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## Prediction Formula of Superstructure for Isolated building Based on 2DOF

### Part 1: Equivalent Height of 2DOF Model

構造-振動

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## MDOF Model, 2DOF Model, Isolated building, Mass distribution, Eigen analysis

#### 1. Introduction

The base-isolated building showed the benefits of restraining the damage of the mainframe and being easy to repair after an earthquake comparing with the seismic structure. Therefore, base-isolated buildings are increasing these years. In general, the period of superstructure is short obviously comparing with the period of isolation layer, therefore superstructure could be regarded as rigid body. However, when the period of superstructure is long (e.g., warehouse or high building), the effect of superstructure is considered necessary.

Fu et al. [1] proposed a prediction equation of superstructure for base-isolated building with steel dampers considering equivalent period ratio between the isolation layer and the superstructure  $T_{eq}/T_{U}$ . Li et al. [2] verified Fu's prediction equation could be used for base-isolated building with oil dampers. However, in Fu et al.<sup>1</sup>'s equation, the effect of the mass ratio between superstructure and isolation layer, the mass distribution, and the number of mass points for superstructure are not considered.

In this paper, a prediction formula of superstructure for isolated building considering the period ratio and mass ratio between superstructure and isolation layer is proposed. In the part 1, the equivalent height of two-degree-of-freedom (2DOF) model of isolation building is investigated. In the part 2, the prediction formula by 2DOF model is proposed. And the application range of Fu's prediction equation is confirmed.

#### 2. Analysis Model and Mass Distribution

Fig. 1 shows the analysis models, (a) is multi-degree-offreedom (MDOF) model of the base-isolated building, where the superstructure has N lump-masses  $m_i$  ( $i = 1 \sim N$ ), and the isolation layer has a lump-mass  $m_0$ .  $k_i$  ( $i = 1 \sim N$ ) and  $k_0$  shows the stiffness of *i* story and isolation layer, respectively. (b) is twodegree-of-freedom (2DOF) model of the base-isolated building considered in this study, where the superstructure is considered as a whole lump-mass  $m_U$  and the isolation layer has a lumpmass  $m_0$ .  $k_U$  is the stiffness of superstructure.

Fig. 2 shows the mass distribution of superstructure for MDOF model, 4 cases ((a)~(b) = Case A ~ Case D) mass distribution are adopted in this paper. Case A is rectangular distribution. The others are trapezoidal distribution. Case B of mass distribution ratio  $m_N/m_1 = 1/3$ . Case C of mass distribution ratio  $m_N/m_1 = 1/6$ . And Case D of mass distribution ratio  $m_N/m_1 = 3$ . Fig. 3 shows the 1<sup>st</sup> mode eigenvector of superstructure for MODF model. And the 1<sup>st</sup> eigenvalue of *i* story  $_1\phi_i = i$  ( $i = 1 \sim N$ ).

Eq. 1 shows the stiffness of superstructure  $k_i$  ( $i = 1 \sim N$ ). Eq. 2 shows the stiffness of isolation layer  $k_0$ .

$$k_{i} = \frac{1}{1} \frac{\omega^{2} \cdot m_{i} \cdot 1}{1} \frac{\phi_{i} + (1 + \phi_{i+1} - 1)}{1} \frac{\phi_{i} - 1}{1} \phi_{i-1}}, \quad k_{0} = \frac{4\pi^{2}}{T_{0}^{2}} \Sigma m$$
(1, 2)

Where,  $_{1}\omega$  is  $1^{\text{st}}$  circular frequency of superstructure,  $_{1}\omega = 2\pi / T_{\text{U}}$ .  $T_{\text{U}}$  is period of superstructure.  $\Sigma m$  total mass of building,  $\Sigma m = m_{\text{U}} + m_{0}$ .  $T_{0}$  is period of isolation layer.





