論文 / 著書情報
Article / Book Information

論題(和文)	
Title(English)	Seismic response prediction for isolated structure using equivalent period ratio
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出典 / Citation	日本建築学会関東支部研究報告集, Ⅰ, , pp. 457-460
Citation(English)	, Ⅰ, , pp. 457-460
発行日 / Pub. date	2023, 2
権利情報	一般社団法人 日本建築学会

2022年度日本建築学会
関東支部研究報告集 I
2023 年 2 月

Seismic response prediction for isolated structure using equivalent period ratio

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Isolated structure, Seismic response prediction, Viscous damper, Equivalent excitation period, Equivalent period ratio

1. Introduction

Setting a base isolation layer consists of laminated rubber bearings and viscous dampers (VDs) between superstructure and foundation structure is an effective isolation method[1]. In recent years, there are numerous studies (e.g. [2-6]) focusing on energy-based seismic design and response prediction for passive controlled structures with VDs. We note a fact that these studies generally use the area of hysteresis loop under simple harmonic load to express the hysteresis energy of viscous damping devices, which is feasible and smart. However, meanwhile, they also used an assumption that the equivalent excitation period of the equivalent simple harmonic load is always equal to the natural period of structure. This assumption can simplify the design and response prediction process, but inevitably lose the accuracy guarantee, especially for the base-isolated structures which can avoid the main frequency range of general ground motion energy distribution. Therefore, to develop a seismic response prediction method with higher accuracy for isolated structures, this paper tries to propose an empirical formula for equivalent period ratio and propose a new energy balance-based prediction method which can consider the influence of the equivalent excitation period.

2. Empirical formula of equivalent period ratio

2.1. Equivalent excitation period

Consider a simple SDOF model with an isolation layer consist of laminated rubber bearings and linear VDs (see Fig.1, in which the subscript f denotes the laminated rubber bearings as a flexible element, k_f is the lateral stiffness of the laminated rubber bearings, c is the damping coefficient of VDs, M is the mass). It is assumed that the laminated rubber bearings are always in elastic state and undamped. The restoring force characteristics of the model are shown in Fig.2, where δ is the displacement and δ_max is the maximum displacement. Q_f is the shearing force of the laminated rubber bearings and $Q_{\mathrm{f,max}} = k_\mathrm{f}\delta_\mathrm{max}$ is the maximum shearing force of the laminated rubber bearings. Subscript d denotes the VDs, Q_d is the shearing force of VDs and $Q_\mathrm{d,max}$ is the maximum shearing force of VDs, Q is the total shearing force and Q_max is the maximum total shearing force. The natural vibration period without dampers is recorded as T_f and the nature vibration frequency without VDs is ω_f. The damping ratio provided from the VDs is $\zeta_\mathrm{d} = cT_\mathrm{f}/(4\pi M)$.

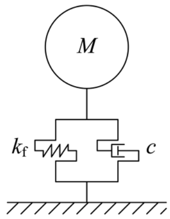
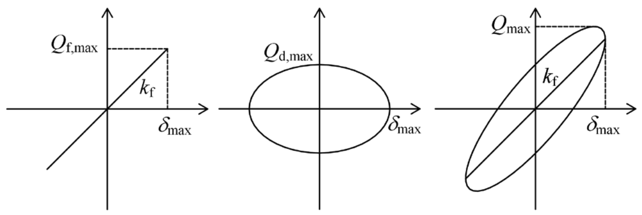


