論文 / 著書情報
Article / Book Information

題目(和文)	
Title(English)	The evolution of cooperation in the unidirectional division of labour on networks with sanction systems
著者(和文)	エムディ シャムス アフィフ ニルジョル
Author(English)	Md Sams Afif Nirjhor
出典(和文)	学位:博士(工学), 学位授与機関:東京工業大学, 報告番号:甲第12758号, 授与年月日:2024年3月26日, 学位の種別:課程博士, 審査員:中丸 麻由子,後藤 美香,辻本 将晴,大和 毅彦,笹原 和俊
Citation(English)	Degree:Doctor (Engineering), Conferring organization: Tokyo Institute of Technology, Report number:甲第12758号, Conferred date:2024/3/26, Degree Type:Course doctor, Examiner:,,,,
学位種別(和文)	博士論文
Type(English)	Doctoral Thesis

The evolution of cooperation in the unidirectional division of labour on networks with sanction systems

MD SAMS AFIF NIRJHOR

Dissertation for the Doctor of Engineering Degree

presented to

Department of Innovation Science

School of Environment and Society, Tokyo Institute of Technology

3-3-6, Shibaura, Minato, Tokyo, 108-0023, Japan

Supervisor:

Professor Mayuko Nakamaru

Acknowledgements

First of all, I express my deepest gratitude to my supervisor Professor Mayuko Nakamaru, for her kind help and support through out my doctoral course. Without her contribution this research would not be possible. I also thank my sub-supervisor Professor Mika Goto for her valuable comments on my research and mentoring. I also thank Professor Kazutoshi Sasahara, Professor Masaharu Tsujimoto, and Professor Takehiko Yamato for their valuable comments on my work.

I also express my gratitude to my parents MD Abul Kalam Azad Munshi and Nasrin Akhter, my spouse Afsari Banu Alpona, for always being supportive throughout my career. Special thanks to my uncle Ifte Hasan, who brought me to Japan and helped me through out my university career as a mentor. I also thank my dear friend Asish Kumar Ghosh for helping me with my programming problems every now and then.

MD SAMS AFIF NIRJHOR

Contents

Chapter 1

Introduction

1.1 Division of labour

Not only the modern world but previous studies have shown that division of labour was present in early human societies as the hunter-gatherer society (Ember, 1978). Even the early societies without government had the division of labour (Chauvin and Ozak, 2017). In pre-industrialized societies, the division of labour was often connected to their evolution (e.g. Nolan and Lenski, 2011; Durkheim, 1893). In the industrialized civilization division of labour creates specialization. Modern countries' economical and political structures are becoming more and more similar to each other and they are adopting the same open global market policy, thus making the division of labour international. This fact is causing more and more interindustry and intraindustry specialization (Krugman, 1981). Dividing and allocating the tasks naturally results in networks (e.g. Mersch, 2016, Fengru and Guitang, 2019), where the people or the groups of people to whom the subtasks are given are present in the nodes of the network and the edges of the network represent the flow of the task. So the study of the division of labour on the networks is important for practical applications. There are largely two types of networks in the division of labour considering the flow of the task, horizontal and vertical (Barley, 1996). In the horizontal division of labour, the divided subtasks are done parallelly by specific experts of those subtasks, often having very little dependency on each other, which is now common in universities and R&Ds. The vertical division of labour where the authority of position and expertise coincide, creating a flow of the task based on hierarchy. The vertical division of labour can explain the hierarchical systems of bureaucracy, and most economic systems prior to the nineteenth century in contrast to the horizontal division of labour which is only dominant in the closed controlled environment (Barley, 1996).

The importance of the division of labour in modern economic systems such as the supply chain is self-evident (e.g. Krugman, 1981; Fengru and Guitang, 2019). The supply chain is now an inter-border and cross-border division of labour which can be depicted by network-focused models (Henderson et al., 2002; Roper and Grimes, 2005). Min and Zhou (2002) explained the linear supply chain to be consisted of suppliers, manufacturers, distributors, retailers, and customers. Farooque et al. (2019) included the resource gathering from environment at the top of the linear chain considered by Min and Zhou (2002), and waste disposal at the end of the linear chain. The manufacturing is now often done through the global production network, which itself has a complex network structure of division of labour (Fengru and Guitang, 2019). The supply, distribution, and retailing all are consisted of complex networks (Ghiani et al., 2004). The waste disposal is also a linear division of labour (Nakamaru et al., 2018). Therefore, although the minimum structure of the supply chain is linear, it often becomes more complex and expanded by the number of roles. According to Lambert and Cooper (2000) and Cooper et. al (1997), the supply chain looks like an uprooted tree, where most of the components of linear division of labour are divided into multiple tiers. Thus studying the supply chain as a division of labour of finite roles on complex networks is required.

The subcontract systems which relies on the many-layered intraproduct specialization and a contractor depends on the specialist subcontractors below him to finish his task or product, have the division of labour (e.g. Tam et al., 2011). The subcontract system is a system where a task is given to a principal contractor along with some monetary benefit, who then keeps some benefit to himself and gives it to some other contractor/contractors according to their specialization often with a less monetary benefit to the later sub-contractor/contractors. The sub-contractors can later duplicate the process in their downstream, thus creating a multilayered sub-contract system (Tam et al., 2011). From the perspective of the principal contractor and other contractors who are again subcontracting their tasks, division of the task, choosing the subcontractors and passing the subtasks to the subcontractors are their subtasks. The fulfilment of those corresponding subtasks are the subcontractors' subtask. This also creates a complex network of the division of labour as showed in Tam et al. (2011). Chiang (2009) pointed out that the cooperation of the specialist sub-contractors as well as the cooperation of the principal contractor in choosing the suitable sub-contractors are essential for a successful sub-contract system.