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#### 3. 1st Mode Period Ratio and 1st Mode Shape Ratio

When the periods of 2DOF and MDOF are close, meanwhile the ratio of top eigenvalue between 2DOF and MDOF is stable, the 1<sup>st</sup> mode shape of 2DOF could be used to predict 1<sup>st</sup> mode of MDOF. Furthermore, the prediction equation of superstructure could be proposed. (Part 2)

#### 3.1. 1<sup>st</sup> Mode Period Ratio

Fig. 4 shows relationship between mass ratio  $m_U/m_0$  and 1<sup>st</sup> mode period ratio  ${}_1T_{2\text{DOF}}/{}_1T_{\text{MDOF}}$ .  ${}_1T_{2\text{DOF}}$  is calculated by Eq. 10 of part 2 and  ${}_1T_{\text{MDOF}}$  is calculated by eigen analysis. (a)~(d) are various number of mass points (5, 15, 30 and 50DOF). (1)~(4) are various mass distribution of superstructure (case A~D). Horizontal axis is  $m_U/m_0$  and vertical axis is  ${}_1T_{2\text{DOF}}/{}_1T_{\text{MDOF}}$ . For legend, solid lines are various  $T_0/T_U$  from 1.0 s to 5.0 s. With  $T_0/T_U$  increases, the gradient color become from red to bule. The dotted line means  $T_0/T_U = 2.0$ .

According to Fig. 4, when number of mass point of superstructure, mass distribution pattern, and mass ratio  $m_{\rm U}/m_0$  are same, with the period ratio  $T_0/T_{\rm U}$  increases, the 1<sup>st</sup> mode period  ${}_{1}T_{2\text{DOF}}/{}_{1}T_{\text{MDOF}}$  decreases. And when  $T_{0}/T_{U} \ge 2.0 \text{ s}$ (general base-isolated building [3], [4]),  $_1T_{2DOF}/_1T_{MDOF} <$ 1.057. In other words, the periods of 2DOF and MDOF models are mostly same for general isolated building. Moreover, when number of mass point of superstructure is same, the 1<sup>st</sup> mode period  $_1T_{2DOF}/_1T_{MDOF}$  of case D < case A < case B < case C, namely with the building centre-of-gravity position decreases, the 1<sup>st</sup> mode period  ${}_{1}T_{2DOF}/{}_{1}T_{MDOF}$ increases. In addition, when mass distribution of superstructure is positive trapezoidal distribution, with the mass ratio  $T_0/T_U$  increases, the 1<sup>st</sup> mode period  ${}_1T_{2DOF}/{}_1T_{MDOF}$ tends to increase and then decrease. Lastly, the effect of mass point on  $1^{\text{st}}$  mode period  ${}_{1}T_{2\text{DOF}}/{}_{1}T_{\text{MDOF}}$  is very small.





#### 3.2. 1<sup>st</sup> Mode Shape Ratio

Fig. 5 shows the relationship between the mass ratio  $m_U/m_0$ and 1<sup>st</sup> mode shape ratio  $_1\phi_{U,2DOF}/_1\phi_{N,MDOF}$ . Where  $_1\phi_{U,2DOF}$ (calculated by Eq. 12 of part 2) and  $_1\phi_{N,MDOF}$  (calculated by eigen analysis) are corresponding values when  $_1\phi_{0,2DOF} =$  $_1\phi_{0,MDOF} = 1$  for 2DOF and MDOF models, respectively. (a) ~ (d) are various number of mass points for superstructure. (a) ~ (d) are 5, 15, 30 and 50DOF. (1) ~ (4) are various mass distribution of superstructure (case A ~ D). Horizontal axis is mass ratio  $m_U/m_0$  and vertical axis is 1<sup>st</sup> mode shape ratio  $_1\phi_{U,2DOF}/_1\phi_{N,MDOF}$ . For legend, solid lines are various period ratio  $T_0/T_U$ . With  $T_0/T_U$  increases, the gradient color become from red to bule. The dotted line means  $T_0/T_U = 2.0$ .

According to Fig. 5, when number of mass point of superstructure, mass distribution pattern, and mass ratio  $m_U/m_0$  are same, with the period ratio  $T_0/T_U$  increases, the 1<sup>st</sup>