Fig.1 SDOF model



(a) Rubber bearings　(b) VDs　(c) Isolation layer

Fig.2 Restoring force characteristics

Since the maximum response of structure is most concerned, it is considered that the ground motion can be expressed by an equivalent simple harmonic load, and the solution under the simple harmonic load is

$$\delta(t) = \delta_\mathrm{max}\sin(\omega_\mathrm{eq}t - \phi) \tag{1}$$

where ω_eq is the equivalent excitation frequency of the equivalent simple harmonic load, t is the time. Then the shearing force of VDs, Q_d can be deduced as

$$\begin{aligned} Q_\mathrm{d}(t) &= cv = c\dot{\delta}(t) = c\omega_\mathrm{eq}\cos(\omega_\mathrm{eq}t - \phi) \\ &= \pm c\omega_\mathrm{eq}\sqrt{\delta_\mathrm{max}^2 - \left[\delta_\mathrm{max}\sin(\omega_\mathrm{eq}t - \phi)\right]^2} \\ &= \pm c\omega_\mathrm{eq}\sqrt{\delta_\mathrm{max}^2 - \delta^2(t)} \end{aligned} \tag{2}$$

where v is the velocity. Based on Eq.(2), the maximum shearing force $Q_\mathrm{d,max}$ can be expressed in two forms:

$$Q_\mathrm{d,max} = cv_\mathrm{max} = c\omega_\mathrm{eq}\delta_\mathrm{max} \tag{3}$$

where v_max is the maximum velocity. Thus, the equivalent excitation frequency ω_eq can be expressed as

$$\omega_\mathrm{eq} = v_\mathrm{max}/\delta_\mathrm{max} \tag{4}$$

or in period form, which is easy to understand and use:

$$T_{eq} = 2\pi\delta_{max} / v_{max} \tag{5}$$

where T_{eq} is the equivalent excitation period, $T_{eq} \equiv 2\pi/\omega_{eq}$. Thus, the value of T_{eq} can be obtained by substituting the time history analysis results of δ_{max} and v_{max} into Eq.(5).

2.2. Time history analysis model

A SDOF model is established (Fig.1) and T_f is 1.0 s, 2.0 s, 3.0 s, 4.0 s, 5.0 s and 6.0 s, respectively. The damping ratio of VDs, ζ_d is 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0, respectively.

Natural records Taft 1952 NS, El Centro 1940 NS and Hachinohe 1968 EW (hereinafter referred to as Taft, El Centro and Hachi, respectively) are selected to do the time history analysis. In addition, Art Hachi is selected as well, which is an artificial earthquake wave using the phase characteristic of Hachinohe 1968 EW and taking pseudo velocity response spectrum $S_{pv} = 100\text{cm/s}$ (damping ratio $\zeta = 5\%$) as the target. The energy spectrum of the earthquake waves is shown in Fig.3.

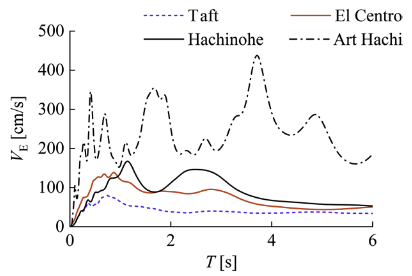


Fig.3 Energy spectrum

2.3. Empirical formula

To explore the relation between T_{eq} and T_f, define the ratio of T_{eq} to T_f as the equivalent period ratio, r_T, which can be expressed as

$$r_T \equiv \frac{T_{eq}}{T_f} = \frac{\omega_f}{\omega_{eq}} = \frac{2\pi}{T_f}\frac{\delta_{max}}{v_{max}} \tag{6}$$

The analysis results of r_T are shown in Fig.4, which is derived by substituting the time history analysis results of δ_{max} and v_{max} into Eq.(7). As can be seen, when $T_f \geq 2.0$ s, r_T has a concave function decreasing trend start from $r_T = 1$ with increase of ζ_d. Additionally, r_T decreases along with increase of period as well. To basically describe and envelope the analysis results, following inverse proportional function as an empirical formula of r_T is proposed in the range of $\zeta_d \leq 1$:

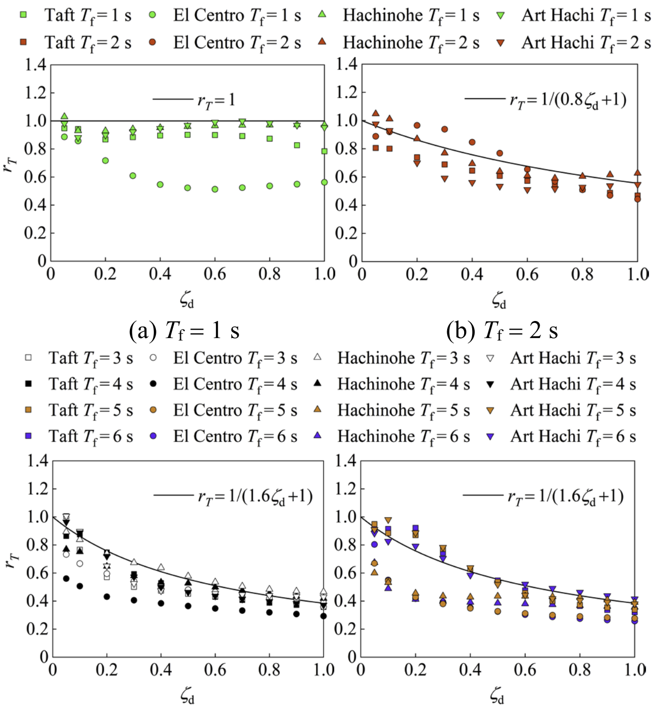
$$r_T = \frac{1}{\lambda\zeta_d + 1} \tag{7}$$

where λ is a periodic coefficient, determined by

$$\lambda = \begin{cases} 0 & T_f \leq 1 \\ 0.8T_f - 0.8 & 1 < T_f \leq 3 \\ 1.6 & T_f > 3 \end{cases} \tag{8}$$

Therefore, the empirical formula of T_{eq} can be expressed as

$$T_{eq} = \frac{T_f}{\lambda\zeta_d + 1} \tag{9}$$



(a) T_f = 1 s (b) T_f = 2 s

(c) T_f = 3 s, 4 s (d) T_f = 5 s, 6 s

Fig.4 Relationship between ζ_d and r_T

The reason why r_T approaches 1.0 with the reduction of damping is that when damping is low, the pseudo velocity response spectrum S_{pv} is approximately equal to the velocity spectrum S_v. Therefore, by replacing T_{eq} with Eq.(6) into Eq.(7), the equivalent period ratio, r_T can be deduced as

$$r_T \equiv \frac{T_{eq}}{T_f} = \frac{2\pi}{T_f}\frac{\delta_{max}}{v_{max}} = \omega_f \frac{S_\delta}{S_v} \cong \omega_f \frac{S_\delta}{S_{pv}} = \omega_f \frac{1}{\omega_f} = 1 \tag{10}$$

where S_δ is the displacement response spectrum.

The decreasing trend along with the increase of period can be interpreted by a better avoidance to the main frequency range of earthquake energy distribution. Further, since viscous damping can extend period, the increase of damping can decrease r_T as well. As a result, the reduction of r_T should be considered for isolated structures with VDs in isolation layer.

3. Prediction for structures with VDs in base isolation layer

3.1. Prediction formulas

In this Chapter, a prediction method considering the equivalent excitation period will be proposed for base-isolated structures with VDs in isolation layer based on the empirical formula of r_T. To simplify the deduction and make the design safer, it is assumed that all the seismic input energy is dissipated by the isolation layer, and the viscous damping energy dissipation of rubber bearings is ignored.

The shear coefficient of rubber bearings α_f, the shear coefficient of VDs α_d and the total shear coefficient α_1 are respectively defined as follows:

$$\alpha_f = \frac{Q_{f,max}}{Mg}, \quad \alpha_d = \frac{Q_{d,max}}{Mg}, \quad \alpha_1 = \frac{Q_{max}}{Mg} \tag{11}$$