The bureaucratic systems in modern countries where the official duties are divided along the subordinates also have the division of labour (e.g. Eisenstadt, 1958; Hinings et al., 1967; Bolin and Härenstam 2008). Most government works can be considered to be a division of labour (Bezes and Le Lidec, 2016).

Cooperation is a key in the division of labour. The division of labour is studied lately in terms of evolutionary game theory. There are some theoretical works dealing with the division of labour (e.g. Kuhn and Stiner, 2006; Henrich and Boyd, 2008; Nakahashi and Feldman, 2014; Powers and Lehmann, 2014; Roithmayr et al., 2015; Zhang et al., 2018; Qin et al., 2020; Zhang et al., 2020). These studies considered the evolution of the division of labour when there are two or three social roles. Kuhn and Stiner (2006) investigated the evolution of division of labour among sex and age in the human society as an effect of the increased dietary diversity which eventually helped the early Homo sapiens to win over other hominin species in Eurasia. Henrich and Boyd (2008) showed that the economic specialization increases production as well as results in further social stratification, which eventually leads to the constant division of labour. Nakahashi and Feldman (2014) concluded that when the population is large and developing new abilities is needed to acquire resources, specialization at the individual level is favored and the division of labour evolves into the group. Nakahashi and Feldman (2014) also showed that the division of labour among genders can evolve based on the importance of learning and the difference in the efficiencies among the genders. Powers and Lehman (2014) showed the evolution of hierarchy in the transition form small egalitarian society to large stratified despotic society. Powers and Lehman (2014) studied whether division of labour is preferred by dividing a population into two groups, one of which has hierarchy and roles of leader and follower, and another has only acephalous population that does not let a leader to rise and do not themselves become leaders. This study concluded that the evolution of leadership and thus division of labour among leader and follower will be stable when the leadership can lead to higher production of resources. Roithmayr et al. (2015) showed the emergence of institutionalized punishment as a key towards human cooperation, thus the evolution of the roles of the punisher state and the citizen, which is eventually a division of labour by itself.

Basically, many previous studies assumed a non-structural division of labour among an infinite number of players (e.g. Zhang et al., 2018; Yuan and Meng, 2022). The evolutionary dynamics of the division of labour on a cycle network has also been studied using the division of labour game in which two players, who play different roles from each other, get a higher payoff than two players who play the same role, and it is assumed that each player can choose one of two roles and plays the game with their neighbors on the cycle network (Zhang et al., 2020). The division of labour is a premise of supply chain, which has been investigated especially in management science, and no studies about supply chain with network structure have been done by the evolutionary game theory yet (e.g. Min and Zhou, 2002).

Why the evolution of cooperation is important in the division of labour? Division of labour is dividing a large task into several smaller subtasks and allocating those subtasks to several people or groups of people and completing that whole task by means of cooperation among those people or groups of people by completion of their own particular subtasks. Thus, without cooperation the labour can not be completed, resulting into unsuccessful division of labour (Baumgardner, 1988). For this simple reason, cooperation is very important in the division of labour. However, division of labour is rather less studied from the context of evolution of cooperation in the human society. This motivated us to investigate the evolution of cooperation in the division of labour from the viewpoint of evolutionary game theory.

1.2 Evolution of cooperation

Evolution of cooperation is still an unsolved problem. Even though mutual cooperation might have had greater benefits, if mutual trust is not present, humans rationally choose mutual defection instead (Tucker, 1983). The prisoner's dilemma (Tucker, 1983) (hereafter, PD game) can describe why individuals fail to cooperate; if both are cooperators, their payoff is R; if both are defectors, their payoff is P. The payoff of a defector playing the PD game with a cooperator (T) is higher than the cooperator (S), and the order of the payoffs is $T > R > P > S$. As a result, players tend to choose defection rather than cooperation regardless of the opponent's choice. Then, this game has its solution in mutual defection, although mutual cooperation would have given a better payoff for both. This gives rise to the problem of mutual defection in social dilemma (Kollock, 1998). However, in spite of this problem being present, cooperation still exists, and even in very large scale. Mutual cooperation is the foundation of human civilization (Turchin, 2016) and our society have prospered through mutual cooperation (Kollock, 1998). Understanding the reasons behind the existence and persistence of cooperation in our society may give us the opportunity to establish cooperation even when and where it is not present.

The theoretical and experimental studies on the evolution of cooperation have investigated what mechanisms promote the evolution of cooperation and hinder defectors and free-riders on the basis of the evolutionary game theory.

