^r,y_t^r)$ is composed of: (1) $d_t^r(p_t^r(d_t^r)-\rho w)-y_t^r w(1-\rho)$; (2) $\mathbf{E}[h^r(y_t^r-d_t^r-\epsilon_t^r)]$; and (3) $\rho\mathbf{E}[V_{t+1}^{r-r}(y_t^r-d_t^r-\epsilon_t^r)]$. The Hessian matrix of part (1), denoted as $\mathbf{H}'$, is $\begin{bmatrix}2\frac{\partial p_t^r(d_t^r)}{\partial d_t^r} & 0\\ 0 & 0\end{bmatrix}$ when $I_t^m<y_t^{m-r\prime}+y_t^r-I_t^r$, and when $I_t^m\geq y_t^{m-r\prime}+y_t^r-I_t^r$, $\mathbf{H}'$ is $\begin{bmatrix}2\frac{\partial p_t^r(d_t^r)}{\partial d_t^r} & \frac{\beta_2}{\alpha_2}\frac{\partial p_t^{m-r\prime}(I_t^m+I_t^r-y_t^r)}{\partial y_t^r}\\ \frac{\beta_2}{\alpha_2}\frac{\partial p_t^{m-r\prime}(I_t^m+I_t^r-y_t^r)}{\partial y_t^r} & 0\end{bmatrix}$. Since $\frac{\partial p_t^r(d_t^r)}{\partial d_t^r}<0$ which is proved in Proposition 3.1, for any vector $\mathbf{x}$, we have $\mathbf{x^T H' x}\leq 0$. Hence, we can get that part

(1) is jointly concave in (d_t^r, y_t^r). Parts (2) and (3) are jointly concave in (y_t^r, d_t^r) can be proved in the same way as Lemma 2.1.

With $J_t^{r-r}(d_t^r, y_t^r)$ is jointly concave, concavity is preserved after maximization, and the feasible set is convex, $V_t^{r-r}(I_t^r)$ is concave in I_t^r. $V_t^{r-r}(I_t^r)$ is decreasing in I_t^r, because $J_t^{r-r}(d_t^r, y_t^r)$ is independent of I_t^r and a larger I_t^r leads to a more restrictive feasible domain and so a smaller maximum objective function value.

Proof of Theorem 3.2.

(a) Under the scenario where $I_t^m < y_t^{m-r\prime} + y_t^r - I_t^r$, $J_t^{r-r}(d_t^r, y_t^r) = d_t^r(p_t^r(d_t^r) - \rho w) - w y_t^r(1-\rho) - \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho\mathbf{E}[V_{t+1}^{r-r}(y_t^r - d_t^r - \epsilon_t^r)]$ and the constraint is $y_t^r - I_t^r \geq 0$. It is obvious that if $I_t^r < y_t^{r-r\star}$, the optimal solution is $(p_t^{r-r\star}, y_t^{r-r\star})$ which is independent of I_t^r. If $I_t^r \geq y_t^{r-r\star}$, $d_t^{r-r\star}(I_t^r) = \arg\max J_t^{r-r}(d_t^r, I_t^r)$, which is derived with $\frac{\partial J_t^{r-r}(d_t^r)}{\partial d_t^r} = 0$, where $\frac{\partial J_t^{r-r}(d_t^r)}{\partial d_t^r} = p_t^r(d_t^r) + d_t^r\frac{\partial p_t^r(d_t^r)}{\partial d_t^r} - \rho w + \frac{\partial \mathbf{E}[h^r(I_t^r - d_t^r - \epsilon_t^r)]}{\partial(I_t^r - d_t^r)} - \rho\frac{\partial \mathbf{E}[V_{t+1}^{r-r}(I_t^r - d_t^r - \epsilon_t^r)]}{\partial(I_t^r - d_t^r)}$. Since $\frac{\partial p_t^r(d_t^r)}{\partial d_t^r} = \frac{-2\alpha_1}{2\alpha_1\alpha_2 - \beta_1\beta_2} < 0$ and $-\mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho\mathbf{E}[V_{t+1}^{r-r}(y_t^r - d_t^r - \epsilon_t^r)]$ is concave in $y_t^r - d_t^r$, we can get that $\frac{\partial d_t^{r-r\star}(I_t^r)}{\partial I_t^r} > 0$ and $\frac{\partial p_t^{r-r\star}(I_t^r)}{\partial I_t^r} < 0$.

(b) Under the scenario where $I_t^m \geq y_t^{m-r\prime} + y_t^r - I_t^r$, $J_t^{r-r}(d_t^r, y_t^r) = d_t^r(p_t^r(d_t^r) - \rho w) - w y_t^r(1-\rho) - \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho\mathbf{E}[V_{t+1}^{r-r}(y_t^r - d_t^r - \epsilon_t^r)]$ and the constraint is $y_t^r - I_t^r \geq 0$. Obtain $d_t^{r-r(1)\star}(I_t^r, I_t^m)$ and $y_t^{r-r\star}(I_t^r, I_t^m)$ with $\frac{\partial J_t^{r-r}(d_t^r, y_t^r)}{\partial d_t^r} = 0$ and $\frac{\partial J_t^{r-r}(d_t^r, y_t^r)}{\partial y_t^r} = 0$, where $\frac{\partial J_t^{r-r}(d_t^r, y_t^r)}{\partial d_t^r} = p_t^r(d_t^r) + d_t^r\frac{\partial p_t^r(d_t^r)}{\partial d_t^r} - \rho w + \frac{\partial \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)]}{\partial(y_t^r - d_t^r)} - \rho\frac{\partial \mathbf{E}[V_{t+1}^{r-r}(y_t^r - d_t^r - \epsilon_t^r)]}{\partial(y_t^r - d_t^r)}$ and $\frac{\partial J_t^{r-r}(d_t^r, y_t^r)}{\partial y_t^r} = -(1-\rho)w - \frac{\partial \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)]}{\partial(y_t^r - d_t^r)} + \rho\frac{\partial \mathbf{E}[V_{t+1}^{r-r}(y_t^r - d_t^r - \epsilon_t^r)]}{\partial(y_t^r - d_t^r)}$. It is obvious that if $I_t^r < y_t^{r-r\star}(I_t^r, I_t^m)$, the optimal solution is $(p_t^{r-r(1)\star}(I_t^r, I_t^m), y_t^{r-r\star}$ $(I_t^r, I_t^m))$. With $p_t^r(d_t^r) = \frac{(1-\theta)\alpha_1 a + \theta\beta_2 a - \beta_2 d_t^{m-r\prime}(I_t^m + I_t^r - y_t^r) - \alpha_1 d_t^r}{\alpha_1\alpha_2 - \beta_1\beta_2}$ and $0 < \frac{\partial d_t^{m-r\prime}(I_t^m + I_t^r - y_t^r)}{\partial(I_t^m + I_t^r - y_t^r)} < 1$, we get $\frac{\partial d_t^{r-r(1)\star}(I_t^r, I_t^m)}{\partial(I_t^m + I_t^r - y_t^r)} = \frac{\partial y_t^{r-r\star}(I_t^r, I_t^m)}{\partial(I_t^m + I_t^r - y_t^r)} = \frac{-\beta_2}{2\alpha_1}\frac{\partial d_t^{m-r\prime}(I_t^m + I_t^r - y_t^r)}{\partial(I_t^m + I_t^r - y_t^r)} < 0$ and $\frac{\partial d_t^{r-r(1)\star}(I_t^r, I_t^m)}{\partial(I_t^m + I_t^r)} = \frac{\partial y_t^{r-r\star}(I_t^r, I_t^m)}{\partial(I_t^m + I_t^r)} < 0$. Then, with $p_t^r(d_t^r)$, we can further get that $\frac{\partial p_t^{r-r(1)\star}(I_t^r, I_t^m)}{\partial(I_t^m + I_t^r)} < 0$. If $I_t^r \geq y_t^{r-r\star}(I_t^r, I_t^m)$, $d_t^{r-r(2)\star}(I_t^r, I_t^m) = \arg\max J_t^{r-r}(d_t^r, I_t^r)$, which is derived with $\frac{\partial J_t^{r-r}(d_t^r)}{\partial d_t^r} = 0$, where $\frac{\partial J_t^{r-r}(d_t^r)}{\partial d_t^r} = p_t^r(d_t^r) + d_t^r\frac{\partial p_t^r(d_t^r)}{\partial d_t^r} - \rho w + \frac{\partial \mathbf{E}[h^r(I_t^r - d_t^r - \epsilon_t^r)]}{\partial(I_t^r - d_t^r)} - \rho\frac{\partial \mathbf{E}[V_{t+1}^{r-r}(I_t^r - d_t^r - \epsilon_t^r)]}{\partial(I_t^r - d_t^r)}$. Since $-\mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho\mathbf{E}[V_{t+1}^{r-r}(y_t^r - d_t^r - \epsilon_t^r)]$ is concave in $y_t^r - d_t^r$, we can get that $\frac{-\beta_2}{2\alpha_1}\frac{\partial d_t^{m-r\prime}(I_t^m)}{\partial I_t^m} < \frac{\partial d_t^{r-r(2)\star}(I_t^r, I_t^m)}{\partial I_t^m} < 0$ and $0 < \frac{\partial d_t^{r-r(2)\star}(I_t^r, I_t^m)}{\partial I_t^r}$. Then, with $p_t^r(d_t^r)$, we can further get that $\frac{\partial p_t^{r-r(2)\star}(I_t^r, I_t^m)}{\partial I_t^r} < 0$ and $\frac{\partial p_t^{r-r(2)\star}(I_t^r, I_t^m)}{\partial I_t^m} < 0$. Hence, Theorem 3.2 is completely proved.

Proof of Theorem 3.3.

Theorem 3.3 can be proved in a similar way to Theorem 2.3.

Proof of Theorem 3.4.

For any y_t^m, obtain $d_t^{m-v}(y_t^m)$ with $\frac{\partial J_t^{m-v}(d_t^m, y_t^m)}{\partial d_t^m} = 0$ where $\frac{\partial J_t^{m-v}(d_t^m, y_t^m)}{\partial d_t^m} = \frac{\theta\alpha_2 a + (1-\theta)\beta_1 a - \beta_1 d_t^r - 2\alpha_2 d_t^m}{\alpha_1} - \rho c + \frac{\partial \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)]}{\partial(y_t^m - d_t^m)} - \rho \frac{\partial \mathbf{E}[V_{t+1}^{m-v}(y_t^m - d_t^m - \epsilon_t^m)]}{\partial(y_t^m - d_t^m)}$. For any y_t^r, $d_t^{r-v}(y_t^r)$ is obtained with $\frac{\partial J_t^{r-v}(d_t^r, y_t^r)}{\partial d_t^r} = \frac{(1-\theta)\alpha_1 a + \theta\beta_2 a - \beta_2 d_t^m - 2\alpha_1 d_t^r}{\alpha_2} - \rho w + \frac{\partial \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)]}{\partial(y_t^r - d_t^r)} - \rho \frac{\partial \mathbf{E}[V_{t+1}^{r-v}(y_t^r - d_t^r - \epsilon_t^r)]}{\partial(y_t^r - d_t^r)}$. Moreover, $\frac{\partial J_t^{m-v}(y_t^m)}{\partial y_t^m} = -c(1-\rho) - \frac{\partial \mathbf{E}[h^m(y_t^m - d_t^{m-v}(y_t^m) - \epsilon_t^m)]}{\partial(y_t^m - d_t^{m-v}(y_t^m))} + \rho \frac{\partial \mathbf{E}[V_{t+1}^{m-v}(y_t^m - d_t^{m-v}(y_t^m) - \epsilon_t^m)]}{\partial(y_t^m - d_t^{m-v}(y_t^m))}$, $\frac{\partial J_t^{r-v}(y_t^r)}{\partial y_t^r} = \rho \frac{\partial \mathbf{E}[V_{t+1}^{r-v}(y_t^r - d_t^{r-v}(y_t^r) - \epsilon_t^r)]}{\partial(y_t^r - d_t^{r-v}(y_t^r))} - \frac{\partial \mathbf{E}[h^r(y_t^r - d_t^{r-v}(y_t^r) - \epsilon_t^r)]}{\partial(y_t^r - d_t^{r-v}(y_t^r))} - w(1-\rho)$. With $\frac{\partial J_t^{m-v}(d_t^m, y_t^m)}{\partial d_t^m} = 0$, $\frac{\partial J_t^{r-v}(d_t^r, y_t^r)}{\partial d_t^r} = 0$, $\frac{\partial J_t^{m-v}(y_t^m)}{\partial y_t^m} = 0$ and $\frac{\partial J_t^{r-v}(y_t^r)}{\partial y_t^r} = 0$, it is obvious that (a) is true.

Under the situation of (b), the optimal solution of (d_t^m, d_t^r) denoted as $(d_t^{m-v\star}, d_t^{r-v\star})$. Then, with $\frac{\partial J_t^{r-v}(d_t^r, y_t^r)}{\partial d_t^r} = 0$ and $\frac{\partial J_t^{r-v}(y_t^r)}{\partial y_t^r} = 0$, we can get that $\frac{\partial d_t^{r-v\star}}{\partial d_t^m} = \frac{-\beta_2}{2\alpha_1} < 0$ and $\frac{\partial y_t^{r-v\star}(I_t^r, I_t^m)}{\partial d_t^{r-v\star}} > 0$. With $\frac{\partial d_t^{r-v\star}}{\partial d_t^m} = \frac{-\beta_2}{2\alpha_1}$ and $\frac{\partial J_t^{m-v}(d_t^m, y_t^m)}{\partial d_t^m}$, we can get that $d_t^{m-v\star}$ is positively correlated with the optimal solution of y_t^m which is $I_t^m + I_t^r - y_t^{r-v\star}(I_t^r, I_t^m)$ under (b). Then, we can further get that $d_t^{r-v\star}$ and $y_t^{r-v\star}(I_t^r, I_t^m)$ are negatively correlated with I_t^r or I_t^m, and $d_t^{m-v\star}$ is positively correlated with I_t^r or I_t^m. With $\frac{\partial d_t^{r-v\star}}{\partial d_t^m} = \frac{-\beta_2}{2\alpha_1}$, $p_t^{m-v(1)\star}(I_t^r, I_t^m) = \frac{\theta\alpha_2 a + (1-\theta)\beta_1 a - \frac{2\alpha_1\alpha_2 - \beta_1\beta_2}{2\alpha_1} d_t^{m-v\star}}{\alpha_1}$ is negatively correlated with I_t^r or I_t^m. Similarly, $p_t^{r-v(1)\star}(I_t^r, I_t^m) = \frac{(1-\theta)\alpha_1 a + \theta\beta_2 a - \frac{\beta_2}{2} d_t^{m-v\star}}{\alpha_1\alpha_2 - \beta_1\beta_2}$. Since $\alpha_i > \beta_i$, $i = 1, 2$ and $d_t^{m-v\star}$ is positively correlated with I_t^r or I_t^m, we get that $p_t^{m-v(1)\star}(I_t^r, I_t^m)$ and $p_t^{r-v(1)\star}(I_t^r, I_t^m)$ are negatively correlated with I_t^r or I_t^m. With $\frac{\partial J_t^{m-v}(y_t^m)}{\partial y_t^m} = 0$, we can get that $y_t^{m-v\star}(I_t^r, I_t^m)$ is positively correlated with $d_t^{m-v\star}$. Hence, (b) is completely proved. Similarly, (c) and (d) can be proved.

Proof of Corollary 2.