Define the reference displacement δ_0 and reference shear coefficient α_0 as [7]

$$\delta_0 = \frac{T_\mathrm{f} V_\mathrm{E}}{2\pi} \tag{12}$$

$$\alpha_0 = \frac{2\pi V_\mathrm{E}}{T_\mathrm{f} g} \tag{13}$$

where V_E is the velocity conversion value of total input energy[7], g is the gravitational acceleration. The relation between V_E and the total input energy $E(t_0)$ is

$$E(t_0) = \frac{M V_\mathrm{E}^2}{2} \tag{14}$$

where t_0 is the end time of earthquake. Since α_0 and δ_0 can conceptually represent the response of isolation layer without VDs[2], $R_\mathrm{D} = \delta_{\max} / \delta_0$ can be defined as the displacement response ratio of the isolation layer with VDs to the uncontrolled isolation layer. Similarly, $R_\mathrm{A} = \alpha_1 / \alpha_0$ is defined as the total shear response ratio.

Based on Higashino and Kitamura's prediction method[3], the prediction formulas of R_D and R_A can be expressed as

$$R_\mathrm{D} = \frac{\delta_{\max}}{\delta_0} = \frac{\alpha_\mathrm{f}}{\alpha_0} = -\pi n_\mathrm{d} \frac{\alpha_\mathrm{d}}{\alpha_0} + \sqrt{\pi^2 n_\mathrm{d}^2 \left(\frac{\alpha_\mathrm{d}}{\alpha_0}\right)^2 + 1} \tag{15}$$

$$R_\mathrm{A} = \frac{\alpha_1}{\alpha_0} = \sqrt{\left(-\pi n_\mathrm{d} \frac{\alpha_\mathrm{d}}{\alpha_0} + \sqrt{\pi^2 n_\mathrm{d}^2 \left(\frac{\alpha_\mathrm{d}}{\alpha_0}\right)^2 + 1}\right)^2 + \left(\frac{\alpha_\mathrm{d}}{\alpha_0}\right)^2} \tag{16}$$

where n_d is the equivalent repetition number of VDs. For preliminary seismic design and response prediction, n_d can be determined by the following empirical formula[3]:

$$n_\mathrm{d} = \begin{cases} 2.0, & T_\mathrm{f} \le 5 \\ -0.33 T_\mathrm{f} + 3.67, & 5 < T_\mathrm{f} < 8 \\ 1.0, & T_\mathrm{f} \ge 8 \end{cases} \tag{17}$$

Based on Eq.(6) and (11), the following can be deduced:

$$r_T = 2\zeta_\mathrm{d} \frac{Q_{\mathrm{f,max}}}{Q_{\mathrm{d,max}}} = 2\zeta_\mathrm{d} \frac{\alpha_\mathrm{f}}{\alpha_\mathrm{d}} \tag{18}$$

By substituting Eq.(15) into Eq.(18), the expressions of α_d with respect to ζ_d can be obtained:

$$\frac{\alpha_\mathrm{d}}{\alpha_0} = \frac{2\zeta_\mathrm{d}}{\sqrt{r^2 + 4\pi n_\mathrm{d} \zeta_\mathrm{d} r_T}} \tag{19}$$

Further, substituting Eq.(19) into Eq.(15) can obtain R_D and $\delta_{\max}$ with respect to ζ_d:

$$R_\mathrm{D} = \frac{\delta_{\max}}{\delta_0} = -\frac{2\pi n_\mathrm{d} \zeta_\mathrm{d}}{\sqrt{r^2 + 4\pi n_\mathrm{d} \zeta_\mathrm{d} r_T}} + \sqrt{\frac{4\pi^2 n_\mathrm{d}^2 \zeta_\mathrm{d}^2}{r^2 + 4\pi n_\mathrm{d} \zeta_\mathrm{d} r_T} + 1} \tag{20}$$

and substituting Eq.(19) into Eq.(16) can obtain R_A and α_1 with respect to ζ_d:

$$R_\mathrm{A} = \frac{\alpha_1}{\alpha_0} = \sqrt{\frac{(8\pi^2 n_\mathrm{d}^2 + 4)\zeta_\mathrm{d}^2 - 4\pi n_\mathrm{d} \zeta_\mathrm{d} \sqrt{4\pi^2 n_\mathrm{d}^2 \zeta_\mathrm{d}^2 + r_T^2 + 4\pi n_\mathrm{d} \zeta_\mathrm{d} r_T}}{r_T^2 + 4\pi n_\mathrm{d} \zeta_\mathrm{d} r_T} + 1} \tag{21}$$