Nowak (2006) and Rand and Nowak (2013) describe that there are five mechanisms to encourage cooperation. These studies consider that through an act of cooperation, a cooperator pays the cost c, and the person to whom the cooperation was done receives a benefit b. [1] Kin selection which shows that the related individuals are more prone to cooperate with each other (Hamilton, 1964). The relatedness which means the probability of sharing a gene, is shown with the relatedness factor r and if $r > c/b$, then cooperation evolves (Hamilton, 1964). [2] Direct reciprocity (e.g. Axelrod and Hamilton, 1981; Nowak and Sigmund, 1993, Press and Dyson, 2012), which concludes that the action to one player

A done by another B player has an effect on A's future action towards B. In the repeated interactions between these two players, cooperation evolves if the probability of having one more future interaction, $w > c/b$. [3] Indirect reciprocity (e.g. Sugden, 1986; Nowak and Sigmund 1998, 2005; Nakamaru and Kawata, 2006; Ohtsuki and Iwasa 2004), where the reputation of some player A of doing the altruistic act, has an effect on others' action toward him/her. This mechanism can evolve cooperation if the probability of knowing the reputation, $q > c/b$. [4] Network reciprocity (e.g. Nowak and May, 1992; Nakamaru et al., 1997, 1998; Ohtsuki et al., 2006; Pacheco et al., 2008; Santos et al., 2008; Su et al., 2022), which includes the idea that when the players are in the nodes of a graph and the edges of the graph represent their interaction, cooperators can create clusters to cooperate with each other, resulting into the evolution of cooperation. This rule dictates that, when The average number of edges from a node, in other words, the average interacting neighbors of a player, $k < b/c$, the cooperation evolves. Finally, [5] group selection (e.g. Sober and Wilson, 1999; Traulsen and Nowak, 2006), which implies the idea that the selection happens not only in the individual level but also group levels. A population is split into groups, where cooperators cooperate with each other in the group basis and have a better payoff thus higher reproduction than the defectors who defect with each other in the groups basis. With higher reproduction, as the population within the cooperator group increases it splits into two and to constrain the overall population, defector groups die off. In this way, even though the defectors are more successful than the cooperators in the individual level, the group selection makes the evolution of cooperation. The condition for evolution of cooperation is $(1 + n/m) < b/c$, where n is the maximum population of a group and m is the number of groups.

In addition to these, the utilization of sanctions to enforce cooperation has also been investigated (e.g. Axelrod, 1986; Sigmund et al., 2001; Boyd et al., 2003; Nakamaru and Iwasa, 2005, 2006; Rand et al., 2010; Sigmund et al., 2010; Shimao and Nakamaru, 2013; Chen et al., 2014, 2015; Roithmayr et al., 2015; Sasaki et al., 2015). Axelrod (1986) discussed the sanctions as an outcome of vengefulness, which is costly to the individual who is applying it on the defector while witnessing a defection. Although the sanctioning cost is way less than the sanction itself, eventually the costly sanctioning can not sustain the evolution of cooperation, and defection wins as a stable strategy if there is no incentive to the sanction givers. However, with the incentive of a second order sanction, which is sanctioning the individuals who chose not to apply sanctions on the defectors, cooperation can evolve even with costly sanctioning. When the sanction is costly to the individual who is giving it, however not directly benefiting the sanction giver, this is called an altruistic punishment or sanction. Boyd et al. (2003) showed that as the problems of altruistic sanction and altruistic cooperation are different from each other, altruistic sanction can evolve through group selection and eventually promote cooperation among non-relatives. Sigmund et al. (2001) showed that sanctioning the defectors works better than rewarding the cooperators in inducing the evolution of cooperation, when the reputation of the sanction giver or the reward giver is available as information. Nakamaru and Iwasa (2005) studied the evolution of altruistic sanction on a lattice-structured population, and showed that the altruistic sanction as a strategy can evolve when the benefit from the cooperation or the fine are large. Nakamaru and Iwasa (2006) found that selfish sanction givers who are defectors themselves, however, give sanction to other defectors by paying a cost, promote the evolution of altruistic sanction givers who are cooperators, and also give sanction to defectors. Rand et al. (2010) studied the effects of anti-social sanctions on the evolution of costly altruistic sanctioning and cooperation. The anti-social sanction means when sanction is given to the cooperators. When the anti-social sanction is present, it hinders the evolution of cooperation through costly sanction (Rand et al., 2010). Shimao and Nakamaru (2013) compared graduated sanctioning which means the severity of sanction gradually changes with the severity of defection, with strict sanctioning where the sanction level changes drastically based on a threshold of cooperation. They found that when a spatial structure is present among the population strict sanctioning works better and when the interaction happens at random, the graduated sanctioning works better towards evolution of cooperation. Chen et al. (2014) discussed the effects of probabilistic sharing of the costly sanctioning as the costly sanctioning itself becomes an altruistic act and thus the players can defect by not choosing to sanction the defectors, which is called the second order defection. The probabilistic sanction means that there is a probability that some of the players are sanction givers and the cost of sanctioning the defectors is shared among all the sanction givers. Chen et al. (2014) showed that the probabilistic sanctioning helps promoting the evolution of cooperation by solving the problem of costly sanctioning. Up until now we have discussed the studies related to so called peer-punishment or where the players sanction their game-mates by paying a cost. However, there is another types of sanction which is called the pool-punishment, where the sanction is given through an institution that acts as an overseer of the system and represents the governing body of the whole population. Sigmund et al. (2010) investigated the application of institutionalized sanction towards the evolution of cooperation. They showed that when there is costly peer sanctioning available and also there are players who do not choose to sanction defectors because of this cost, pool-punishment or institutionalized sanctioning works better in the evolution of cooperation than the peer-punishment. However, pool-punishers are needed to be hired by the taxation of the population, which can give rise to another second order defection where the defectors do not want to hire the pool-punishment enforcer.

Sasaki et al. (2015) discussed the problem of defection in hiring the pool-punishment enforcer can be eliminated through rewarding the participants who are cooperators and pave the way for a stable pool-punishment system.