If optimal pricing and inventory decisions under different power structures are the same, then problems $\max\limits_{d_t^m} d_t^m(p_t^m(d_t^m) - \rho c)$ and $\max\limits_{d_t^r} d_t^r(p_t^r(d_t^r) - \rho w)$ under different power structures must have the same optimal solutions, denoted as $(d_t^{m\star}, d_t^{r\star})$. By solving $\max\limits_{d_t^m} d_t^m(p_t^m(d_t^m) - \rho c)$ and $\max\limits_{d_t^r} d_t^r(p_t^r(d_t^r) - \rho w)$, we get that: under MS, $d_t^{m\star} = \frac{\beta_1(1-\theta)a + 2\alpha_2\theta a + \beta_1\alpha_2\rho w - \rho c(2\alpha_1\alpha_2 - \beta_1\beta_2)}{4\alpha_2}$, and $d_t^{r\star} = \frac{(1-\theta)a - \alpha_2\rho w}{2} + \frac{\beta_2}{2}\frac{2\alpha_2\theta a + \beta_1(1-\theta)a + \beta_1\alpha_2\rho w + \rho c(2\alpha_1\alpha_2 - \beta_1\beta_2)}{4\alpha_1\alpha_2 - 2\beta_1\beta_2}$; $d_t^{m\star} = \frac{\theta a - \alpha_1\rho c}{2} + \frac{\beta_1}{2}\frac{2\alpha_1(1-\theta)a + \beta_2\theta a + \beta_2\alpha_1\rho c + \rho w(2\alpha_1\alpha_2 - \beta_1\beta_2)}{4\alpha_1\alpha_2 - 2\beta_1\beta_2}$, and $d_t^{r\star} = \frac{\beta_2\theta a + 2\alpha_1(1-\theta)a + \beta_2\alpha_1\rho c - \rho w(2\alpha_1\alpha_2 - \beta_1\beta_2)}{4\alpha_1}$ under RS; $d_t^{m\star} = \frac{2\alpha_1(1-\theta)a + \beta_2\theta a + \beta_2\alpha_1\rho c + 2\alpha_1\alpha_2\rho w}{4\alpha_1\alpha_2 - \beta_1\beta_2}\frac{\beta_1}{2} + \frac{\theta a - \alpha_1\rho c}{2}$, and $d_t^{r\star} = \frac{\beta_2}{2}\frac{2\alpha_2\theta a + \beta_1(1-\theta)a + \beta_1\alpha_2\rho w + 2\alpha_1\alpha_2\rho c}{4\alpha_1\alpha_2 - \beta_1\beta_2} + \frac{(1-\theta)a - \alpha_2\rho w}{2}$ under VN. It is obvious that $(d_t^{m\star}, d_t^{r\star})$ under different power structures are different. Therefore, the optimal pricing and inventory decisions vary in different power structures.

Proof of Theorem 3.5

Theorem 3.5 can be proved in a similar way to Theorem 3.4.

Chapter 4

Comparison of Dynamic and Static Pricing Strategies in a Dual-channel Supply Chain with Inventory Control

4.1 Introduction

For retailers and manufacturers, the main motivation to practice dynamic pricing can be mitigating the imbalance between supply and demand caused by demand uncertainty (e.g., Dell changes prices to promote the sale of products whose inventory was building beyond prescribed levels), capturing the maximum of consumer surplus (e.g., in September 2000, Amazon charged different prices for the same DVD products based on customers' profiles and purchase histories), gaining competitive advantage in sale (e.g., Nojima, a big electronics retailer in Japan, was introducing dynamic pricing at its 182 stores across Japan in 2019 to compete with its rivals), and so on (Byrnes, 2003; Grewal et al., 2004; Yuma, 2019). Regardless of the motivation for dynamic pricing, one of the critical concerns for retailers and manufacturers is whether dynamic pricing performs better than traditional static pricing in terms of profitability. In this study, we concentrate on comparing the performance of dynamic pricing motivated by demand uncertainty with static pricing in the retail and manufacturing industries where inventory control is traditionally used to alleviate the adverse impact of the inherent uncertainty in demand on a firm's profitability.

Existing research on comparing dynamic and static pricing in a system with inventory control has focused on the situation where products are distributed by a single selling channel. Compared with the single-channel supply chain, since either of the channels can adopt dynamic or static pricing, four different pricing strategies exist in a dual-channel supply chain. That is, both the traditional retail and direct online channels adopt dynamic pricing strategy (DD strategy); the traditional retail channel adopts dynamic pricing strategy while the online direct channel adopts static pricing strategy (DS strategy); the traditional retail channel adopts static pricing strategy while the direct online channel adopts dynamic pricing strategy (SD strategy); and both the traditional retail and direct online channels adopt static pricing strategy (SS strategy). Moreover, horizontal price competition is introduced in the dual-channel supply chain which may affect the performance of dynamic or static pricing.

Motivated by the above observations, we have the following question: Does dynamic pricing perform better than static pricing in a dual-channel supply chain which is equipped with inventory control policy to deal with demand uncertainty? To answer this question, we consider a dual-channel supply chain where a manufacturer sells a single type of products with its own online channel and a traditional retail channel over a short selling season consisting of two periods. An important reason of studying a two-period dual-channel supply chain is that, in this era of rapidly changing technology, the life-cycles of products such as PCs, digital cameras, mobile phones, etc., have become shorter (Kuo and Huang, 2012; Maiti and Giri, 2017). Another important reason is for analytical tractability. In each period, demand at both channels is stochastic and sensitive to selling prices. The manufacturer and retailer play a Stackelberg game where the manufacturer acting as the leader decides the production quantity and online price, and the retailer acting as the follower decides the order quantity and retail price. As the optimal joint pricing and inventory control policy under DD strategy has been investigated in Chapter 2, in this chapter, models are developed with stochastic dynamic programming to derive the optimal joint pricing and inventory control policies under the left three pricing strategies (i.e., DS strategy, SD strategy, and SS strategy) with the goal of maximizing manufacturer and retailer's respective total expected discounted profits over two periods. The optimal pricing and inventory policies under different pricing strategies are analyzed and compared. Finally, the performance of dynamic pricing and static pricing is studied with numerical examples.

Our results provide several insights into the joint pricing and inventory management for members in a dual-channel supply chain. First, integrating dynamic pricing strategy into inventory management may underperform the traditional inventory control policy with static pricing strategy in coping with additive demand uncertainty. The performance of dynamic pricing strategy is affected by market parameters including the degree of demand uncertainty, market size, customers' channel preference, price sensitivity to demand in a channel, and the cross-channel price sensitivity. This illustrates that dual-channel supply chain members should think carefully about implementing dynamic pricing in dealing with demand uncertainty, and it is necessary to take market parameters into account when deciding whether to adopt dynamic pricing strategy. Specifically, (i) if the degree of demand uncertainty in the retail channel is low, it would be better for retailer to take dynamic or static pricing strategy and manufacturer to take static pricing strategy, otherwise it would be better for retailer to take dynamic pricing strategy and manufacturer to use static pricing strategy; (ii) if customers' preference for the direct channel is not particularly high, it would be better for retailer to take dynamic pricing strategy and manufacturer to adopt static pricing strategy when the market size is small, and it would be better for retailer to take dynamic pricing strategy and manufacturer to adopt dynamic or static pricing strategy when the market size is large; (iii) if customers' preference for the direct channel is particularly high, it would be better for both members to take the same dynamic or static pricing strategy when the market size is small, and it would be better for retailer to take dynamic pricing strategy and manufacturer to use dynamic or static pricing strategy when the market size is large; (iv) retailer should adopt dynamic pricing strategy if the price sensitivity to demand in a channel is low or the cross-channel price sensitivity is high, and manufacturer should adopt static pricing strategy if the price sensitivity to demand in a channel is high or the cross-channel price sensitivity is low. Second, if both channel members choose to take dynamic pricing strategy, one channel member should pay close attention to its inventory level as well as the changes in the selling price of the other channel member to decide whether to charge a list price or reduced price. This is caused by the horizontal price competition between the channel members. Moreover, no matter what pricing strategy the member adopts, the manufacturer should concern itself with the retailer's inventory information in addition to its own inventory information. However, the retailer only needs to focus on its own inventory information under the pricing strategies where the manufacturer takes static pricing strategy.

The rest of this chapter is organized as follows: literature review is presented in Section 4.2. In Section 4.3, we make notations for the joint pricing and inventory control problems of a dual-channel supply chain under different pricing strategies and establish models for these problems. Section 4.4 presents the optimal joint pricing and inventory control policies under different pricing strategies and the comparisons of these optimal policies. Numerical examples are carried out to illustrate the theoretical results and compare the performance of different pricing strategies in Section 4.5. Finally in Section 4.6, we summarize the main findings and point out future research directions.

4.2 Literature review

The related literature to our work mainly includes the following streams: (i) comparison of static pricing and dynamic pricing with inventory control; (ii) dynamic versus static pricing in a duopoly system or a dual-channel supply chain.

4.2.1 Comparison of static pricing and dynamic pricing with inventory control

Research on comparing inventory-based dynamic pricing strategy with static pricing strategy has received considerable attention. Some works (e.g., Chen et al., 2010; Li et al., 2015; Herbon and Khmelnitsky, 2017; Duan et al., 2018) have centered on the continuous-review systems where price can be adjusted and inventory can be replenished at any time or EOQ setting where firms are allowed to vary the selling prices continuously (Transchel and Minner, 2009; Chen et al., 2015; Stamatopoulos et al., 2019; Bray and Stamatopoulos, 2022).

Our research is related to the studies of the periodic-review systems where pricing and inventory decisions are made periodically. Federgruen and Heching (1999) is one of earlier works of studying the joint dynamic pricing and inventory control policy of a periodic-review system where the demand is stochastic and price sensitive and exploring the benefits of the proposed inventory-based dynamic pricing policy. Through an extensive numerical study, they find that a firm can benefit from implementing the inventory-based dynamic pricing policy, and the benefits increase as the degree of demand uncertainty increases. Recently, Gayon and Dallery (2007) consider a capacitated production system where the replenishment process is partially controlled. With the help of a numerical study, they demonstrate that dynamic pricing might be much more beneficial when the production is not totally controlled. Yin and Rajaram (2007) investigate the benefits of dynamic pricing in a system where demand fluctuates and depends on exogenous factors. Their results show that it is more beneficial to take dynamic pricing in a Markovian demand environment with a high fixed ordering cost or with high demand variability. Feng (2010) compares the performance of dynamic pricing with that of static pricing in the cases where supply is uncertain or limited, and finds that in these cases, firms may obtain significant profit improvements by using dynamic pricing. Yang and Zhang (2014) demonstrate that the value of dynamic pricing is significant for a firm which replenishes and sells a product under the scarcity effect of inventory. Bernstein et al. (2016) focus on a firm with a positive lead time and show that the value of inventory-based dynamic pricing increases with lead time and can be substantial when the lead time is long. Gong et al. (2022) find that dynamic pricing is very valuable for a firm constrained by a total minimum commitment contract with numerical studies when the committed quantity is moderate or large.

All of the above works consider the products are distributed with a single channel. In this work, we aim to explore the benefit of inventory-based dynamic pricing strategy in a dual-channel setting. A related problem was considered by Lamas and Chevalier (2018), who study the optimal dynamic pricing and inventory problem of two firms which compete with each other on selling prices and compare dynamic pricing with static pricing under the competitive environment. The relationship between the two firms corresponds to a Bertrand competition, and each firm is assumed to select prices from a discrete set in every period. Differs from the work of Lamas and Chevalier (2018), we focus on a manufacturer-retailer dual-channel system where supply-demand relationship and competition relationship coexist between the two firms. Considering, in many cases, the manufacturer is more powerful than the retailer in a dual-channel supply chain, we formulate the competition relationship between the manufacturer and retailer as Stackelberg game models where the manufacturer is the leader and the retailer is the follower.

4.2.2 Dynamic versus static pricing in a duopoly system or a dual-channel supply chain

In the second stream of research, the works mainly focus on a duopoly system where firms cannot replenish the product during the planning horizon (Xu and Hopp, 2006; Liu and Zhang, 2013; Sato and Sawaki, 2013; Sun et al., 2020) or a dual-channel supply chain where the demand is deterministic (Zhang et al., 2017; Wang and Sun, 2019).

For a duopoly system with fixed capacity, Xu and Hopp (2006) consider the customer arrival rate follows a geometric Brownian motion and find that, compared with static pricing, dynamic pricing is beneficial when competition is not too intense. In the recent works of this stream, Liu and Zhang (2013) consider dynamic versus pricing problem between two competing firms when customers are strategic. Their results show that, compared with dynamic pricing competition, a unilateral price commitment game in which one firm commits to static pricing and the other firm dynamically changes prices benefits both firms. Sato and Sawaki (2013) consider a firm adopts continuous-time dynamic pricing, and its competitor adopts static pricing. They find that dynamic pricing is not always effective. Sun et al. (2020) examine the performance of static and dynamic pricing strategies in the presence of social influence over two periods. Their results show that firms prefer dynamic pricing when the social influence is either relatively weak or sufficiently strong.

For a dual-channel supply chain, Zhang et al. (2017) consider the manufacturer sells products through an online retailer and an exogenous distribution channel under different pricing modes (static wholesale price and retail price, static wholesale price and dynamic retail price, dynamic wholesale price and retail price) and compare the performance of these pricing mode combinations with numerical studies. They find that the supply chain efficiency is the lowest when both members set static retail prices while the highest when only the online retailer charges a fixed price, and the manufacturer will be better off if it adjusts wholesale price dynamically. Wang and Sun (2019) compare the performance of dynamic wholesale pricing with static wholesale pricing in a dual-channel green supply chain where one manufacturer produces a green product and sells it through the traditional retail channel and its direct online channel. Their results show that the manufacturer prefers to adopt the dynamic wholesale pricing strategy in most cases and prefers the static one only when the consumers in both channels have relatively high energy efficiency perceptions. In the works of Zhang et al. (2017) and Wang and Sun (2019), the demand is deterministic, and the changes in pricing decisions are caused by the evolution of goodwill and the evolution of energy efficiency level, respectively. In this study, we consider the manufacturer and retailer face demand uncertainty which affects their decisions and profits, and they can replenish products during the planning horizon.

The main contributions of this chapter are summarized as follows. First, existing literature on the performance comparison of dynamic pricing and static pricing in a system where inventory control policy is designed to deal with demand uncertainty has mostly focused on the setting where products are distributed through a single channel. To the best of our knowledge, we are the first to compare the profit performance of dynamic pricing and static pricing in a two-stage system where products are distributed via the manufacturer's direct online channel as well as the traditional retail channel. Second, this chapter explores the effects of market parameters including market size, customers' channel preference, price sensitivities and demand variety on the results of performance comparison of dynamic pricing and static pricing in a multi-period dual-channel supply chain. Third, we analyze and compare the properties of optimal joint pricing and inventory policies for a multi-period dual-channel supply chain under different pricing strategies. By considering demand uncertainty and different pricing strategies in a multi-period setting, this study complements existing research on the joint pricing and inventory management where most of the works are conducted on single-period pricing and inventory models.

4.3 Models

The model description and assumptions under DD strategy are referred to Section 2.3 of Chapter 2. The difference between the model description and assumptions under the other three pricing strategies and those under DD strategy only lies in the point where the member who takes dynamic pricing strategy decides its selling price, and the member who takes static pricing strategy decides its fixed price only at the beginning of the planning horizon. Denote p_t^m as the dynamic online price, p_t^r as the dynamic retail price, p^m as the fixed online price and p^r as the fixed retail price, $t = 1, 2$.