3.2. Prediction accuracy evaluation

A five-story shear structure model in which the bottom story is replaced by an isolation layer is established (Fig.5). In the original structure, the mass of each story is uniformly distributed. The lateral stiffness is trapezoidal distribution, and the lateral stiffness of top story is 0.5 times of the bottom story. The natural vibration period of the original structure is 0.3 s. The specific parameters of each story of the original structure are shown in Table.1. Laminated rubber bearings in isolation layer are elastic and undamped. The mass of the isolation layer is 980,000 kg, so the total mass of the structure M = 4,900,000 kg. The total lateral stiffness of the laminated rubber bearings is 21500kN/m and 12000kN/m respectively, and the corresponding natural vibration period, $T_\mathrm{f} = 4\pi^2 M / k_\mathrm{f}$ is 3.0s and 4.0s respectively. The damping ratio of VDs, $\zeta_\mathrm{d} = cT_\mathrm{f} / (4\pi M)$ is 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0, respectively. The viscous damping of the rubber bearings and superstructure is set as 0 in this study.

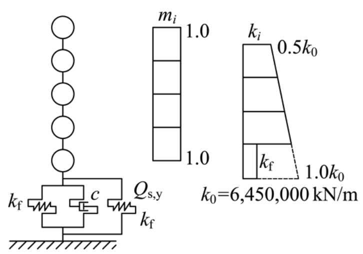


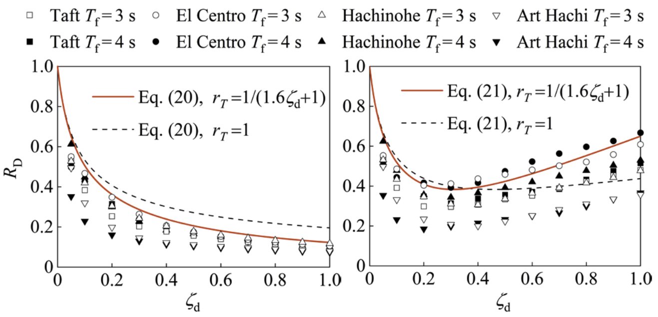
Fig.5 Analysis model

Table.1 Original structure parameters

Story	Mass/kg	Stiffness/(kN·m^{-1})
5	980,000	3,225,000
4	980,000	4,031,250
3	980,000	4,837,500
2	980,000	5,643,750
1	980,000	6,450,000

Fig.6(a) shows a comparison between the analysis results of R_D and the curve of Eq.(20) setting $n_\mathrm{d} = 2.0$ determined by Eq.(17). As can be seen, the proposed method can envelope the analysis results of R_D better than the existing method in a wide range. With the increase of damping, the deviation between the analysis results and existing method increases, which means a low and conservative accuracy for structures. Oppositely, the envelope performance of the proposed method basically remains good and unchanged in $\zeta_\mathrm{d} \le 1.0$.

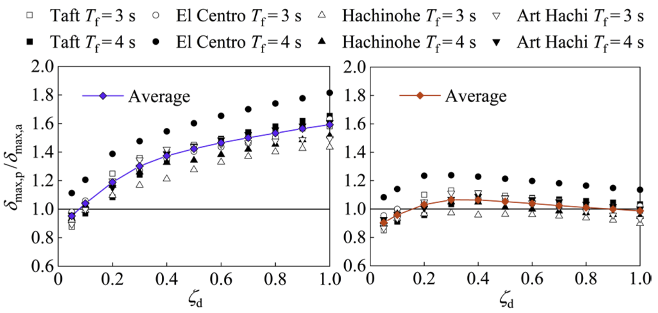
Fig.6(b) shows a comparison between the analysis results of R_A and the curve of Eq.(21) setting $n_\mathrm{d} = 2.0$ determined by Eq.(17). It can be seen from Fig.6(b) that with the increase of damping, the existing method gradually make the prediction curve obviously lower than the analysis results from $\zeta_\mathrm{d} = 0.3$. But the proposed method can basically predict the upper limit of the analysis results of R_D in the wide range of damping. In addition, there is an obvious minimum point in the distribution of R_A. Correspondingly, the curve of the proposed method also has an extreme value, which can describe the characteristics of the minimum values well.