To conclude, there are 6 mechanisms to promote the evolution of cooperation, kin Selection, direct reciprocity, indirect reciprocity, network reciprocity, group selection and sanctions. In this thesis, we focus on the evolution of cooperation through sanction systems.

1.3 Purpose of research

Many previous theoretical studies about the evolution of cooperation considered that two or more players who have the same position as well as have the same payoff matrix regarding those choices. Rather, in reality, players take different positions, which the asymmetric game can describe. For instance, parental investment to care for their offspring was analyzed by the asymmetric game by means of evolutionary game theory (Maynard Smith, 1977). It is because the fitness of the male is different from the female in parental care; for example, the female reproduces larger gametes incurring a larger cost, but the male reproduces smaller gametes incurring a smaller cost. This asymmetry is caused by sexual role, depending on the survivorship, the number of eggs and the chance to mate with other females. Another example of the asymmetric relationship among players is cooperation among different social roles where hierarchical social relationships exist (Henrich and Boyd, 2008; Powers and Lehmann, 2014; Roithmayr et al., 2015). The division of labour is one of the other examples of the asymmetric relationship between players (Kuhn and Stiner, 2006; Henrich and Boyd, 2008; Nakahashi and Feldman, 2014). Because, as different players in different roles obtain different benefits and costs, the asymmetric game can be used for this system. Szolnoki and Szabó (2007) studied the evolution of cooperation in the asymmetric Prisoner's Dilemma, where the asymmetry is present in the strategy adaption rule of the players. McAvoy and Hauert (2015) studied the evolution of cooperation in the asymmetric evolutionary games of social dilemma based on two types of asymmetry, ecological and genotypic. However, the evolution of cooperation in the division of labour from the context of asymmetric games has not been thoroughly studied previously, so in this thesis, we investigate that domain.

Many previous studies on evolutionary games on networks considered when each node represents a single player (e.g. Boyd and Richerson, 1989; McAvoy and Hauert, 2015; Zhang et al., 2020; Su et al., 2022). Boyd and Richerson (1989) studied the repeated PD game on cycle network. Zhang et al. (2020) studied the evolution of the division of labour in the circular network. Su et al. (2022) considered the effects of the properties such as directionality, regularity, orientation, etc. of graphs on the evolution of cooperation. A recent paper by Sharma et al. (2023) studied the effects of self-loop on the fitness of the players on a network based on evolutionary graph theory. However, because of the specialization happening in our world, in the division of labour, often the candidate for a subtask is not a single player, but a group of players, from which one should be chosen. For example in the supply chain there can be many competing farms for the same task (e.g. Fengru and Guitang, 2019). In the sub-contract systems there can be many possible subcontractor for the same task from whom one should be chosen by the main contractor (Chiang, 2009). However, the evolution of cooperation in the division of labour on networks where the nodes of the network represent a group of players is an under-investigated field. In our thesis, we consider the network of groups for the division of labour. We used the replicator equation because it was easier to model such scenarios with it.

Previous studies have mostly studied the evolution of the division of labour with two or three roles. In reality, there are often more than two or three social roles in network structures in the division of labour as discussed in the previous section of the division of labour. For example, the supply chain has several stakeholders and consists of several suppliers, manufacturers, distributors, retailers, and customers on the basis (e.g. Min and Zhou, 2002; Fengru and Guitang, 2019). However, the division of labour with network structure where different roles are represented by different nodes, is largely unexplored in terms of evolutionary game theory.

To our knowledge, Nakamaru et al.(2018) was the first to explore the linear division of labour, where each subtask must be performed in sequence in a line, using the evolutionary game theory. They take the industrial dumping system in Japan as an example. The model structure is shown in the figure 1.1. In the industrial dumping system, five subtasks are needed to treat the waste completely. Then, there are companies which are experts in one of five subtasks. A company in subtask 1 asks a company in subtask 2 to treat the waste, paying a monetary cost. The company in subtask 2 accepts the offer and receives the money from the company in subtask 1. After the company finishes treating the waste with paying the cost of treatment, the company asks a company in subtask 3 to treat it, giving money to the company in subtask 3. This process continues. If a company in subtask 4 asked a company in subtask 5 to treat the waste and the company in subtask 5 accepts the offer and completes the task, the industrial waste is treated successfully and completely. If a company in one subtask, for example, dumps the waste illegally, all companies have damage from the illegal dumping. Nakamaru et al.(2018) considered three subtasks to make the model simple, assuming that a large number of players are in