Therefore, similar to the problem under DD strategy in Chapter 2, the problem under DS strategy where retailer takes dynamic pricing strategy while manufacturer takes static pricing strategy can be formulated as (4.1) and (4.2), the problem under DS strategy where retailer takes static pricing strategy while manufacturer takes dynamic pricing strategy can be formulated as (4.3) and (4.4), and the problem under DS strategy where both manufacturer and retailer take static pricing strategy can be formulated as (4.5) and (4.6).

$$V_t^{m-DS}(I_t^m) = \begin{cases} \max\limits_{(p^m, y_t^m) \ when \ t=1; \ y_t^m \ when \ t>1} & J_t^{m-DS}(p^m, y_t^m) \\ s.\ t. \quad y_t^m - I_t^m + y_t^r - I_t^r \geq 0 \end{cases} \tag{4.1}$$

where $J_t^{m-DS}(p^m, y_t^m) = d_t^m(p^m - \rho c) + (w - c)(y_t^r - I_t^r) - cy_t^m(1 - \rho) - \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] + \rho\mathbf{E}[V_{t+1}^{m-DS}(y_t^m - d_t^m - \epsilon_t^m)]$.

$$V_t^{r-DS}(I_t^r) = \begin{cases} \max\limits_{p_t^r, y_t^r} \quad J_t^{r-DS}(p_t^r, y_t^r) \\ s.\ t. \quad y_t^r - I_t^r \geq 0 \end{cases} \tag{4.2}$$

where $J_t^{r-DS}(p_t^r, y_t^r) = d_t^r(p_t^r - \rho w) - y_t^r w(1 - \rho) - \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho\mathbf{E}[V_{t+1}^{r-DS}(y_t^r - d_t^r - \epsilon_t^r)]$.

$$V_t^{m-SD}(I_t^m) = \begin{cases} \max\limits_{p_t^m, y_t^m} \quad J_t^{m-SD}(p_t^m, y_t^m) \\ s.\ t. \quad y_t^m - I_t^m + y_t^r - I_t^r \geq 0 \end{cases} \tag{4.3}$$

where $J_t^{m-SD}(p_t^m, y_t^m) = d_t^m(p_t^m - \rho c) + (w - c)(y_t^r - I_t^r) - cy_t^m(1 - \rho) - \mathbf{E}[h^m(y_t^m -$

$d_t^m - \epsilon_t^m)] + \rho \mathbf{E}[V_{t+1}^{m-DS}(y_t^m - d_t^m - \epsilon_t^m)]$.

$$V_t^{r-SD}(I_t^r) = \begin{cases} \max\limits_{(p^r, y_t^r) \text{ } when \text{ } t=1; \text{ } y_t^r \text{ } when \text{ } t>1} & J_t^{r-SD}(p^r, y_t^r) \\ s.\ t. \quad y_t^r - I_t^r > 0 \end{cases} \tag{4.4}$$

where $J_t^{r-SD}(p^r, y_t^r) = d_t^r(p^r - \rho w) - y_t^r w(1-\rho) - \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho \mathbf{E}[V_{t+1}^{r-SD}(y_t^r - d_t^r - \epsilon_t^r)]$.

$$V_t^{m-SS}(I_t^m) = \begin{cases} \max\limits_{(p^m, y_t^m) \text{ } when \text{ } t=1; \text{ } y_t^m \text{ } when \text{ } t>1} & J_t^{m-SS}(p^m, y_t^m) \\ s.\ t. \quad y_t^m - I_t^m + y_t^r - I_t^r \geq 0 \end{cases} \tag{4.5}$$

where $J_t^{m-SS}(p^m, y_t^m) = d^m(p^m - \rho c) + (w - c)(y_t^r - I_t^r) - c y_t^m(1-\rho) - \mathbf{E}[h^m(y_t^m - d^m - \epsilon_t^m)] + \rho \mathbf{E}[V_{t+1}^{m-SS}(y_t^m - d^m - \epsilon_t^m)]$.

$$V_t^{r-SS}(I_t^r) = \begin{cases} \max\limits_{(p^r, y_t^r) \text{ } when \text{ } t=1; \text{ } y_t^r \text{ } when \text{ } t>1} & J_t^{r-SS}(p^r, y_t^r) \\ s.\ t. \quad y_t^r - I_t^r \geq 0 \end{cases} \tag{4.6}$$

where $J_t^{r-SS}(p^r, y_t^r) = d^r(p^r - \rho w) - y_t^r w(1-\rho) - \mathbf{E}[h^r(y_t^r - d^r - \epsilon_t^r)] + \rho \mathbf{E}[V_{t+1}^{r-SS}(y_t^r - d^r - \epsilon_t^r)]$.

4.4 Analysis of the optimal joint pricing and inventory control policies under different pricing strategies

4.4.1 Analysis of the optimal joint pricing and inventory control policy under DS strategy

Since Lemma 2.1, Theorem 2.1 and Proposition 2.1 of the retailer's problem under DD strategy are true for any given manufacturer's decisions (p_t^m, y_t^m), they are undoubtedly true for the retailer's problem under DS strategy where $p_t^m = p^m$. This implies that no matter what pricing strategy the manufacturer takes, retailer's optimal response policy is a base-stock-list-price policy. Therefore, for any given (p^m, y_t^m), retailer's optimal response decisions can be defined as follows:

$(d_t^{r-DS\prime}, y_t^{r-DS\prime}) = \arg\max J_t^{r-DS}(d_t^r, y_t^r)$, $d_t^{r-DS\prime}(I_t^r) = \arg\max_{d_t^r} J_t^{r-DS}(d_t^r, I_t^r)$. The manufacturer's problem formulation under DS strategy, can be represented as the following model.

$$V_t^{m-DS}(I_t^m) = \begin{cases} \max\limits_{(d_t^m, y_t^m)\ when\ t=1;\ y_t^m\ when\ t>1} & J_t^{m-DS}(d_t^m, y_t^m) \\ s.\ t. \quad y_t^m - I_t^m + y_t^r - I_t^r \geq 0 \end{cases} \tag{4.7}$$

where $J_t^{m-DS}(d_t^m, y_t^m) = d_t^m(p^m(d_t^m) - \rho c) + (w-c)(y_t^r - I_t^r) - cy_t^m(1-\rho) - \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] + \rho\mathbf{E}[V_{t+1}^{m-DS}(y_t^m - d_t^m - \epsilon_t^m)]$, d_t^r and y_t^r are retailer's optimal response decisions. Specifically, d_t^m is a variable in each period t because $d_t^m = \theta a - \alpha_1 p^m + \beta p_t^r$ where p^m is decided in period 1 and (4.7) is solved with the sequence $t = 2, 1$.

Lemma 4.1. $J_t^{m-DS}(d_t^m, y_t^m)$ is jointly concave in (d_t^m, y_t^m).

Based on Lemma 4.1, the manufacturer's optimal pricing and inventory control policy under DS strategy exists and can be derived as follows.

Step 1: For $t = 2$, with any given p^m, define $y_2^{m-DS\prime} = \arg\max_{y_2^m} J_2^{m-DS}(d_2^m, y_2^m)$ where (d_2^r, y_2^r) is $(d_2^{r-DS\prime}, y_2^{r-DS\prime})$, and $y_2^{m-DS\prime}(I_2^r) = \arg\max_{y_2^m} J_2^{m-DS}(d_2^m, y_2^m)$ where (d_2^r, y_2^r) is $(d_2^{r-DS\prime}(I_2^r), I_2^r)$.

Step 2: For $t = 1$, since I_1^m and I_1^r are assumed to be zero, $y_1^m - I_1^m + y_1^{r-DS\prime} - I_1^r \geq 0$ and $y_1^{r-DS\prime} - I_1^r \geq 0$. Define $(d_1^{m-DS\star}, y_1^{m-DS\star}) = \arg\max J_1^{m-DS}(d_1^m, y_1^m)$ where (d_1^r, y_1^r) is $(d_1^{r-DS\prime}, y_1^{r-DS\prime})$.

Step 3: Get the optimal pricing decision $p^{m-DS\star}$ by substituting $d_1^{m-DS\star}$ into $p_1^m(d_1^m)$. Get $y_2^{m-DS\star}$ and $y_2^{m-DS\star}(I_2^r)$ by substituting $p^{m-DS\star}$ into $y_2^{m-DS\prime}$ and $y_2^{m-DS\prime}(I_2^r)$, respectively.

With the manufacturer's optimal decisions and the retailer's optimal response decisions, we can make the structural analysis of the dual-channel supply chain's optimal policy under DS strategy which is demonstrated in the following theorem.

Theorem 4.1. The dual-channel supply chain's optimal pricing and inventory control policy under DS strategy is:

(a) if $I_t^r < y_t^{r-DS\star}$ and $I_t^m < y_t^{m-DS\star} + y_t^{r-DS\star} - I_t^r$, the optimal decisions of the retailer and manufacturer are $(p_t^{r-DS\star}, y_t^{r-DS\star})$ and $(p^{m-DS\star}, y_t^{m-DS\star})$, where $y_t^{r-DS\star}$, $p_t^{r-DS\star}$, $y_t^{m-DS\star}$ and $p^{m-DS\star}$ are independent of I_t^m and I_t^r .

(b) if $I_t^r < y_t^{r-DS\star}$ and $I_t^m \geq y_t^{m-DS\star} + y_t^{r-DS\star} - I_t^r$, the optimal decisions of the retailer and manufacturer are $(p_t^{r-DS\star}, y_t^{r-DS\star})$ and $(p^{m-DS\star}, I_t^m + I_t^r - y_t^{r-DS\star})$.

(c) if $I_t^r \geq y_t^{r-DS\star}$ and $I_t^m < y_t^{m-DS\star}(I_t^r)$, the optimal decisions of the retailer and manufacturer are $(p_t^{r-DS\star}(I_t^r), I_t^r)$ and $(p^{m-DS\star}, y_t^{m-DS\star}(I_t^r))$, where $p_t^{r-DS\star}(I_t^r)$ and $y_t^{m-DS\star}(I_t^r)$ decrease as I_t^r increases.

(d) if $I_t^r \geq y_t^{r-DS\star}$ and $I_t^m \geq y_t^{m-DS\star}(I_t^r)$, the optimal decisions of the retailer and manufacturer are $(p_t^{r-DS\star}(I_t^r), I_t^r)$ and $(p^{m-DS\star}, I_t^m)$.

Theorem 4.1 indicates that, under DS strategy, one member should increase its inventory level to its base stock level through producing or placing an order if its initial inventory level is below its base stock level, otherwise it should keep its inventory at the initial level. Moreover, manufacturer's base stock level is affected by the retailer's initial inventory level, while retailer's base stock level is independent of its initial inventory level. Retailer should charge a list price if its initial inventory level is below its base stock level, otherwise it should reduce the retail price.

4.4.2 Analysis of the optimal joint pricing and inventory control policy under SD strategy

Under SD strategy, for any manufacturer's decisions (p_t^m, y_t^m), retailer's problem is solved to get the optimal response functions. Since d_t^r has an inverse function $p^r(d_t^r) = \frac{(1-\theta)a+\beta p_t^m - d_t^r}{\alpha_2}$ when p_t^m is given, the retailer's problem can be rewritten as the following model.

$$V_t^{r-SD}(I_t^r) = \begin{cases} \max\limits_{(d_t^r, y_t^r) \ when \ t=1; \ y_t^r \ when \ t>1} & J_t^{r-SD}(d_t^r, y_t^r) \\ s.\ t. \quad y_t^r - I_t^r \geq 0 \end{cases} \tag{4.8}$$

where $J_t^{r-SD}(d_t^r, y_t^r) = d_t^r(p^r(d_t^r) - \rho w) - y_t^r w(1-\rho) - \mathbf{E}[h^r(y_t^r - d_t^r - \epsilon_t^r)] + \rho\mathbf{E}[V_{t+1}^{r-SD}(y_t^r - d_t^r - \epsilon_t^r)]$. Specifically, d_t^r is a variable in each period t because $d_t^r = (1-\theta)a - \alpha_2 p^r + \beta p_t^m$ where p^r is decided in period 1 and (4.8) is solved with the sequence $t = 2, 1$.

Lemma 4.2. For any manufacturer's decisions (p_t^m, y_t^m), $J_t^{r-SD}(d_t^r, y_t^r)$ is jointly concave in (d_t^r, y_t^r).

Based on Lemma 4.2, the retailer's optimal response pricing and inventory decisions exist and can be obtained with the following steps.

Step 1: For $t = 2$, with any given p^r and (p_2^m, y_2^m), define $y_2^{r-SD\prime} = \arg\max\limits_{y_2^r} J_2^{r-SD}$ (d_2^r, y_2^r). The optimal response decision of y_2^r is $y_2^{r-SD\prime}$ if $I_2^r < y_2^{r-SD\prime}$, otherwise I_2^r

is optimal.

Step 2: For $t = 1$, since I_1^r is assumed to be zero, $y_1^r - I_1^r \geq 0$. With any given (p_1^m, y_1^m), define $(d_1^{r-SD\prime}, y_1^{r-SD\prime}) = \arg\max J_1^{r-SD}(d_1^r, y_1^r)$. Get retailer's optimal price response decision $p^{r-SD\prime}$ by substituting $d_1^{r-SD\prime}$ into $p^r(d_t^r)$.

Proposition 4.1. The mean demand d_t^m has an inverse function $p_t^m(d_t^m) = \frac{\theta a + \beta p^{r-SD\prime} - d_t^m}{\alpha_1}$, which is strictly decreasing.

Proposition 4.1 indicates that optimizing the price p_t^m in period t is equivalent to optimizing the mean demand d_t^m. Hence, the manufacturer's problem under SD strategy can be represented as (4.9).

$$V_t^{m-SD}(I_t^m) = \begin{cases} \max\limits_{d_t^m, y_t^m} \quad J_t^{m-SD}(d_t^m, y_t^m) \\ s.\ t. \quad y_t^m - I_t^m + y_t^r - I_t^r \geq 0 \end{cases} \tag{4.9}$$

where $J_t^{m-SD}(d_t^m, y_t^m) = d_t^m(p_t^m(d_t^m) - \rho c) + (w - c)(y_t^r - I_t^r) - c y_t^m(1-\rho) - \mathbf{E}[h^m(y_t^m - d_t^m - \epsilon_t^m)] + \rho \mathbf{E}[V_{t+1}^{m-SD}(y_t^m - d_t^m - \epsilon_t^m)]$, p^r and y_t^r are retailer's optimal response decisions.

Similar to DD strategy, the manufacturer's optimal decisions under SD strategy exist and can be obtained as follows. Under the scenario where $I_t^r < y_t^{r-SD\prime}$, define $(d_t^{m-SD(1)\star}, y_t^{m-SD(1)\star}) = \arg\max J_t^{m-SD}(d_t^m, y_t^m)$ and $d_t^{m-SD(1)\star}(I_t^r, I_t^m) = \arg\max\limits_{d_t^m} J_t^{m-SD}(d_t^m, I_t^m + I_t^r - y_t^r)$. Get $p_t^{m-SD(1)\star}$ and $p_t^{m-SD(1)\star}(I_t^r, I_t^m)$ by respectively substituting $d_t^{m-SD(1)\star}$ and $d_t^{m-SD(1)\star}(I_t^r, I_t^m)$ into $p_t^m(d_t^m)$. Under the scenario where $I_t^r \geq y_t^{r-SD\prime}$, define $(d_t^{m-SD(2)\star}, y_t^{m-SD(2)\star}) = \arg\max J_t^{m-SD}(d_t^m, y_t^m)$ and $d_t^{m-SD(2)\star}(I_t^m) = \arg\max J_t^{m-SD}(d_t^m, I_t^m)$. Get $p_t^{m-SD(2)\star}$ and $p_t^{m-SD(2)\star}(I_t^m)$ by respectively substituting $d_t^{m-SD(2)\star}$ and $d_t^{m-SD(2)\star}(I_t^m)$ into $p_t^m(d_t^m)$.

With the manufacturer's optimal decisions and the retailer's optimal response decisions, we can make the structural analysis of the dual-channel supply chain's optimal policy under SD strategy which is shown in Theorem 4.2.

Theorem 4.2. The dual-channel supply chain's optimal pricing and inventory control policy under SD strategy is:

(a) if $I_t^r < y_t^{r-SD(1)\star}$ and $I_t^m < y_t^{m-SD(1)\star} + y_t^{r-SD(1)\star} - I_t^r$, the optimal decisions of the retailer and manufacturer are $(p^{r-SD(1)\star}, y_t^{r-SD(1)\star})$ and $(p_t^{m-SD(1)\star}, y_t^{m-SD(1)\star})$, where $y_t^{r-SD(1)\star}$, $p^{r-SD(1)\star}$, $y_t^{m-SD(1)\star}$ and $p_t^{m-SD(1)\star}$ are independent of I_t^r and I_t^m.