(a) R_D (b) R_A

Fig.6 Comparison of analysis value and prediction curve

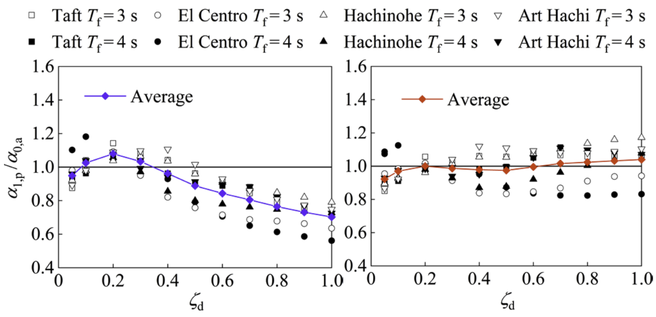
Fig.7 shows a comparison between the displacement ratio of prediction result to analysis result, $\delta_{max,p}/\delta_{max,a}$. To eliminate the influence of n_d, the values of n_d used in the prediction process are the analysis results. As can be seen, the displacement prediction accuracy of the existing method decreases with the increase of ζ_d, and the average value of $\delta_{max,p}/\delta_{max,a}$ is larger than 120% when $\zeta_d \geq 0.2$, which will let the prediction be too conservative. However, the average value of $\delta_{max,p}/\delta_{max,a}$ of the proposed method is belong to 90%-110% from $\zeta_d = 0.05$ to $\zeta_d = 1$. Therefore, the proposed method can keep a high prediction accuracy for displacement prediction in the range of $\zeta_d \leq 1.0$.



(a) Existing method (b) Proposed method

Fig.7 Prediction accuracy of δ_{max}

Fig.8 shows a comparison between the total shear coefficient ratio of prediction result to analysis result, $\alpha_{1,p}/\alpha_{1,a}$. To eliminate the influence of n_d, the values of n_d used in the prediction process are the analysis results. As can be seen, in general, since the average values of the proposed method is closer to 1 from $\zeta_d = 0.05$ to $\zeta_d = 1$, it can be considered that this new method has a better prediction accuracy. When $\zeta_d > 0.4$, the average values of the existing method continue to decrease and become less than 80%, which represents a low accuracy and unsafe prediction in this damping range. However, the average values of the proposed method maintain a good value belonging to 95%-105%, which means that the proposed prediction method can keep a high accuracy for shear coefficient prediction in a wide range.



(a) Existing method (b) Proposed method

Fig.8 Prediction accuracy of α_1

4. Conclusions

In this paper, an empirical formula for the equivalent period ratio of SDOF system with viscous damping is proposed by using equivalent simple harmonic load. Based on the empirical formula, a seismic response prediction method which can consider the influence of the equivalent excitation period for structures with VDs in isolation layer is proposed and the prediction formulas are deduced. The prediction accuracy of the proposed method is verified by dynamic time history analysis. Following conclusions are obtained:

1) The equivalent period ratio decreases with the increase of damping or period. The proposed empirical formula can describe the value of the equivalent period ratio well.

2) The existing prediction method makes the displacement prediction too conservative and makes the prediction accuracy of shearing force not good, even in dangerous side.

3) The proposed method using the empirical formula of equivalent period ratio can obtain obviously better accuracy for seismic response prediction of isolation layer in a wide range of the damping ratio of VDs.

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