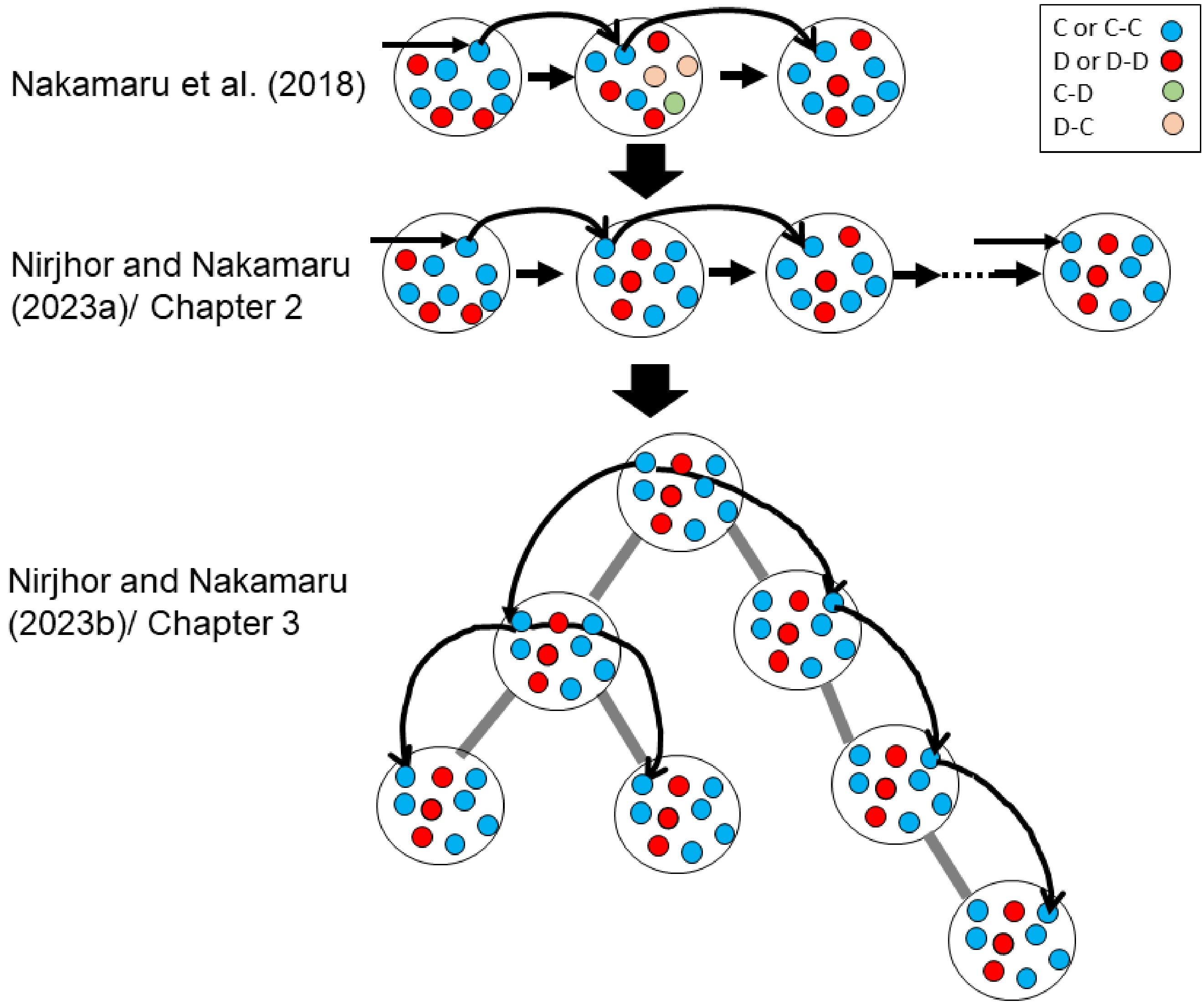

Figure 1.1: Relation of this thesis with Nakamaru et al.(2018). C is for a cooperator, D is for a defector. In Nakamaru et al. (2018) there was only three roles, however in the 2nd group there were 4 types of strategy. In Nirjhor and Nakamaru (2023a) which corresponds to the chapter 2 in this thesis there are finite number of roles in the linear division of labour. In Nirjhor and Nakamaru (2023b) which corresponds to the chapter 3 in this thesis it is a division of labour on a finite unidirectional tree graph.

each group where one subtask is assigned to players. A player in the group with subtask 1 chooses a player from the group of subtask 2 and asks the player in subtask 2 to treat the industrial waste properly, giving money to the player. The players for subtask 2 two can have 4 different strategies, they can treat the waste and give it to the chosen player in the group for subtask 3, in other words cooperate-cooperate (C-C); or they can treat the waste but do not give it to the chosen player in the group for subtask 3 and illegally dump, in other words cooperate-defect (C-D); they can choose to not treat the waste but give it to the chosen player in the group for subtask 3, in other words defect-cooperate (D-C); and finally they can choose to neither treat the waste and nor give it to the chosen player in the group for subtask 3, and illegally dump it, in other words defect-defect (D-D). If the player in subtask 2 finishes treating the waste and the player asks a player in subtask 3 to treat the waste properly, then the task flows to the group of subtask 3. If the player in subtask 3 finishes treating it, paying the cost of treatment, the division of labour is completed. Players update their strategy, imitating the strategy of a player with a higher payoff in the group of the same subtask. However, if a player in one of the subtasks does not treat the waste and illegally dumps it, all players in the three subtasks suffer the damage from the illegal dumping and a restoration fee is imposed on all players in all subtasks. As they regard treating the waste properly as cooperation and dumping it illegally as defection, this system can be studied by means of "the evolution of cooperation." As the payoffs of different players in different groups are asymmetric, this system can be modelled by the replicator equations for asymmetric games. They made concrete assumptions to fit the real industrial dumping system, and investigated if either of two existing sanction systems, namely the producer responsibility system and actor responsibility system, can promote the evolution of cooperation by means of the replicator equations for asymmetric games. In the former system if defection happens in the linear chain, whoever defects, the player in the first group gets punished by the supervision. In the later system, if defection happens, the defector is detected and gets punished by the supervision. It was shown that the sanction systems, especially the producer responsibility system when it is almost impossible to monitor and detect defectors, can promote cooperation more than the actor responsibility system. Hereafter, the former sanction system is called the first-role sanction system; the latter is the defector sanction system.