(b) if $I_t^r < y_t^{r-SD(1)\star}(I_t^r, I_t^m)$ and $I_t^m \geq y_t^{m-SD(1)\star} + y_t^{r-SD(1)\star} - I_t^r$, the optimal decisions of the retailer and manufacturer are $(p^{r-SD(1)\star}, y_t^{r-SD(1)\star}(I_t^r, I_t^m))$ and $(p_t^{m-SD(1)\star}(I_t^r, I_t^m), I_t^m + I_t^r - y_t^{r-SD(1)\star}(I_t^r, I_t^m))$, where $y_t^{r-SD(1)\star}(I_t^r, I_t^m)$ and $p_t^{m-SD(1)\star}(I_t^r, I_t^m)$ decrease as either I_t^r or I_t^m increases.

(c) if $I_t^r \geq y_t^{r-SD(2)\star}$ and $I_t^m < y_t^{m-SD(2)\star}$, the optimal decisions of the retailer and manufacturer are $(p^{r-SD(1)\star}, I_t^r)$ and $(p_t^{m-SD(2)\star}, y_t^{m-SD(2)\star})$, where $p_t^{m-SD(2)\star}$ and $y_t^{m-SD(2)\star}$ are independent of I_t^r and I_t^m.

(d) if $I_t^r \geq y_t^{r-SD(2)\star}(I_t^m)$ and $I_t^m \geq y_t^{m-SD(2)\star}$, the optimal decisions of the retailer and manufacturer are $(p^{r-SD(1)\star}, I_t^r)$ and $(p_t^{m-SD(2)\star}(I_t^m), I_t^m)$, where $y_t^{r-SD(2)\star}(I_t^m)$ and $p_t^{m-SD(2)\star}(I_t^m)$ decrease as I_t^m increases.

Theorem 4.2 indicates that, under SD strategy, one member should increase its inventory level to its base stock level through producing or placing an order if its initial inventory level is below its base stock level, otherwise it should keep its inventory at the initial level. Moreover, manufacturer's base stock level and retailer's base stock level may be affected by members' initial inventory levels. Manufacturer should charge a list price if its initial inventory level is below its base stock level, otherwise it should reduce the online price.

4.4.3 Analysis of the optimal joint pricing and inventory control policy under SS strategy

Similar to SD strategy, with any given (p^m, y_t^m), retailer's optimal response decisions under SS strategy can be obtained as follows:

Step 1: For $t = 2$, with any given p^r and (p^m, y_2^m), define $y_2^{r-SS\prime} = \arg\max_{y_2^r} J_2^{r-SS}(d_2^r, y_2^r)$. The optimal response decision of y_2^r is $y_2^{r-SS\prime}$ if $I_2^r < y_2^{r-SS\prime}$, otherwise I_2^r is optimal.

Step 2: For $t = 1$, since I_1^r is assumed to be zero, $y_1^r - I_1^r \geq 0$. With any given (p^m, y_1^m), define $(d_1^{r-SS\prime}, y_1^{r-SS\prime}) = \arg\max J_1^{r-SS}(d_1^r, y_1^r)$. Get retailer's optimal price response decision $p^{r-SS\prime}$ by substituting $d_1^{r-SS\prime}$ into $p^r(d_t^r)$.

Obviously, d^m has an inverse function $p^m(d^m) = \frac{\theta a + \beta p^{r-SD\prime} - d^m}{\alpha_1}$, which is strictly decreasing. Therefore, the decision variable p^m of the manufacturer's problem under SS strategy can be replaced by d^m. Then, the manufacturer's optimal pricing and inventory decisions can be derived as follows. Define $y_2^{m-SS\star} = \arg\max_{y_2^m} J_2^{m-SS}(d^m, y_2^m)$. Define $(d_1^{m-SS\star}, y_1^{m-SS\star}) = \arg\max J_1^{m-SS}(d_m^r, y_m^r)$. Get manufacturer's optimal price decision $p^{m-SS\star}$ by substituting $d_1^{m-SS\star}$ into $p^m(d^m)$.

After substituting the manufacturer's optimal decisions into the retailer's optimal response decisions, we can make the structural analysis of the dual-channel supply chain's optimal policy under SS strategy.

Theorem 4.3. The dual-channel supply chain's optimal pricing and inventory control policy under SS strategy is:

(a) if $I_t^r < y_t^{r-SS\star}$ and $I_t^m < y_t^{m-SS\star} + y_t^{r-SS\star} - I_t^r$, the optimal decisions of the retailer and manufacturer are $(p^{r-SS\star}, y_t^{r-SS\star})$ and $(p^{m-SS\star}, y_t^{m-SS\star})$, where $y_t^{r-SS\star}$, $p^{r-SS\star}$, $y_t^{m-SS\star}$ and $p^{m-SS\star}$ are independent of I_t^r and I_t^m .

(b) if $I_t^r < y_t^{r-SS\star}$ and $I_t^m \geq y_t^{m-SS\star} + y_t^{r-SS\star} - I_t^r$, the optimal decisions of the retailer and manufacturer are $(p^{r-SS\star}, y_t^{r-SS\star})$ and $(p^{m-SS\star}, I_t^m + I_t^r - y_t^{r-SS\star})$.

(c) if $I_t^r \geq y_t^{r-SS\star}$ and $I_t^m < y_t^{m-SS\star}$, the optimal decisions of the retailer and manufacturer are $(p^{r-SS\star}, I_t^r)$ and $(p^{m-SS\star}, y_t^{m-SS\star})$.

(d) if $I_t^r \geq y_t^{r-SS\star}$ and $I_t^m \geq y_t^{m-SS\star}$, the optimal decisions of the retailer and manufacturer are $(p^{r-SS\star}, I_t^r)$ and $(p^{m-SS\star}, I_t^m)$.

Theorem 4.3 indicates that, under SS strategy, one member should increase its inventory level to its base stock level through producing or placing an order if its initial inventory level is below its base stock level, otherwise it should keep its inventory at the initial level. Moreover, manufacturer's base stock level is affected by the retailer's initial inventory level when the retailer's initial inventory level is below its base stock level.

4.4.4 Comparison study

By comparing and analyzing optimal pricing and inventory policies under different pricing strategies, we have the following corollaries.

Corollary 4.1. The structural properties of the optimal inventory policies under different pricing strategies are the same. Specifically, a member is optimal to produce or order up to its base stock level if its starting inventory level is below its base stock level, otherwise it should not produce or order.

Corollary 4.2. The structural properties of the optimal pricing policies under different pricing strategies vary.

In detail, under DD strategy, manufacturer and retailer are optimal to charge list prices if their initial inventory levels are both below their respective base stock levels, otherwise they should reduce the selling prices. This is caused by the price competition between the manufacturer and retailer. For example, if the manufacturer's initial inventory level is above its base stock level, it is optimal to lower its selling price to stimulate the demand and reduce the stock. Since customers are sensitive to the prices, if the retailer doesn't reduce the retail price, the reduced selling price of the manufacturer would less customers' demand for the products sold by the retailer, and as a result the retailer's profit would be reduced. Therefore, to compete with the manufacturer and counter the negative effect of the manufacturer's reduced price on its profit, the retailer would choose to reduce the retail price at the same time. Under the left pricing strategies, the member who adopts dynamic pricing is optimal to charge a list price if its initial inventory level is below its base stock level, otherwise it should reduce its selling price.

Corollary 4.2 indicates that in the manufacturer-retailer dual-channel system, if both of the channel members take dynamic pricing strategy, one channel member should pay close attention to its inventory level as well as the changes in the selling price of the other channel member to decide whether to charge a list price or reduced price.

Corollary 4.3. Under different pricing strategies, dynamic prices may be affected by members' initial inventory levels. The influence rules of initial inventory levels on dynamic prices are shown in Table 4.1.

Table 4.1
The influence rules of members' initial inventory levels on dynamic prices

Pricing strategy	Case a[1]	Case b	Case c	Case d
DD	$I_t^m(\circ), I_t^r(\circ)$[2]	$I_t^m(-), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$	$I_t^m(-), I_t^r(-)$
DS	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(\circ), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$
SD	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(-), I_t^r(-)$	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(-), I_t^r(\circ)$

[1] Case a is the situation where members' initial inventory levels, I_t^m and I_t^r, are both below their own base stock levels; Case b is the situation where I_t^r is below retailer's base stock level while I_t^m is above manufacturer's base stock level; Case c is the situation where I_t^r is above retailer's base stock level while I_t^m is below manufacturer's base stock level; Case d is the situation where I_t^m and I_t^r are both above members' own base stock levels. Representations of Cases a to d in Tables 4.2 and 4.3 are the same.

[2] $I_t^m(\circ)$ or $I_t^r(\circ)$ denotes dynamic price is independent of I_t^m or I_t^r. $I_t^m(-)$ or $I_t^r(-)$ denotes dynamic price is negatively correlated with I_t^m or I_t^r.

Corollary 4.3 shows that dynamic prices are independent of or negatively affected by members' initial inventory levels. For either of the members in a dual-channel supply chain who adopts dynamic pricing strategy, when its initial inventory level is above its base stock level, it should charge a reduced price by taking its initial inventory level into account. The higher the initial inventory level, the lower the selling price. This property is also possessed by the optimal joint pricing and inventory control policy of a system with a single distribution channel (Federgruen and Heching, 1999; Elmaghraby and Keskinocak, 2003; Chen and Simchi-Levi, 2012). It is interesting to find that in a dual-channel system, when both channel members adopt dynamic pricing strategy, higher initial inventory level of the manufacturer or retailer results in lower selling prices of the manufacturer and retailer. Moreover, when manufacturer takes dynamic pricing strategy, dynamic prices are negatively affected by the manufacturer's initial inventory level as well as the retailer's initial inventory level if the manufacturer's initial inventory level is above its base stock level and the retailer's initial inventory level is below its base stock level. This is caused by the supply-demand relationship between the manufacturer and the retailer.

Corollary 4.4. Under different pricing strategies, base stock levels may be affected by members' initial inventory levels. The influence rules of initial inventory levels on base stock levels are shown in Table 4.2 and Table 4.3.

Table 4.2
The influence rules of members' initial inventory levels on manufacturer's base stock level

Pricing strategy	Case a	Case b	Case c	Case d
DD	$I_t^m(\circ), I_t^r(-)$[1]	$I_t^m(\circ), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$
DS	$I_t^m(\circ), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$
SD	$I_t^m(\circ), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(\circ), I_t^r(\circ)$
SS	$I_t^m(\circ), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(\circ), I_t^r(\circ)$

[1] $I_t^m(\circ)$ or $I_t^r(\circ)$ denotes manufacturer's base stock level is independent of I_t^m or I_t^r. $I_t^m(-)$ or $I_t^r(-)$ denotes manufacturer's base stock level is negatively correlated with I_t^m or I_t^r.

Table 4.3
The influence rules of members' initial inventory levels on retailer's base stock level

Pricing strategy	Case a	Case b	Case c	Case d
DD	$I_t^m(\circ), I_t^r(\circ)$[1]	$I_t^m(-), I_t^r(-)$	$I_t^m(\circ), I_t^r(-)$	$I_t^m(-), I_t^r(-)$
DS	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(\circ), I_t^r(\circ)$
SD	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(-), I_t^r(-)$	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(-), I_t^r(\circ)$
SS	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(\circ), I_t^r(\circ)$	$I_t^m(\circ), I_t^r(\circ)$

[1] $I_t^m(\circ)$ or $I_t^r(\circ)$ denotes retailer's base stock level is independent of I_t^m or I_t^r. $I_t^m(-)$ or $I_t^r(-)$ denotes retailer's base stock level is negatively correlated with I_t^m or I_t^r.

Corollary 4.4 shows that under different pricing strategies, members' base stock levels may be independent of or negatively affected by their initial inventory levels. Particularly, manufacturer's base stock level is independent of its initial inventory level under different pricing strategies, and retailer's base stock level is independent of members' initial inventory levels when manufacturer takes static pricing. Moreover, the influence rules on manufacturer's base stock level are not affected by manufacturer's pricing strategy behavior.

Corollaries 4.3 and 4.4 also indicate that, under different pricing strategies, the information on retailer's initial inventory level is necessary for the manufacturer to obtain its base stock levels and make its production and pricing decisions. Under the pricing strategies where the manufacturer takes static pricing strategy, the information on manufacturer's initial inventory level is not required for the retailer to obtain its base stock levels as well as make the ordering and pricing decisions. Therefore, in the joint pricing and inventory management of the dual-channel system, the manufacturer should concern itself with the retailer's inventory information in addition to its own inventory information regardless of the pricing strategies. However, the retailer only needs to focus on its own inventory information under the pricing strategies where the manufacturer takes static pricing strategy.

4.5 Numerical studies

In this section, we use numerical experiments to illustrate the proposed theoretical results and compare members expected discounted profits under different pricing strategies. The initial values of parameters for the dual-channel supply chain are set as follows: $c = c_m = 10$, $w = c_r = 15$, $a = 200$, $\theta = 0.4$, $\alpha_1 = 4$, $\alpha_2 = 6$, $\beta = 2$, $\rho = 0.9$, $h^m(x) = h^r(x) = x^+ + 23x^-$, $\epsilon_t^m \in [-20, 20]$, $\epsilon_t^r \in [-20, 20]$.

We first analyze the optimal pricing and inventory decisions of period 2 under different pricing strategies to demonstrate the characterized structural properties of the optimal joint pricing and inventory polices of a dual-channel supply chain. Then, with the calculated policies of period 2, we calculate the optimal decisions of period 1 and members' expected discounted profits under different pricing strategies to get some managerial insights. The optimal pricing and inventory policies of period 2 under different pricing strategies are shown in Tables 4.4 to 4.7.

Table 4.4
Optimal pricing and inventory policy of period 2 under DD strategy

	Case a [1]	Case b	Case c	Case d
p_2^m	21.36	$35.92 - 0.14(I_2^m + I_2^r)$	$22.61 - 0.03I_2^r$	$31.48 - 0.15I_2^m - 0.05I_2^r$
y_2^m	53.33	$1.14(I_2^m + I_2^r) - 66.75$	$60.21 - 0.11I_2^r$	I_2^m
p_2^r	21.06	$23.49 - 0.02(I_2^m + I_2^r)$	$26.99 - 0.12I_2^r$	$29.42 - 0.04I_2^m - 0.12I_2^r$
y_2^r	52.2	$66.75 - 0.14(I_2^m + I_2^r)$	I_2^r	I_2^r

[1] Case a: $I_2^r < 52.2$, $I_2^m < 105.53 - I_2^r$; Case b: $I_2^r < 66.75 - 0.14(I_2^m + I_2^r)$, $I_2^m \geq 105.53 - I_2^r$; Case c: $I_2^r \geq 53.45 - 0.03I_2^r$, $I_2^m < 60.21 - 0.11I_2^r$; Case d: $I_2^r \geq 62.32 - 0.15I_2^m - 0.05I_2^r$, $I_2^m \geq 60.21 - 0.11I_2^r$.

Table 4.5
Optimal pricing and inventory policy of period 2 for any p^m under DS strategy

	Case a[1]	Case b	Case c	Case d
y_2^m	$131.67 - 3.67p^m$	$I_2^m + I_2^r - p^m - 30.83$	$138.27 - 3.45p^m - 0.21I_2^r$	I_2^m
p_2^r	$17.5 + 0.17p^m$	$17.5 + 0.17p^m$	$20.8 + 0.27p^m - 0.11I_2^r$	$20.8 + 0.27p^m - 0.11I_2^r$
y_2^r	$30.83 + p^m$	$30.83 + p^m$	I_2^r	I_2^r

[1] Case a: $I_2^r < 30.83 + p^m$, $I_2^m < 162.5 - 2.67p^m - I_2^r$; Case b: $I_2^r < 30.83 + p^m$, $I_2^m \geq 162.5 - 2.67p^m - I_2^r$; Case c: $I_2^r \geq 30.83 + p^m$, $I_2^m < 138.27 - 3.45p^m - 0.21I_2^r$; Case d: $I_2^r \geq 30.83 + p^m$, $I_2^m \geq 138.27 - 3.45p^m - 0.21I_2^r$.