mode shape  $_1\phi_{U,2DOF}/_1\phi_{N,MDOF}$  trends to close 1. Moreover, when number of mass point of superstructure, mass distribution pattern, and mass ratio  $T_0/T_U$  are same, with the mass ratio  $m_{\rm U}/m_0$  increases, 1<sup>st</sup> mode shape ratio  $1\phi_{\rm U,2DOF}/1\phi_{\rm N,MDOF}$  trends to increase. And when  $T_0/T_{\rm U} \ge 2.0$  s (general base-isolated building [3], [4]), the effect of mass ratio  $m_U/m_0$  on  $_1\phi_{U,2DOF}/_1\phi_{N,MDOF}$  is not obvious. In other words, the effect of mass ratio  $m_U/m_0$  on 1<sup>st</sup> mode shape ratio  $1\phi_{U,2DOF}/1\phi_{N,MDOF}$  is not considered necessary for general baseisolated building. In addition, when number of mass point of superstructure, mass ratio and period ratio are same, the 1st mode shape  $_1\phi_{U,2DOF}/_1\phi_{N,MDOF}$  of case D > case A > case B > case C, namely with the centre-of-gravity position decreases, the 1<sup>st</sup> mode shape ratio  $_1\phi_{U,2DOF}/_1\phi_{N,MDOF}$  increases. Lastly, the effect of number of mass point on 1st mode shape  $1\phi_{\rm U,2DOF}/1\phi_{\rm N,MDOF}$  is very small.





#### 4. Equivalent Height

In this chapter the equivalent height ratio  $\lambda_{eq}$  is proposed. As shown in Fig.6, which is 1<sup>st</sup> mode eigenvector of isolated building, the equivalent height ratio  $\lambda_{eq}$  is the ratio between equivalent height of 2DOF model  $H_{eq}$  and the height of MDOF *H*.  $H_{eq}$  is corresponding height of MDOF model when corresponding eigenvalue are same ( $_{1}\phi_{U,2DOF} = _{1}\phi_{eq,MDOF}$ ). Where, the eigenvalues of isolation layer of two type models are same and uniformly 1. And the equivalent height ratio  $\lambda_{eq}$ is shown in Eq. 3.



Fig. 6 1st Mode Eigenvector of Isolated Building

$$\lambda_{\rm eq} = \frac{H_{\rm eq}}{H} = \frac{{}_{1}\phi_{\rm U,2DOF} - {}_{1}\phi_{\rm 0,2DOF}}{{}_{1}\phi_{\rm N,MDOF} - {}_{1}\phi_{\rm 0,MDOF}} = \frac{{}_{1}\phi_{\rm U,2DOF} - 1}{{}_{1}\phi_{\rm N,MDOF} - 1}$$
(3)

Fig.7 & 8 shows the relationship between period ratio  $T_0$ / $T_U$  and equivalent height ratio  $\lambda_{eq}$ . Fig. 7 shows the results when mass ratio  $m_U/m_0 = 2$ . Fig. 8 shows the results when number of mass points of superstructure is 50. (1)~(4) are various mass distribution of superstructure from case A to case D. For legend, solid lines are various number of mass points of superstructure (Fig. 7) or mass period (Fig. 8). With number of mass points of superstructure (Fig. 7) or mass period  $m_U/m_0$  (Fig. 8) increases, the gradient color become from red to bule.

According to Fig.7, when mass distribution and period ratio  $T_0 / T_U$  are same, the effect of number of mass points on equivalent height ratio  $\lambda_{eq}$  are not significant. Moreover, when mass distribution and number of mass points are same, with  $T_0 / T_U$  increases,  $\lambda_{eq}$  is increase. In addition, when number of mass points and  $T_0 / T_U$  are same,  $\lambda_{eq}$  of case D > case A > case B > case C. In general, period ratio  $T_0 / T_U \ge 2$  s for isolated building, therefore,  $\lambda_{eq} \ge 0.55$  (case C).

Similarly, according to Fig.8, when mass distribution and period ratio are same, the effect of mass ratio  $m_U/m_0$  on equivalent height ratio  $\lambda_{eq}$  are not significant. In addition, when number of mass points and  $T_0/T_U$  are same,  $\lambda_{eq}$  of case D > case A > case B > case C = 0.55. In summary, for safety reasons, to let  $\lambda_{eq} = 0.50$ .

#### 5. Conclusion

In this paper, a prediction formula of superstructure for isolated building considering the period ratio and mass ratio between superstructure and isolation layer was proposed. In this part 1, the periods of 2DOF and MDOF models were mostly same for general isolated building (period ratio  $T_0/T_U \ge 2$  s) was verified. And the equivalent height ratio  $\lambda_{eq} (H_{eq}/H) = 0.5$  was confirmed.



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