1.4 The structure of the thesis

In this thesis, we investigate the evolution of cooperation in the division of labour on two types of networks of groups, first a linear graph and then a unidirectional tree graph. In this chapter, in the following sections we shall discuss some literature on the division of labour and evolution of cooperation which will eventually make the purpose and coordination of our study clear. In other words, the literature review will emphasize on the importance of the studies on evolution of cooperation in the division of labour on networks when the subtasks are allocated to groups.

In the following chapter 2 we expand the study of evolution of cooperation in the division of labour on a linear chain with finite number of roles which are allocated to a finite number of groups of only cooperators and defectors. Our model is simpler than Nakamaru et al. (2018) because they have considered 4 different strategies for one of the groups, whereas we consider only possible strategies for all the groups. In doing so, we fill in the gaps of applicability of the former study of Nakamaru et al. (2018), as the finite number of roles in a linear chain allows us to model many more types of linear division of labour. Then in chapter 3 we further expand the study of the evolution of cooperation in the division of labour on a finite unidirectional tree graph. The study on the tree graph allows us to model many more finite branched unidirectional system of the division of labour and we come to understand that the linear division of labour can be considered as a special case of the more general division of labour on a tree graph. In both of these studies we consider two types of institutionalized sanction system as Nakamaru et al. (2018). First of these sanction systems is called the defector sanction system in both of the studies, where the defector is caught and sanctioned directly. The second is where there is a specific group who is sanctioned when any defection happens, which is called the first role sanction system in the linear division of labour model and the premier sanction system in the division of labour on a tree graph model. In both of these sanction systems we do not consider the traditional payment to the sanction giving institution by the players for the sake of simplicity. To make it more clear we attach a flow-chart through figure 1.4, which shows the exact coordination

Chapter 2

Linear graph study

2.1 Introduction

In this chapter, we generalize the three groups model studied by Nakamaru et al. (2018) into any countable number of groups which are in line. We assume that players in one group play one role and that a player in an upstream group interacts with a player in a downstream group. We also generalize the model which can be applied to other systems besides the industrial damping system and make a simple assumption. We will investigate whether the sanction systems can promote the evolution of cooperation or not.

There are some evolutionary game theoretical studies that seem similar to our framework. The effect of network structures on creating cooperation in asymmetric social interactions has been studied (e.g. McAvoy and Hauert, 2015; Su et al., 2022). Their studies seem to include ours, however, it is not true. In their studies, each player is in each vertex, a player imitates the strategy of the neighbour who is a partner in the game if the payoff is higher than that of the player. For example, Su et al. (2022) concentrated on how edge orientation, directionality, regularity, and properties of graphs will change the evolution of cooperation, assuming that the cooperation cost and benefit for all players are the same. While in our study, each group is in the line and has a large number of players. A player imitates the strategy of others in the group. A player chosen randomly from an upstream group never plays the game with another player in the same group but does with a player chosen randomly in the downstream group. The players in different roles have different cooperation costs and benefits. Therefore, from the viewpoint of mathematical modeling, our study proposes a model which the previous studies about the evolution of cooperation in the network structure have not assumed.

We have only compared our study with the previous ones which assumed that each player is located at each node of the network. There are studies about the evolution of cooperation assuming that each group is located at each node of the two dimensional lattice, players move to their neighbouring groups (e.g. Wakano et al., 2009). These studies did not assume a player in a group plays the game with another in a different group.

In our model, players play the game sequentially but our game is different from the sequential game studied previously in economics where the information of the previous player is available to the latter player (e.g. Varian, 1994; Sessa et al., 2020). In "the two-person sequential game", for example, a player chooses the strategy after the opponent chooses the strategy, and upon that information changes his or her strategy (Varian, 1994; Sessa et al., 2020). After two players choose their strategy, they can receive the payoff.

Therefore, our study gives a new perspective that the division of labour can be studied from the viewpoint of "the evolution of cooperation." Moreover, we can propose a model which the previous studies had not assumed and analyzed from the viewpoint of the mathematical models in "the evolution of cooperation."

2.2 Three Models

2.2.1 Baseline system

We present the baseline system (see figure 2.1), where there are n roles ($n \in \mathbf{N}$ and $n \geq 2$). Here $\mathbf{N}$ is the set of natural numbers. For $n = 1$, there is no linear division of labour. There are n groups in the whole population. Each group is allocated to one role, and the group size is infinite. Each group consists of cooperators and defectors. We define cooperators as players who only cooperate, and defectors as players who only defect. We do not assume a mixed strategy in which players choose either cooperation or defection by probability. The frequency of cooperators in group i is i_c and the frequency of defectors is i_d. Here, $i_c + i_d = 1$. It is assumed that one player chosen randomly from the group i interacts with a player chosen randomly from the group $i+1$ ($1 \leq i < n$).