Table 4.6
Optimal pricing and inventory policy of period 2 for any p^r under SD strategy

	Case a[1]	Case b	Case c	Case d
p_2^m	$16.25 + 0.25p^r$	$39.33 - 0.12p^r - 0.27(I_2^m + I_2^r)$	$15 + 0.25p^r$	$20 + 0.39p^r - 0.14I_2^m$
y_2^m	$31.67 + p^r$	$1.23(I_2^m + I_2^r) + 6.54p^r - 214.49$	$36.67 + p^r$	I_2^m
y_2^r	$168.33 - 5.5p^r$	$214.49 - 6.54p^r - 0.23(I_2^m + I_2^r)$	d I_2^r	I_2^r

[1] Case a: $I_2^r < 168.33 - 5.5p^r$, $I_2^m < 200 - 4.5p^r - I_2^r$; Case b: $I_2^r < 214.49 - 6.54p^r - 0.23(I_2^m + I_2^r)$, $I_2^m \geq 200 - 4.5p^r - I_2^r$; Case c: $I_2^r \geq 165.83 - 5.5p^r$, $I_2^m < 36.67 + p^r$; Case d: $I_2^r \geq 175.83 - 5.23p^r - 0.27I_2^m$, $I_2^m \geq 36.67 + p^r$.

Tables 4.4 to 4.7 further identify that: (i) under DD strategy, higher members' initial inventory levels would lead to lower selling prices of dual channels when manufacturer's initial inventory level is above its base stock level, (ii) under DS strategy, higher retailer's initial inventory level would cause lower retail price when retailer's initial inventory level is above its base stock level, and (iii) under SD strategy, higher manufacturer's initial inventory level would lead to lower online price when manufacturer's initial inventory level is above its base stock level.

Table 4.7
Optimal inventory decisions of period 2 under SS strategy

	Case a [1]	Case b	Case c	Case d
y_2^m	$96.67 - 4p^m + 2p^r$	$I_2^m + I_2^r + 6p^r - 2p^m - 135.83$	$96.67 - 4p^m + 2p^r$	I_2^m
y_2^r	$135.83 - 6p^r + 2p^m$	$135.83 - 6p^r + 2p^m$	I_2^r	I_2^r

[1] Case a: $I_2^r < 135.83 - 6p^r + 2p^m$, $I_2^m < 232.5 - 4p^r - 2p^m - I_2^r$; Case b: $I_2^r < 135.83 - 6p^r + 2p^m$, $I_2^m \geq 232.5 - 4p^r - 2p^m - I_2^r$; Case c: $I_2^r \geq 135.83 - 6p^r + 2p^m$, $I_2^m < 96.67 - 4p^m + 2p^r$; Case d: $I_2^r \geq 135.83 - 6p^r + 2p^m$, $I_2^m \geq 96.67 - 4p^m + 2p^r$.

The optimal prices and inventory decisions of period 1 and members' expected discounted profits under different pricing strategies are shown in Table 4.8. From Table 4.8, we can find that, for the four different pricing strategies, members' optimal decisions of period 2 are under case a when $\epsilon_t^m \in [-20, 20]$ and $\epsilon_t^r \in [-20, 20]$. Moreover, manufacturer's expected discounted profits under different pricing strategies have the following order $E[V_1^{m-DD}] = E[V_1^{m-DS}] = E[V_1^{m-SS}] > E[V_1^{m-SD}]$, and retailer's expected discounted profits under different pricing strategies have the following order $E[V_1^{r-SD}] > E[V_1^{r-DD}] = E[V_1^{r-SS}] = E[V_1^{r-SD}]$.

Table 4.8
Optimal prices, inventory decisions and expected discounted profits under different pricing strategies

	p_2^m	y_2^m	p_2^r	y_2^r	p_1^m	y_1^m	p_1^r	y_1^r	$E[V_1^m]$	$E[V_1^r]$
DD	21.36	53.33	21.06	52.2	21.36	53.33	21.06	52.2	1146.62	333.63
DS	21.36	53.33	21.06	52.2	21.36	53.33	21.06	52.2	1146.62	333.63
SD	21.55	52.87	21.2	51.73	21.4	53.48	21.2	51.42	1144.86	335.9
SS	21.36	53.33	21.06	52.2	21.36	53.33	21.06	52.2	1146.62	333.63

To better understand the relationship of members' expected discounted profits under different pricing strategies, we further calculate members' expected discounted profits with different values of market parameters including demand uncertainties (ϵ_t^m and ϵ_t^r), market size a, customers' preference for the direct online channel θ, and price sensitivities (α_1, α_2, and β). The effects of demand uncertainties, market size and channel preference, and price sensitivities on the relationship of members' profits under different pricing strategies are displayed in Section 4.5.1, Section 4.5.2, and Section 4.5.3 respectively.

4.5.1 Effects of demand varieties

We calculate members' expected discounted profits with different demand varieties including $\epsilon_t^m \in [-40, 40]$ and $\epsilon_t^r \in [-20, 20]$; $\epsilon_t^m \in [-20, 20]$ and $\epsilon_t^r \in [-40, 40]$; and $\epsilon_t^m \in [-40, 40]$ and $\epsilon_t^r \in [-40, 40]$. The calculated results of members' expected discounted profits with different demand varieties are shown in Table 4.9.

Table 4.9
Members' expected discounted profits with different uncertainty degrees of the demand

	$\epsilon_t^m \in [-40, 40]$, $\epsilon_t^r \in [-20, 20]$		$\epsilon_t^m \in [-20, 20]$, $\epsilon_t^r \in [-40, 40]$		$\epsilon_t^m, \epsilon_t^r \in [-40, 40]$	
	$E[V_1^m]$	$E[V_1^r]$	$E[V_1^m]$	$E[V_1^r]$	$E[V_1^m]$	$E[V_1^r]$
DD	1076.95	333.63	1225.88	248.27	1154.55	248.27
DS	1076.95	333.63	1226	248.3	1155.07	243.65
SD	1075.19	335.9	1224.93	250.08	1153.03	250.25
SS	1076.95	333.63	1226.18	248.11	1155.26	243.48

Tables 4.8 and 4.9 show that, the relationship of members' profits under different pricing strategies is affected by the degree of demand uncertainty. Moreover, for the situations of $\epsilon_t^r \in [-20, 20]$, if the manufacturer takes dynamic pricing strategy (DD or SD), it is optimal for the retailer to choose static pricing strategy (SD) because of $E[V_1^{r-SD}] > E[V_1^{r-DD}]$, and if the manufacturer uses static pricing strategy (DS or SS), the retailer's optimal strategy is dynamic or static pricing strategy (DS or SS) because of $E[V_1^{r-SS}] = E[V_1^{r-DS}]$. As for the manufacturer, if the retailer uses dynamic pricing strategy (DD or DS), it is optimal to choose dynamic or static pricing strategy (DD or DS) because of $E[V_1^{m-DD}] = E[V_1^{m-DS}]$, and if the retailer takes static pricing strategy (SD or SS), its optimal strategy is static pricing strategy (SS) because of $E[V_1^{m-SS}] > E[V_1^{m-SD}]$. For the situations of $\epsilon_t^r \in [-40, 40]$, if the manufacturer takes dynamic pricing strategy (DD or SD), it is optimal for the retailer to choose static pricing strategy (SD) because of $E[V_1^{r-SD}] > E[V_1^{r-DD}]$, and if the manufacturer uses static pricing strategy (DS or SS), the retailer's optimal strategy is dynamic pricing strategy (DS) because of $E[V_1^{r-DS}] > E[V_1^{r-SS}]$. As for the manufacturer, if the retailer takes dynamic pricing strategy (DD or DS), it is optimal to choose static pricing strategy (DS) because of $E[V_1^{m-DS}] > E[V_1^{m-DD}]$, and if the retailer takes static pricing strategy (SD or SS), it is optimal to choose static pricing strategy (SS) because of $E[V_1^{m-SS}] > E[V_1^{m-SD}]$. Considering manufacturer and retailer are rational and independent in deciding which pricing strategy to take, the

strategic interaction of the manufacturer and retailer in choosing pricing strategy is a two-player strategic game where the manufacturer and retailer are the players, {dynamic pricing strategy, static pricing strategy} is the set of pure strategies of the manufacturer or retailer, and the expected discounted profits under four different pricing strategies (DD, DS, SD and SS) are the players' payoffs. Therefore, from the above interpretation of results in Tables 4.8 and 4.9, we can find that, DS strategy and SS strategy are the Nash equilibriums of the two-player strategic game when $\epsilon_t^r \in [-20, 20]$, and DS strategy is the Nash equilibrium of the two-player strategic game when $\epsilon_t^r \in [-40, 40]$. This implies that DS or SS strategy is a Nash equilibrium for the dual-channel supply chain when the degree of demand uncertainty in the retail channel is low, and DS strategy is a Nash equilibrium for the dual-channel supply chain when the degree of demand uncertainty in the retail channel is high.

We can also find, if the manufacturer has the power to control the pricing strategy choice of the other member and only focuses on its own performances under different pricing strategies, it will think SS strategy is the best while SD is the worst. And if the retailer has the power to control the pricing strategy choice of the other member and only focuses on its own performances under different pricing strategies, it will think SD strategy is the best while SS is the worst. Therefore, if the manufacturer and retailer are self-interested and dependent in choosing pricing strategy, the manufacturer would prefer static pricing strategy and hope the retailer also takes static pricing strategy, and the retailer would prefer static pricing strategy and hope the manufacturer takes dynamic pricing strategy. This causes confliction between the manufacturer and retailer. To solve this confliction, the manufacturer can take a portion of its profit to compensate the retailer for choosing static pricing strategy if the relative benefit for manufacturer from SS is higher than the relative benefit for retailer from SD, otherwise the retailer can take a portion of its profit to compensate the manufacturer for choosing dynamic pricing strategy.

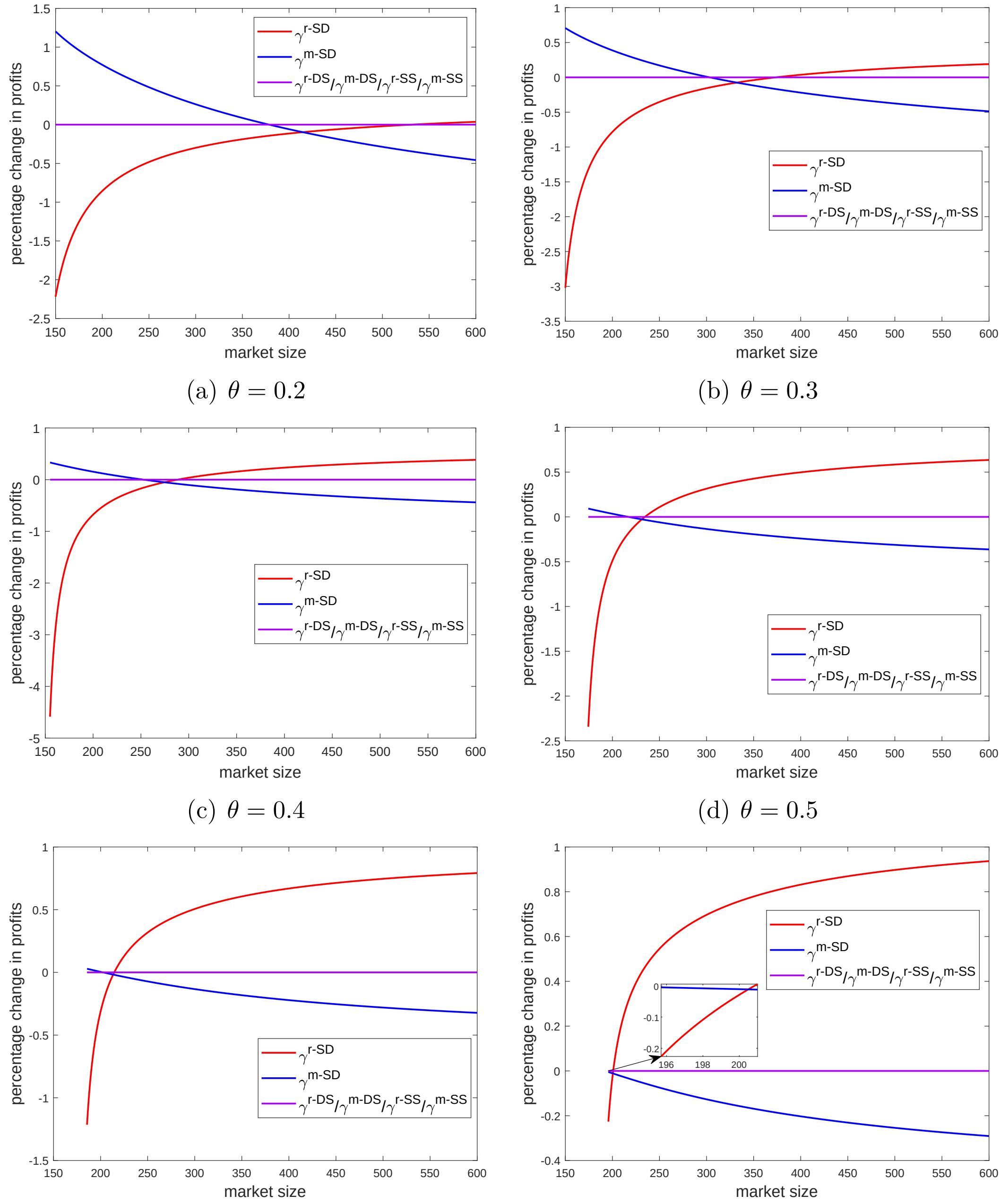
4.5.2 Effects of market size and channel preference

In this subsection, for convenience, we use percentage change in members' profits from one pricing strategy to another pricing strategy to quantify the relationship of members' profits under different pricing strategies. We let $\gamma^{r-x} =$

$\frac{E[V_1^{r-DD}]-E[V_1^{r-x}]}{E[V_1^{r-x}]} \times 100\%$ and $\gamma^{m-x} = \frac{E[V_1^{m-DD}]-E[V_1^{m-x}]}{E[V_1^{m-x}]} \times 100\%$ denote percentage change in retailer's profit and manufacturer's profit from x strategy to DD strategy, where $x \in \{DS, SD, SS\}$; let $\eta^{r-x} = \frac{E[V_1^{r-DS}]-E[V_1^{r-x}]}{E[V_1^{r-x}]} \times 100\%$ and $\eta^{m-x} = \frac{E[V_1^{m-DS}]-E[V_1^{m-x}]}{E[V_1^{m-x}]} \times 100\%$ denote the percentage change in retailer's profit and manufacturer's profit from x strategy to DS strategy, where $x \in \{SD, SS\}$; and let $\lambda^{r-x} = \frac{E[V_1^{r-SD}]-E[V_1^{r-x}]}{E[V_1^{r-x}]} \times 100\%$ and $\lambda^{m-x} = \frac{E[V_1^{m-SD}]-E[V_1^{m-x}]}{E[V_1^{m-x}]} \times 100\%$ denote the percentage change in retailer's profit and manufacturer's profit from x strategy to SD strategy, where $x \in \{SS\}$.

Through calculating the optimal pricing and inventory decisions at different values of market size and channel preference, we find that, for any value of channel preference, when the market size is larger than a specified value, the optimal decisions of each period under different pricing strategies belong to case a where members should charge list prices and replenish inventories. Therefore, we first investigate the effects of market size and channel preference θ on percentage change in profits under case a, which are shown in Fig.4.1 and Fig.4.2.