We consider that the final product or service is produced through the division of labour; if players in all roles cooperate together to produce the product or service, the final product or service can be completed (figure 2.1a). A cooperator in an upstream group produces and modifies the product or service, paying a cost of cooperation, and gives it to a player in a downstream group. Let x_i be defined as a cost of cooperation by a cooperator in the group i. The value of the product or service is regarded as the benefit to the player in the group $i+1$, b_{i+1}. In this case, the net benefit of cooperators in the

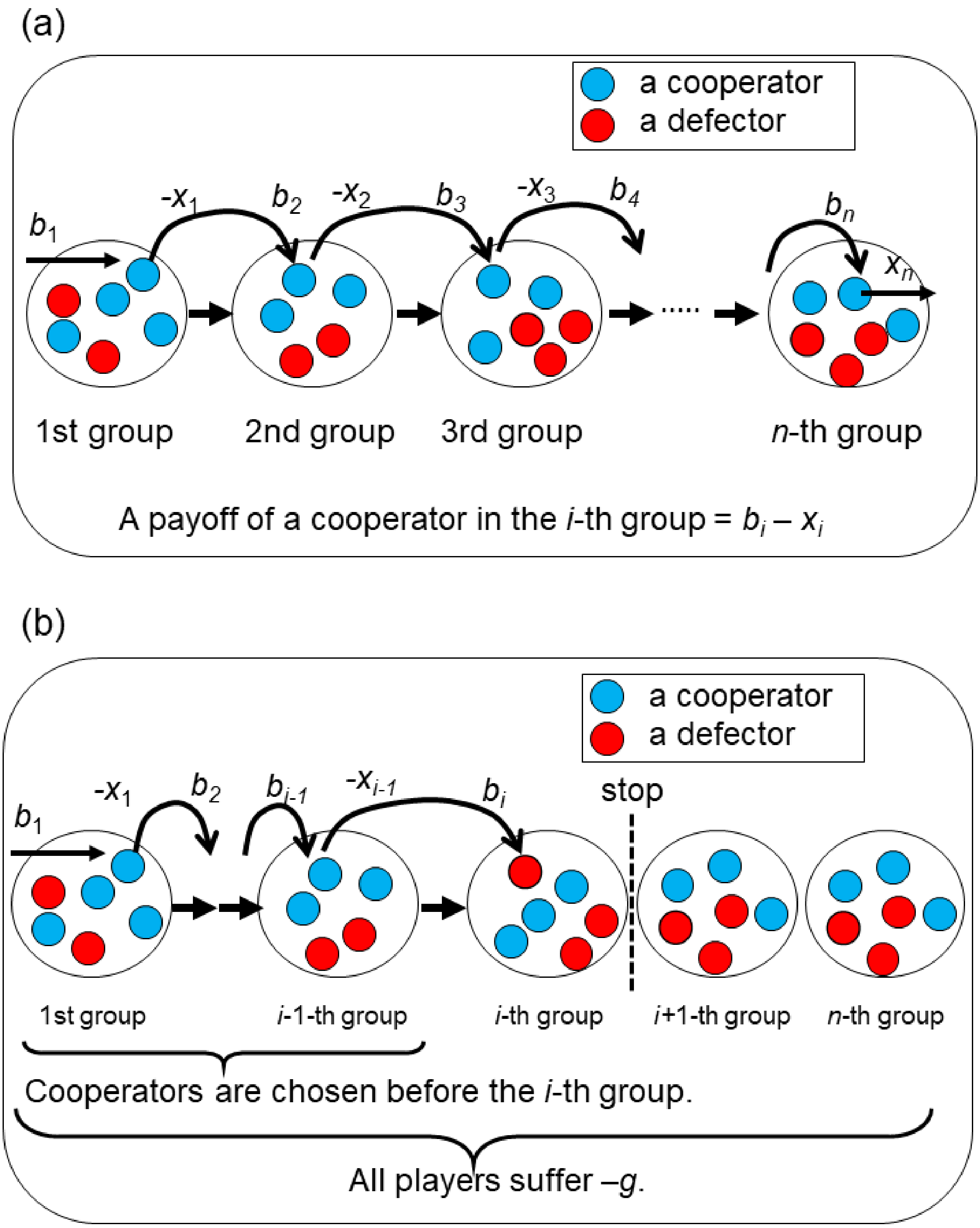

Figure 2.1: (a) Linear graph showing n division of labour when cooperators are chosen in all groups. The payoff of a cooperator in the ith group is $x_{i-1}-x_i$. (b) Linear graph showing n division of labour when cooperators are chosen before the ith group and then a defector is chosen in the ith group. Once a defector is chosen, the whole system is broken. As a result, all players suffer $-g$.

group i is $b_i - x_i$ (Table 2.1).

For the player in the 1st group, the benefit comes from the source. The net benefit of the cooperator in group 1 is $b_1 - x_1$. In the nth group, a cooperator pays a cost, x_n, to produce the final product. and the payoff of the cooperator is $b_n - x_n$, where the player sells the final product and gets benefit b_n.

Table 2.1: The payoff matrix in the baseline system for a player in the ith group

Cases	All being cooperator except the ith	A defector before ith	A defector after ith
ith player cooperator	$b_i - x_i$	$-g$	$b_i - x_i - g$
ith player defector	$b_i - g$	$-g$	$b_i - g$

If a defector is chosen randomly from group i, the defector receives the benefit b_i and s/he does not cooperate with a player in group $i+1$ (figure 2.1b). As a result, the division of labour fails and the whole system is broken down. As a result, all players in all the groups get the same negative effect g in their payoff ; g can be interpreted as a bad reputation because the incomplete work gives a bad reputation to all members in the division of labour. The payoff of the defector is $b_i - g$ (Table 2.1). After the defector is chosen from group i, the players in the latter groups will not choose either cooperation or defection, and there can only be one acting defector player in the chain of the linear division of labour here. Therefore, the net payoff of players after defection occurs is $-g$ (Table 2.1).