The results shown in Fig.4.1 and Fig.4.2 reveal that, when members are optimal to charge list prices and replenish inventory in each period, DD, DS, and SS strategies have the same performance in terms of profitability. However, the performance of DD, DS, and SS strategies differs from that of SD strategy. Specifically, as shown in Fig.4.1(a)-(e), when customers' preference for the direct channel θ is relatively low, with the increase in market size, the percentage change in manufacturer's profit from SD strategy to DD strategy decreases from positive to negative values, while the percentage change in retailer's profit from SD strategy to DD strategy increases from negative to positive values. Moreover, there exists a situation where both of them are negative values. These imply that, when customers' preference for the direct channel is relatively low, as the market size increases, SD strategy first underperforms other strategies for manufacturer but outperforms other strategies for retailer, then it performs better than other strategies for both members, and lastly it outperforms other strategies for manufacturer but underperforms other strategies for retailer. Fig.4.1(f) shows that there exist situations where SD strategy first performs better than other strategies for both members, and then it outperforms other strategies for manufacturer but underperforms other strategies for retailer as



(a) $\theta = 0.2$ (b) $\theta = 0.3$

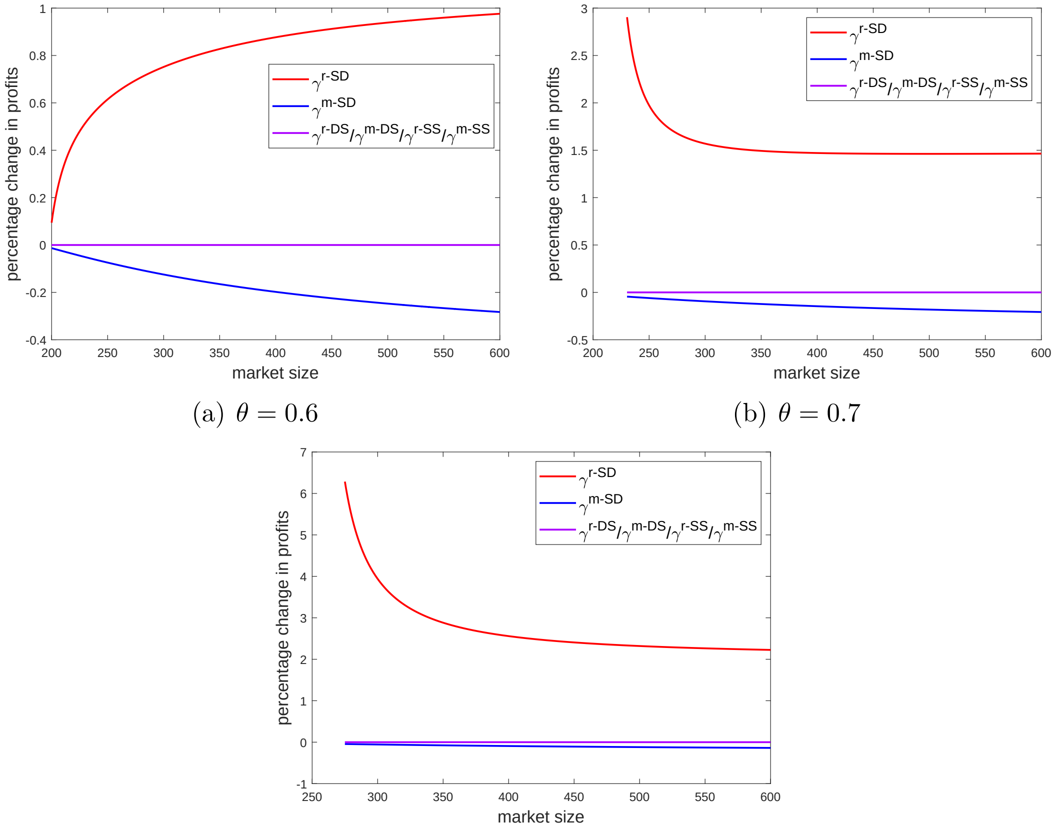
(c) $\theta = 0.4$ (d) $\theta = 0.5$

(e) $\theta = 0.55$ (f) $\theta = 0.59$

Fig. 4.1. Effects of market size and channel preference on percentage change in profits under case a

the market size increases. Fig.4.2(a)-(c) indicate that, when customers' preference for the direct channel is relatively high, for any market size under case a, SD strategy outperforms other strategies for manufacturer but underperforms other strategies for retailer.

Similar to the analysis in Section 4.5.1, from Fig.4.1 and Fig.4.2, we can also get that if customers' preference for the direct channel θ is low, DS or SS strategy is a Nash equilibrium for the dual-channel supply chain when the market size is small,



(a) $\theta = 0.6$ (b) $\theta = 0.7$

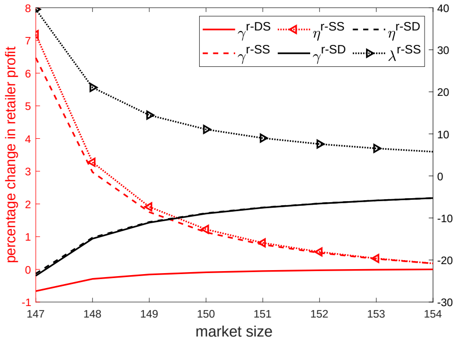
(c) $\theta = 0.8$

Fig. 4.2. Effects of market size and channel preference on percentage change in profits under case a

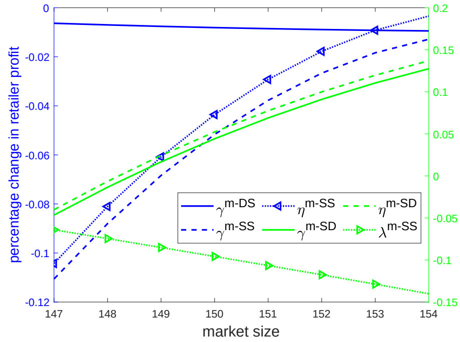
DS or DD strategy is a Nash equilibrium for the dual-channel supply chain when the market size is large, and SD strategy is the optimal strategy for the dual-channel supply chain within a certain range of market size. Moreover, DS or DD strategy is a Nash equilibrium for the dual-channel supply chain if customers' preference for the direct channel θ is high.

Next, we analyze the effects of market size and channel preference θ when market size is below the value that ensures pricing and inventory decisions under case a are optimal. After observing the patterns of percentage change in profits at different channel preferences θ in Fig.4.1 and Fig.4.2, we select four channel preferences ($\theta = 0.4$, $\theta = 0.59$, $\theta = 0.6$, and $\theta = 0.8$) as representative samples for analysis. Effects of market size and channel preference on percentage change in profits at the four different channel preferences are shown in Fig.4.3.

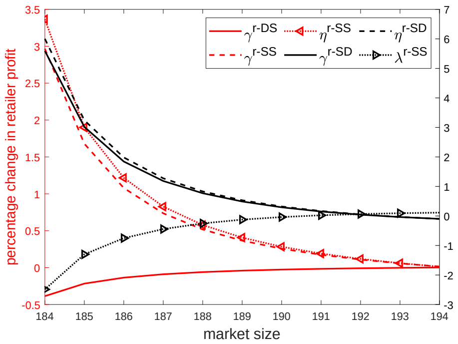
From Fig.4.3, we see that values of percentage change in profits from DD strategy to DS strategy (γ^{r-DS} and γ^{m-DS}) are close to zero, which is also indirectly reflected



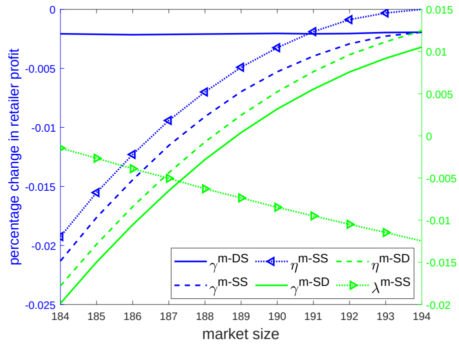
(a) Change in retailer profit with $\theta = 0.4$



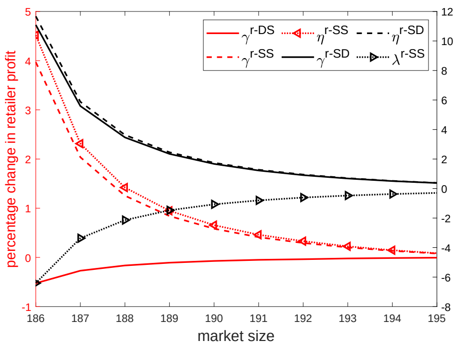
(b) Change in manufacturer profit with $\theta = 0.4$



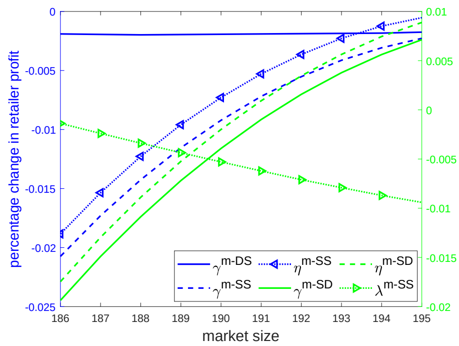
(c) Change in retailer profit with $\theta = 0.59$



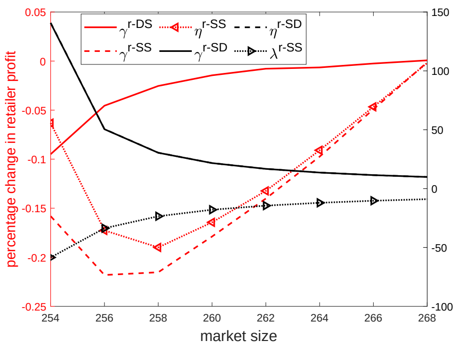
(d) Change in manufacturer profit with $\theta = 0.59$



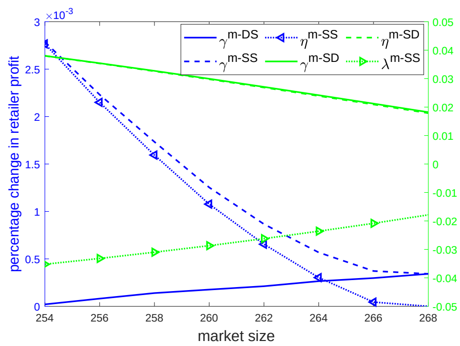
(e) Change in retailer profit with $\theta = 0.6$



(f) Change in manufacturer profit with $\theta = 0.6$



(g) Change in retailer profit with $\theta = 0.8$



(h) Change in manufacturer profit with $\theta = 0.8$

Fig. 4.3. Effects of market size and channel preference on percentage change in profits when market size is below the value that ensures decisions under case a are optimal

by the observation that the line of percentage change in profits from DD strategy to SS or SD strategy is relatively close to the line of percentage change in profits from DS strategy to SS or SD strategy (see, e.g., lines of γ^{r-SS} and η^{r-SS}). Besides, as shown in Fig.4.3, γ^{r-DS} is less than zero, and γ^{m-DS} is also less than zero except the case of $\theta = 0.8$. This indicates that the performance of DS strategy is nearly equal to or even better than that of DD strategy for manufacturer and retailer.

From Fig.4.3, we get the comparison results of members profits under different pricing strategies when market size is below the value that ensures pricing and inventory decisions under case a are optimal. For convince, the results are presented in Table 4.10, where $x < y$ denotes a member's expected discounted profit under x strategy is lower than that under y strategy, $x, y \in \{DD, DS, SD, SS\}$, and $U; V$ means the relationship of members' profits first is U and then becomes V as the market size increases. Specifically, (i) in the case of $\theta = 0.4$, SD strategy performs better than other strategies for retailer, SS strategy performs better than other strategies for manufacturer, and SD strategy performs better than DD and DS strategies for both manufacturer and retailer; (ii) in the case of $\theta = 0.59$, DS strategy outperforms other strategies for retailer when the market size is relatively low, SD strategy outperforms other strategies for retailer when the market size is relatively high, and SS strategy outperforms other strategies for manufacturer; (iii) in the case of $\theta = 0.6$, DS strategy outperforms other strategies for retailer, SS strategy outperforms other strategies for manufacturer, and SS strategy outperforms SD strategy for both members; (iv) in the case of $\theta = 0.8$, SS strategy outperforms other strategies for retailer, DD strategy outperforms other strategies for manufacturer, and SS strategy outperforms better than SD strategy for both members. Table 11 also shows that DS strategy is the Nash equilibrium for the dual-channel supply chain when $\theta = 0.4$, $\theta = 0.59$, and $\theta = 0.6$, and DD or SS strategy is the Nash equilibrium for the dual-channel supply chain when $\theta = 0.8$.

Table 4.10
Relationship of manufacturer's profits under different pricing strategies

θ	Relationship of manufacturer's profits
0.4	$DD < DS < SD < SS$; $SD < DD < DS < SS$
0.59	$DD < DS < SD < SS$; $SD < DD < DS < SS$
0.6	$DD < DS < SD < SS$; $SD < DD < DS < SS$
0.8	$SD < SS < DS < DD$

Table 4.11
Relationship of retailer's profits under different pricing strategies

θ	Relationship of retailer's profits
0.4	$SS < DD < DS < SD$
0.59	$SD < SS < DD < DS$; $SS < DD < DS < SD$
0.6	$SD < SS < DD < DS$
0.8	$SD < DD < DS < SS$

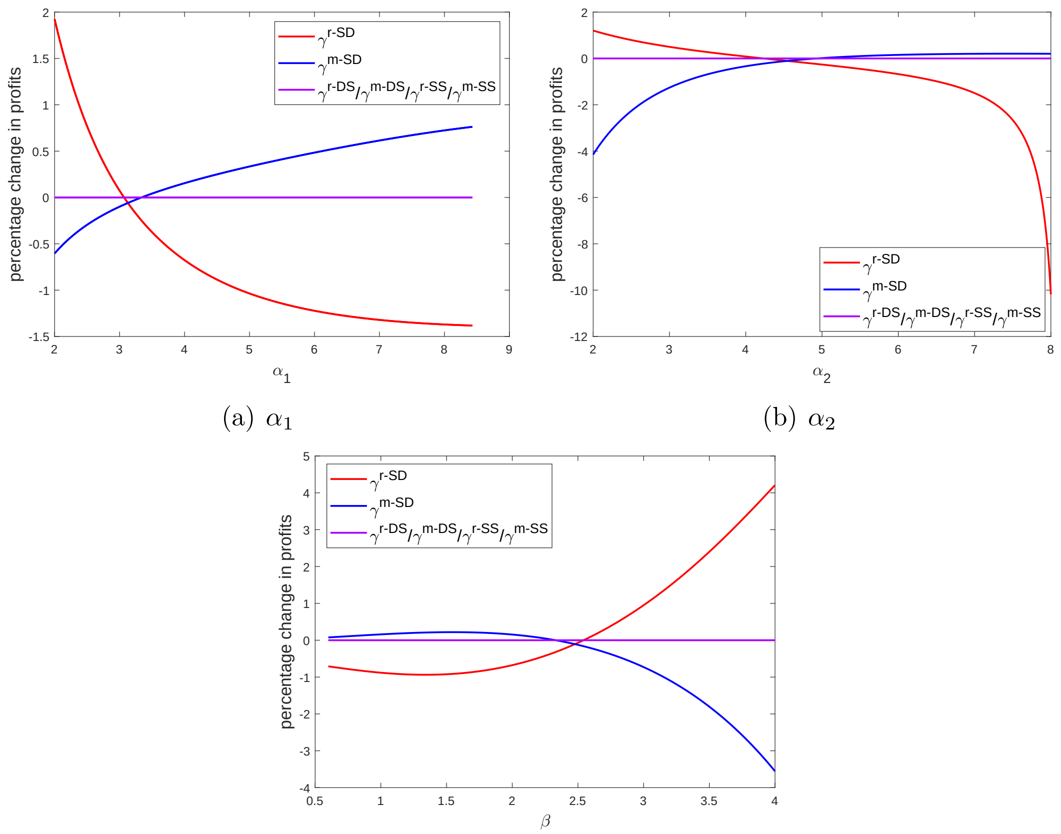
Fig.4.1, Fig.4.2, and Fig.4.3 show that the Nash equilibrium of manufacturer and retailer in choosing pricing strategy is affected by market size and customers' channel preference. If customers' preference for the direct channel is low, with the increase in market size, the Nash equilibrium for the dual-channel supply chain has the following order: DS strategy, DS/SS strategy, SD strategy, DS/DD strategy. If customers' preference for the direct channel is high, with the increase in market size, the Nash equilibrium changes from DS strategy into DS/DD strategy. And if customers' preference for the direct channel is particularly high, the Nash equilibrium changes from DD/SS strategy into DS/DD strategy as the market size increases.

From Fig.4.1, Fig.4.2, and Fig.4.3, we can also find that, the value of percentage change in retailer's profit is always larger than that of percentage change in manufacturer's profit. This implies that, the effect of changing pricing strategy on the follower's profit is larger than that on the leader's profit in a dual-channel supply chain. This is contributed to the fact that the follower needs to make decisions after the leader.

4.5.3 Effects of price sensitivities

In this subsection, we examine how the price sensitivity to demand in online channel α_1, the price sensitivity to demand in retail channel α_2, and the cross-channel price sensitivity β affect the relationship of members profits under different pricing strategies, respectively.

As shown in Fig.4.4, the best pricing strategy for manufacturer or retailer may change as any of the price sensitivities (α_1, α_2, and β) changes. Specifically, with the increase in α_1 or α_2, the best pricing strategy for the manufacturer may change from SD strategy into DD/DS/SS strategy, while the best pricing strategy for the retailer may change from DD/DS/SS strategy into SD strategy. With the increase



(a) α_1

(b) α_2

(c) β

Fig. 4.4. Effects of price sensitivities on percentage change in profits

in β, the best pricing strategy for the manufacturer may change from DD/DS/SS strategy into SD strategy, while the best pricing strategy for the retailer may change from SD strategy into DD/DS/SS strategy. Moreover, SD strategy can be the best pricing strategy for both manufacturer and retailer.

Fig.4.4 demonstrates that, DS or DD strategy is the Nash equilibrium for the dual-channel supply chain with low price sensitivity to demand in a channel or with high cross-channel price sensitivity, DS or SS strategy is the Nash equilibrium for the dual-channel supply chain with high price sensitivity to demand in a channel or with low cross-channel price sensitivity, and SD strategy may be the best pricing strategy for the dual-channel supply chain when the price sensitivities change in a certain range.

4.6 Conclusions

This chapter considers a dual-channel supply chain with one manufacturer and one retailer where demand is stochastic and price sensitive, and both members can take dynamic or static pricing strategy. The main purpose of this chapter is to investigate and compare the optimal joint pricing and inventory control policies of a dual-channel supply chain under four different pricing strategies (i.e., DD strategy, DS strategy, SD strategy and SS strategy).

We find that, under each of the four different pricing strategies, the optimal inventory control policy of a dual-channel supply chain is a base-stock type, that is, the member is optimal to replenish its inventory up to its base stock level if its starting inventory level is below its base stock level, otherwise it should not replenish. The structural properties of the optimal pricing policy under DD strategy are different from that under the other pricing strategies. Specifically, under DD strategy, members are optimal to charge list prices if their initial inventory levels are both below their respective base stock levels, otherwise they should reduce selling prices. Under the other pricing strategies, the member who adopts dynamic pricing is optimal to charge a list price if its initial inventory level is below its base stock level, otherwise it should reduce its selling price. Moreover, members' optimal dynamic prices and base stock levels are independent of or negatively correlated with members' starting inventory levels. From the numerical analysis, we find that, the performance comparison results of different pricing strategies are affected by the degree of demand uncertainty in the retail channel, customers' channel preference, market size, and price sensitivities. For the situation where the manufacturer and retailer are rational and independent in choosing dynamic or static pricing strategy, DS or SS strategy is a Nash equilibrium for the dual-channel supply chain if the degree of demand uncertainty in the retail channel is low, otherwise DS strategy is a Nash equilibrium. If customers' preference for the direct channel is low, the Nash equilibrium changes as the market size increases in the following sequence: DS strategy, DS or SS strategy, SD strategy, DS or DD strategy. With the increase in the market size, the Nash equilibrium changes from DS strategy into DS or DD strategy if customers' preference for the direct channel is high and changes from DD or SS strategy into DS or DD strategy if customers' preference for the direct channel is particularly high. DS or DD strategy is a Nash equilibrium if the price sensitivity to demand in a channel is low or the cross-channel price sensitivity is high, and DS or SS strategy is a Nash equilibrium if the price sensitivity to demand in a channel is high or the cross-channel price sensitivity is low. For the situation where supply chain members are rational and dependent in choosing pricing strategy, the manufacturer should take a portion of its profit to compensate the retailer for choosing static pricing strategy if the relative benefit for manufacturer from SS is higher than the relative benefit for retailer from SD, otherwise the retailer can take a portion of its profit to compensate the manufacturer for choosing dynamic pricing strategy. The amount of compensation depends on the negotiation between the manufacturer and retailer.

4.7 Appendices

Proof of Lemma 4.1

Since $V_3^{m-DS} = (c_m - c)I_3^m$ is concave in I_3^m, similar to Lemma 2.1, $J_2^{m-DS}(d_2^m, y_2^m)$ is jointly concave in (d_2^m, y_2^m) can be proved. Next, we will prove $J_1^{m-DS}(d_1^m, y_1^m)$ is jointly concave in (d_1^m, y_1^m).

$J_1^{m-DS}(d_1^m, y_1^m)$ consists of three parts: (1) $d_1^m(p^m(d_1^m) - \rho c) + (w - c)(y_1^r - I_1^r) - cy_1^m(1 - \rho)$; (2) $-\mathbf{E}[h^m(y_1^m - d_1^m - \epsilon_1^m)]$; and (3) $\rho\mathbf{E}[V_2^{m-DS}(y_1^m - d_1^m - \epsilon_1^m)]$. Similar to Lemma 2.1, we can get that parts (1) and (2) are jointly concave in (d_1^m, y_1^m). For (3), $V_2^{m-DS}(y_1^m - d_1^m - \epsilon_1^m) = \max_{y_2^m} J_2^{m-DS}(d_2^m, y_2^m)$, where $J_2^{m-DS}(d_2^m, y_2^m) = d_2^m(p^m(d_2^m) - \rho c) + (w - c)(y_2^r - I_2^r) - cy_2^m(1 - \rho) - \mathbf{E}[h^m(y_2^m - d_2^m - \epsilon_2^m)] + \rho\mathbf{E}[V_3^{m-DS}(y_2^m - d_2^m - \epsilon_2^m)]$ and $y_2^m - I_2^m + y_2^r - I_2^r \geq 0$. It is obvious that the optimal solution of y_2^m should be $I_2^m + I_2^r - y_2^r$ or be derived with $\frac{\partial J_2^{m-DS}(d_2^m, y_2^m)}{\partial y_2^m} = -c(1 - \rho) - \frac{\partial \mathbf{E}[h^m(y_2^m - d_2^m - \epsilon_2^m)]}{\partial (y_2^m - d_2^m)} + \rho\frac{\partial \mathbf{E}[V_3^{m-DS}(y_2^m - d_2^m - \epsilon_2^m)]}{\partial (y_2^m - d_2^m)} = 0$. As Proposition 2.1 is also true under DS strategy, that is both of d_2^m and d_1^m are negatively correlated with p^m, we can get that d_2^m in $J_2^{m-DS}(d_2^m, y_2^m)$ is positively correlated with d_1^m. Therefore, when the optimal solution of y_2^m is derived with $\frac{\partial J_2^{m-DS}(d_2^m, y_2^m)}{\partial y_2^m} = 0$, we can get that $V_2^{m-DS}(y_1^m - d_1^m - \epsilon_1^m)$ is concave in d_1^m. When the optimal solution of y_2^m is $I_2^m + I_2^r - y_2^r$, we can get that $V_2^{m-DS}(y_1^m - d_1^m - \epsilon_1^m)$ is concave in y_1^m and d_1^m. Since expectation preserves concavity, we get $\rho\mathbf{E}[V_2^{m-DS}(y_1^m - d_1^m - \epsilon_1^m)]$ is jointly concave in (d_1^m, y_1^m). Hence, Lemma 4.1 is completely proved.

Proof of Theorem 4.1

It is obvious that Theorem 4.1 is true for $t = 1$. Since $J_2^{r-DS}(d_2^r, y_2^r)$ is jointly concave in (d_2^r, y_2^r) and $J_2^{m-DS}(d_2^m, y_2^m)$ is jointly concave in (d_2^m, y_2^m), similar to Theorem 2.3, Theorem 4.1 for $t = 2$ in which p^m is given can be easily proved.

Proof of Lemma 4.2

Since $V_3^{r-SD} = (c_r - w)I_3^r$ is concave in I_3^r, similar to Lemma 4.2, $J_2^{r-SD}(d_2^r, y_2^r)$ is jointly concave in (d_2^r, y_2^r) can be proved. Next, we will prove $J_1^{r-SD}(d_1^r, y_1^r)$ is jointly concave in (d_1^r, y_1^r) for any given (p_1^m, y_1^m).

$J_1^{r-SD}(d_1^r, y_1^r)$ consists of three parts: (1) $d_1^r(p^r(d_1^r) - \rho w) - y_1^r w(1 - \rho)$, (2) $-\mathbf{E}[h^r(y_1^r - d_1^r - \epsilon_1^r)]$, and (3) $\rho\mathbf{E}[V_2^{r-SD}(y_1^r - d_1^r - \epsilon_1^r)]$. Similar to Lemma 2.1, we can get that parts (1) and (2) are jointly concave in (d_1^r, y_1^r). For (3), $V_2^{r-SD}(y_1^r - d_1^r - \epsilon_1^r) = \max_{y_2^r} J_2^{r-SD}(d_2^r, y_2^r)$, where $J_2^{r-SD}(d_2^r, y_2^r) = d_2^r(p^r(d_2^r) - \rho w) - y_2^r w(1-\rho) - \mathbf{E}[h^r(y_2^r - d_2^r - \epsilon_2^r)] + \rho\mathbf{E}[V_3^{r-SD}(y_2^r - d_2^r - \epsilon_2^r)]$ and $y_2^r - I_2^r \geq 0$. It is obvious that the optimal solution of y_2^r should be I_2^r or be derived with $\frac{\partial J_2^{r-SD}(d_2^r, y_2^r)}{\partial y_2^r} = -w(1 - \rho) - \frac{\partial \mathbf{E}[h^r(y_2^r - d_2^r - \epsilon_2^r)]}{\partial(y_2^r - d_2^r)} + \rho\frac{\partial \mathbf{E}[V_3^{r-SD}(y_2^r - d_2^r - \epsilon_2^r)]}{\partial(y_2^r - d_2^r)} = 0$. As both of d_2^r and d_1^r are negatively correlated with p^r, we can get that d_2^r in $J_2^{r-SD}(d_2^r, y_2^r)$ is positively correlated with d_1^r. Therefore, when the optimal solution of y_2^r is derived with $\frac{\partial J_2^{r-SD}(d_2^r, y_2^r)}{\partial y_2^r} = 0$, we can get that $V_2^{r-SD}(y_1^r - d_1^r - \epsilon_1^r)$ is concave in d_1^r. When the optimal solution of y_2^r is I_2^r, we can get that $V_2^{r-SD}(y_1^r - d_1^r - \epsilon_1^r)$ is concave in y_1^r and d_1^r. Since expectation preserves concavity, we get $\rho\mathbf{E}[V_2^{r-SD}(y_1^r - d_1^r - \epsilon_1^r)]$ is jointly concave in (d_1^r, y_1^r). Hence, Lemma 4.2 is completely proved.

Proof of Proposition 4.1

For $t = 2$, with any given p^r, it is obvious that $p_2^m(d_2^m) = \frac{\theta a + \beta p^r - d_2^m}{\alpha_1}$ is strictly decreasing in d_2^m. The decision variable p_2^m can be replaced with d_2^m. For any (p_2^m, y_2^m), $y_2^{r-SD\prime}$ is derived with $\frac{\partial J_2^{r-SD}(d_2^r, y_2^r)}{\partial y_2^r} = -w(1-\rho) - \frac{\partial \mathbf{E}[h^r(y_2^r - d_2^r - \epsilon_2^r)]}{\partial(y_2^r - d_2^r)} + \rho\frac{\partial \mathbf{E}[V_3^{r-SD}(y_2^r - d_2^r - \epsilon_2^r)]}{\partial(y_2^r - d_2^r)} = 0$. In view of $\frac{\partial J_2^{r-SD}(d_2^r, y_2^r)}{\partial y_2^r}$ and $d_t^r = (1 - \theta)a - \alpha_2 p^r + \beta p_t^m$, we get that $\frac{\partial y_2^{r-SD\prime}}{\partial p_2^m} = \frac{\partial d_2^r}{\partial p_2^m} = \beta$.

(1) Under the scenario where $I_2^r < y_2^{r-SD\prime}$, $J_2^{m-SD}(d_2^m, y_2^m) = d_2^m(p_2^m(d_2^m) - \rho c) + (w - c)(y_2^{r-SD\prime} - I_2^r) - c y_2^m(1 - \rho) - \mathbf{E}[h^m(y_2^m - d_2^m - \epsilon_2^m)] + \rho\mathbf{E}[V_3^{m-SD}(y_2^m - d_2^m - \epsilon_2^m)]$. Get $(d_2^{m-SD(1)\prime}, y_2^{m-SD(1)\prime})$ with $\frac{\partial J_2^{m-SD}(d_2^m, y_2^m)}{\partial d_2^m} = 0$ and $\frac{\partial J_2^{m-SD}(d_2^m, y_2^m)}{\partial y_2^m} = 0$, where

$\frac{\partial J_2^{m-SD}(d_2^m,y_2^m)}{\partial d_2^m} = \frac{\theta a+\beta p^r-2d_2^m}{\alpha_1} - \rho c - (w-c)\frac{\beta}{\alpha_1} + \frac{\partial \mathbf{E}[h^m(y_2^m-d_2^m-\epsilon_2^m)]}{\partial(y_2^m-d_2^m)} - \rho\frac{\partial \mathbf{E}[V_3^{m-SD}(y_2^m-d_2^m-\epsilon_2^m)]}{\partial(y_2^m-d_2^m)}$ and $\frac{\partial J_2^{m-SD}(d_2^m,y_2^m)}{\partial y_2^m} = -(1-\rho)c - \frac{\partial \mathbf{E}[h^m(y_2^m-d_2^m-\epsilon_2^m)]}{\partial(y_2^m-d_2^m)} + \rho\frac{\partial \mathbf{E}[V_3^{m-SD}(y_2^m-d_2^m-\epsilon_2^m)]}{\partial(y_2^m-d_2^m)}$. If $I_2^m < y_2^{m-SD(1)\prime} + y_2^{r-SD\prime} - I_2^r$, $(d_2^{m-SD(1)\prime}, y_2^{m-SD(1)\prime})$ is the optimal solution. Obviously, $\frac{\partial d_2^{m-SD(1)\prime}}{\partial p^r} = \frac{\beta}{2}$.

Then, with $d_2^r = (1-\theta)a - \alpha_2 p^r + \frac{\beta}{\alpha_1}p_2^m(d_2^m)$, we get $\frac{\partial d_2^{r-SD(1)\prime}}{\partial p^r} = \frac{\beta^2-2\alpha_1\alpha_2}{2\alpha_1} < 0$. If $I_2^m \geq y_2^{m-SD(1)\prime} + y_2^{r-SD\prime} - I_2^r$, the optimal solution is $(d_2^{m-SD(1)\prime}(I_2^r, I_2^m), I_2^r + I_2^m - y_2^{r-SD\prime})$. With $\frac{\partial y_2^{r-SD\prime}}{\partial p_2^m} = \beta$, we get $0 < \frac{\partial d_2^{m-SD(1)\prime}(I_2^r,I_2^m)}{\partial p^r} < \frac{\beta}{2}$. Similarly, we can get $\frac{\beta^2-2\alpha_1\alpha_2}{2\alpha_1} < \frac{\partial d_2^{r-SD(1)\prime}(I_2^r,I_2^m)}{\partial p^r} < \frac{\beta^2-\alpha_1\alpha_2}{\alpha_1} < 0$.

(2) Under the scenario where $I_2^r \geq y_2^{r-SD\prime}$, $J_2^{m-SD}(d_2^m, y_2^m) = d_2^m(p_2^m(d_2^m) - \rho c) - cy_2^m(1-\rho) - \mathbf{E}[h^m(y_2^m - d_2^m - \epsilon_2^m)] + \rho\mathbf{E}[V_3^{m-SD}(y_2^m - d_2^m - \epsilon_2^m)]$. $\frac{\partial J_2^{m-SD}(d_2^m,y_2^m)}{\partial d_2^m} = \frac{\theta a+\beta p^r-2d_2^m}{\alpha_1} - \rho c + \frac{\partial \mathbf{E}[h^m(y_2^m-d_2^m-\epsilon_2^m)]}{\partial(y_2^m-d_2^m)} - \rho\frac{\partial \mathbf{E}[V_3^{m-SD}(y_2^m-d_2^m-\epsilon_2^m)]}{\partial(y_2^m-d_2^m)}$ and $\frac{\partial J_2^{m-SD}(d_2^m,y_2^m)}{\partial y_2^m} = -(1-\rho)c - \frac{\partial \mathbf{E}[h^m(y_2^m-d_2^m-\epsilon_2^m)]}{\partial(y_2^m-d_2^m)} + \rho\frac{\partial \mathbf{E}[V_3^{m-SD}(y_2^m-d_2^m-\epsilon_2^m)]}{\partial(y_2^m-d_2^m)}$. Get $(d_2^{m-SD(2)\prime}, y_2^{m-SD(2)\prime})$ with $\frac{\partial J_2^{m-SD}(d_2^m,y_2^m)}{\partial d_2^m} = 0$ and $\frac{\partial J_2^{m-SD}(d_2^m,y_2^m)}{\partial y_2^m} = 0$. If $I_2^m < y_2^{m-SD(2)\prime}$, the optimal solution of (d_2^m, y_2^m) is $(d_2^{m-SD(2)\prime}, y_2^{m-SD(2)\prime})$. If $I_2^m \geq y_2^{m-SD(2)\prime}$, the optimal solution of (d_2^m, y_2^m) is $(d_2^{m-SD(2)\prime}(I_2^m), I_2^m)$. Similar to (1), we get $\frac{\partial d_2^{r-SD(2)\prime}}{\partial p^r} = \frac{\beta^2-2\alpha_1\alpha_2}{2\alpha_1} < 0$ and $\frac{\beta^2-2\alpha_1\alpha_2}{2\alpha_1} < \frac{\partial d_2^{r-SD(2)\prime}(I_2^m)}{\partial p^r} < \frac{\beta^2-\alpha_1\alpha_2}{\alpha_1} < 0$.

With (1) and (2) which show that the optimal solution of d_2^r is negatively correlated with p^r, we can get that $V_2^{r-SD}(I_2^r)$ is concave in p^r.

For $t = 1$, $J_1^{r-SD}(d_1^r, y_1^r) = d_1^r(p^r(d_1^r) - \rho w) - y_1^r w(1-\rho) - \mathbf{E}[h^r(y_1^r - d_1^r - \epsilon_1^r)] + \rho\mathbf{E}[V_2^{r-SD}(y_1^r - d_1^r - \epsilon_1^r)]$. For any (p_1^m, y_1^m), get $(d_1^{r-SD\prime}, y_1^{r-SD\prime})$ with $\frac{\partial J_1^{r-SD}(d_1^r,y_1^r)}{\partial d_1^r} = 0$ and $\frac{\partial J_1^{r-SD}(d_1^r,y_1^r)}{\partial y_1^r} = 0$, where $\frac{\partial J_1^{r-SD}(d_1^r,y_1^r)}{\partial d_1^r} = \frac{(1-\theta)a+\beta p_1^m-2d_1^r}{\alpha_2} - \rho w + \frac{\partial \mathbf{E}[h^r(y_1^r-d_1^r-\epsilon_1^r)]}{\partial(y_1^r-d_1^r)} - \rho\frac{\partial \mathbf{E}[V_2^{r-SD}(y_1^r-d_1^r-\epsilon_1^r)]}{\partial(y_1^r-d_1^r)} + \rho\frac{\partial \mathbf{E}[V_2^{r-SD}(y_1^r-d_1^r-\epsilon_1^r)]}{\partial p^r}\frac{\partial p^r}{\partial d_1^r}$ and $\frac{\partial J_1^{r-SD}(d_1^r,y_1^r)}{\partial d_1^r} = -\frac{\partial \mathbf{E}[h^r(y_1^r-d_1^r-\epsilon_1^r)]}{\partial(y_1^r-d_1^r)} - (1-\rho)w + \rho\frac{\partial \mathbf{E}[V_2^{r-SD}(y_1^r-d_1^r-\epsilon_1^r)]}{\partial(y_1^r-d_1^r)}$. Then, with $V_2^{r-SD}(I_2^r)$ is concave in p^r and $\frac{\partial p^r}{\partial d_1^r} = -\frac{1}{\alpha_2}$, we can get that $0 < \frac{\partial d_1^{r-SD\prime}}{\partial p_1^m} < \frac{\beta}{2}$. Then, with $p_t^r(d_t^r) = \frac{(1-\theta)a+\beta p_1^m-2d_1^r}{\alpha_2}$ and $p_t^m(d_t^m) = \frac{\theta a+\beta p^{r-SD\prime}-d_t^m}{\alpha_1}$, we can further get that $\frac{p_1^m(d_1^m)}{d_1^m} < 0$. Hence, Proposition 4.1 is completely proved.

Proof of Theorem 4.2

It is obvious that Theorem 4.2 is true for $t = 1$. For $t = 2$ in which p^r is given, with any given (p_2^m, y_2^m), $\frac{\partial y_2^{r-SD\prime}}{\partial p_2^m} = \beta$ which has been proved in Proposition 4.1.

(1) If $I_2^r < y_2^{r-SD\prime}$, $(d_2^{m-SD(1)\star}, y_2^{m-SD(1)\star})$ is obtained with $\frac{\partial J_2^{m-SD}(d_2^m,y_2^m)}{\partial d_2^m} = 0$ and $\frac{\partial J_2^{m-SD}(d_2^m,y_2^m)}{\partial y_2^m} = 0$, where $\frac{\partial J_2^{m-SD}(d_2^m,y_2^m)}{\partial d_2^m} = \frac{\theta a+\beta p^r-2d_2^m}{\alpha_1} - \rho c - (w-c)\frac{\beta}{\alpha_1} + \frac{\partial \mathbf{E}[h^m(y_2^m-d_2^m-\epsilon_2^m)]}{\partial(y_2^m-d_2^m)} - \rho\frac{\partial \mathbf{E}[V_3^{m-SD}(y_2^m-d_2^m-\epsilon_2^m)]}{\partial(y_2^m-d_2^m)}$ and $\frac{\partial J_2^{m-SD}(d_2^m,y_2^m)}{\partial y_2^m} = -\frac{\partial \mathbf{E}[h^m(y_2^m-d_2^m-\epsilon_2^m)]}{\partial(y_2^m-d_2^m)} -$

$(1-\rho)c + \rho\frac{\partial \mathbf{E}[V_3^{m-SD}(y_2^m - d_2^m - \epsilon_2^m)]}{\partial(y_2^m - d_2^m)}$.

It is obvious that, if $I_2^m < y_2^{m-SD(1)\star} + y_2^{r-SD\prime} - I_2^r$, the optimal solution ($p_2^{m-SD(1)\star}$, $y_2^{m-SD(1)\star}$) is independent of I_2^r and I_2^m. If $I_2^m \geq y_2^{m-SD(1)\star} + y_2^{r-SD\prime} - I_2^r$, the optimal solution of y_2^m is $I_2^r + I_2^m - y_2^{r-SD\prime}$. Since $-\mathbf{E}[h^m(y_2^m - d_2^m - \epsilon_2^m)] + \rho\mathbf{E}[V_3^{m-SD}(y_2^m - d_2^m - \epsilon_2^m)]$ is concave in $y_2^m - d_2^m$ and $\frac{\partial y_2^{r-SD\prime}}{\partial p_2^m} = \beta$, we get that the optimal solution $d_2^{m-SD(1)\star}(I_2^r, I_2^m)$ is positively correlated with I_2^r and I_2^m. Then, with $p_2^m(d_2^m) = \frac{\theta a + \beta p^r - d_2^m}{\alpha_1}$ and $\frac{\partial y_2^{r-SD\prime}}{\partial p_2^m} = \beta$, Theorem 4.2-(a) and 4.2-(b) are proved.

(2) If $I_2^r \geq y_2^{r-SD\prime}$, $(d_2^{m-SD(2)\star}, y_2^{m-SD(2)\star})$ is obtained with $\frac{\partial J_2^{m-SD}(d_2^m, y_2^m)}{\partial d_2^m} = 0$ and $\frac{\partial J_2^{m-SD}(d_2^m, y_2^m)}{\partial y_2^m} = 0$, where $\frac{\partial J_2^{m-SD}(d_2^m, y_2^m)}{\partial d_2^m} = \frac{\theta a + \beta p^r - 2d_2^m}{\alpha_1} - \rho c + \frac{\partial \mathbf{E}[h^m(y_2^m - d_2^m - \epsilon_2^m)]}{\partial(y_2^m - d_2^m)} - \rho\frac{\partial \mathbf{E}[V_3^{m-SD}(y_2^m - d_2^m - \epsilon_2^m)]}{\partial(y_2^m - d_2^m)}$ and $\frac{\partial J_2^{m-SD}(d_2^m, y_2^m)}{\partial y_2^m} = -\frac{\partial \mathbf{E}[h^m(y_2^m - d_2^m - \epsilon_2^m)]}{\partial(y_2^m - d_2^m)} + \rho\frac{\partial \mathbf{E}[V_3^{m-SD}(y_2^m - d_2^m - \epsilon_2^m)]}{\partial(y_2^m - d_2^m)} - (1-\rho)c$.

It is obvious that, if $I_2^m < y_2^{m-SD(2)\star}$, the optimal solution $(p_2^{m-SD(2)\star}, y_2^{m-SD(2)\star})$ is independent of I_2^r and I_2^m. If $I_2^m \geq y_2^{m-SD(2)\star}$, the optimal solution $d_2^{m-SD(2)\prime}(I_2^m)$ is positively correlated with I_2^m due to the concavity of $-\mathbf{E}[h^m(y_2^m - d_2^m - \epsilon_2^m)] + \rho\mathbf{E}[V_3^{m-SD}(y_2^m - d_2^m - \epsilon_2^m)]$. Then, with $p_2^m(d_2^m) = \frac{\theta a + \beta p^r - d_2^m}{\alpha_1}$ and $\frac{\partial y_2^{r-SD\prime}}{\partial p_2^m} = \beta$, Theorem 4.2-(c) and 4.2-(d) are proved.

Proof of Theorem 4.3

Similar to Theorem 4.2, Theorem 4.3 can be easily proved.

Chapter 5

Conclusions

This dissertation focuses on the joint pricing and inventory control problems of a multi-period dual-channel supply chain with stochastic and price-sensitive demand. The problem faced by either of the members in the dual-channel supply chain is to determine their joint pricing and inventory control policy to maximize their respective total expected profits over the finite planning horizon with multiple periods. To analyze the structural properties of the optimal joint pricing and inventory control policies, we establish mathematical models for the problem faced by either of the members with stochastic dynamic programming, introduce a transformation technique to facilitate the analysis, and formulate the interaction relationship between the two dual-channel members with game theory. Moreover, numerical studies are conducted to get some managerial insights. In Chapters 2 and 3, the optimal joint dynamic pricing and inventory control policies of a dual-channel supply chain under different channel power structures are analyzed and compared. The effects of the dual-channel setting and the channel power structure on the optimal joint dynamic pricing and inventory control policy are clarified. In Chapter 4, the optimal joint pricing and inventory control policies of a dual-channel supply chain under different pricing strategies are analyzed and compared. Moreover, the performance of dynamic pricing and static pricing in the dual-channel supply chain with inventory control is compared, and the effects of market parameters on the comparison results are clarified.

Results of this dissertation can provide several insights into the joint pricing and inventory management for a dual-channel supply chain where the demand is stochastic with additive form. First, the optimal joint dynamic pricing and inventory control policy belongs to a base-stock-list-price type, that is, a member should produce or order goods only if its starting inventory level is below its base stock level, and list prices should be charged only when both members' starting inventory levels are below their respective base stock levels. Moreover, as long as one member's starting inventory level is above its base stock level, both members should reduce prices. The higher the initial inventory level exceeds the base stock level, the lower the reduced prices. The base stock levels of manufacturer and retailer are independent of or correlated with their initial inventory levels. Second, the above described structural properties of the optimal joint dynamic pricing and inventory control policy are not affected by the channel power structure as well as the dual-channel setting. Third, whether the member takes dynamic pricing strategy or not, the optimal inventory control policy of a dual-channel supply chain is a base-stock type, that is, the member is optimal to replenish its inventory up to its base stock level if its starting inventory level is below its base stock level, otherwise it should not replenish. When only one member chooses to take dynamic pricing strategy, the member who adopts dynamic pricing is optimal to charge a list price if its initial inventory level is below its base stock level, otherwise it should reduce its selling price. Last, integrating dynamic pricing strategy into inventory management may underperform the traditional inventory control policy with static pricing strategy in coping with additive demand uncertainty. In addition, the performance of dynamic pricing strategy is affected by market parameters including the degree of demand uncertainty, market size, customers' channel preference, price sensitivity to demand in a channel, and the cross-channel price sensitivity.

In this dissertation, the demand is assumed to be only dependent on the current selling prices. However, it has been empirically observed in some industries and recognized in the psychological and behavioral studies that the demand may be affected by the past selling prices or the presented inventory levels. Therefore, an interesting research direction can be the joint pricing and inventory management of a dual-channel supply chain with the effect of historical prices or inventory level on demand. Moreover, we consider the price-demand relationship and the random demand distribution are given before the decision-making. However, in many situations such as the period of the introduction of a new product, the demand information may be unknown when the decisions are made. Hence, it would be interesting to study the joint pricing and inventory management of a dual-channel supply chain with demand learning.

There are two types of price competition in a dual-channel supply chain, one is the horizontal price competition, and the other is the vertical price competition. In this dissertation, we only consider the horizontal price competition by assuming the wholesale price is determined in advance and fixed during the planning horizon. It would be interesting to investigate the joint pricing and inventory management of a multi-period dual-channel supply chain when considering the wholesale price as a decision variable.

Moreover, this dissertation considers the joint pricing and inventory management of a dual-channel supply chain with durable products. An interesting future research direction can be the joint pricing and inventory management of a dual-channel supply chain where perishable products with fixed shelf lives are sold. Compared with the dual-channel supply chain with durable products, the joint pricing and inventory control problem of a dual-channel supply chain with perishable products is more complicated, because perishable products can be differentiated by their ages which leads to the competition between the new and old products. Other future research directions based on this study can be considering the uncertainty in the supply side, considering the fixed ordering cost, and so on.

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