Table 2.1 indicates that, when $g < x_i$, the game can be the PD game. Otherwise, mutual cooperation is always the best. In other words, when $g > x_i$ for all is, the cooperators' payoff is higher than the defectors. Therefore, we assume that $g < x_i$, which means that the baseline system has a dilemma situation.

For calculating the payoff matrix of a player in group i we need to consider three cases. First, the probability that all of the players in the rest of the groups are cooperators in the chain of the division of labour is defined as c_i, which is the product of the frequencies of the cooperators of the groups except i. Therefore, $c_i = \Pi_{j=1}^{i-1} j_c \Pi_{j=i+1}^{n} j_c$. Second, there are defectors in the groups before the group i, with probability d_{ib}. Thus, it is $d_{ib} = 1 - c_{ib} = 1 - \Pi_{j=1}^{i-1} j_c$, where c_{ib} is the probability that all the players chosen from the previous groups are cooperators. Third, the probability that defectors in the groups after the group i, is d_{ia}, where no defectors are in the previous groups. Thus, $d_{ia} = c_{ib}(1 - c_{ia}) = \Pi_{j=1}^{i-1} j_c (1 - \Pi_{j=i+1}^{n} j_c)$, where, c_{ia} is the probability of not having defectors after i. Here, $c_i + d_{ib} + d_{ia} = 1$.

After each player randomly chosen from each group interacts with a player randomly chosen from the next downstream group, the expected payoff of each player in each group can be calculated. Then, within each group, players decided to imitate a strategy of others, proportional to the expected payoff relative to the total payoff in the group. Here the random change of the strategy or mutation does not occur. Therefore, this interaction can be described by the replicator dynamics of the asymmetric game without mutation (Hofbauer and Sigmund, 1998).

The replicator equation of a cooperator in the group 1 in the baseline system is as follows,

$$\frac{d1_c}{dt} = 1_c(1 - 1_c)(P_{1_c} - P_{1_d}) = 1_c(1 - 1_c)\{c_1 g - x_1\}, \tag{2.1}$$

where the average payoff of cooperator in the group 1, $P_{1_c} = c_1(b_1 - x_1) + d_{1a}(b_1 - x_1 - g)$ and the average payoff of the defector in the group 1, $P_{1_d} = b_1 - g$.

The replicator equation for cooperators in the group i is (when $1 < i < n$) as follows,

$$\frac{di_c}{dt} = i_c(1 - i_c)(P_{i_c} - P_{i_d}) = i_c(1 - i_c)\{c_i g - x_i(1 - d_{ib})\}, \tag{2.2}$$

where the average payoff of the cooperators in the group i is, $P_{i_c} = c_i(b_i - x_i) + d_{ib}(-g) + d_{ia}(b_i - x_i - g)$ and the average payoff of the defectors in the group the group i is, $P_{i_d} = c_i(b_i - x_i - g) + d_{ib}(-g) + d_{ia}(b_i - x_i - g)$.

The replicator equation for the cooperators in the group n is as follows,

$$\frac{dn_c}{dt} = n_c(1 - n_c)(P_{n_c} - P_{n_d}) = n_c(1 - n_c)\{c_n(g - x_n)\}, \tag{2.3}$$

where the average payoff of the cooperators in the group n, $P_{n_c} = c_n(b_n - x_n) + d_{nb}(-g)$ and the average payoff of the defectors in the group n, $P_{n_d} = c_n(b_n - x_n - g) + d_{nb}(-g)$.

2.2.2 The defector sanction system

Next we focus on two sanction systems, namely the defector sanction system, and the first role sanction system. In the defector sanction system, the defector in the chain of the linear division of labour gets punished with a fine f, where $(f > 0)$ and the finding probability of the defector is ρ. In some types of linear division of labour, where monitoring a defector is too hard, where ρ is very low compared to other parameters.

For the defector sanction system, the payoffs are given in a similar way as the baseline except the sanction; adding the sanction of ρf to the defector's payoff. In the defector sanction system, the payoff matrix for a player in group i is in Table 2.2.

The replicator equation for the cooperators in the group 1 is as follows,

$$\frac{d1_c}{dt} = 1_c(1 - 1_c)(P_{1_c} - P_{1_d}) = 1_c(1 - 1_c)\{c_1 g + \rho f - x_1\}, \tag{2.4}$$

where the average payoff of the cooperators in the group 1, $P_{1_c} = b_1 - x_1 - d_{1a} g$ and the average payoff

Table 2.2: The payoff matrix in the defector sanction system for a player in the ith group

Cases	All being cooperator except the ith	A defector before ith	A defector after ith
ith player cooperator	$b_i - x_i$	$-g$	$b_i - x_i - g$
ith player defector	$b_i - g - \rho f$	$-g$	$b_i - g - \rho f$

of the defectors in the group 1, $P_{1_d} = b_1 - g - \rho f$.

In the defector sanction system, the replicator equation for the cooperators in the group i is (when $1 < i < n$) as follows,

$$\frac{di_c}{dt} = i_c(1 - i_c)(P_{i_c} - P_{i_d}) = i_c(1 - i_c)\{c_i g + (\rho f - x_i)(1 - d_{ib})\}, \tag{2.5}$$

where the average payoff of cooperators in the group i is, $P_{i_c} = c_i(b_i - x_i) + d_{ib}(-g) + d_{ia}(b_i - x_i - g)$ and the average payoff of the defectors in the group i is, $P_{i_d} = c_i(b_i - g - \rho f) + d_{ib}(-g) + d_{ia}(b_i - g - \rho f)$.

In the defector sanction system, the replicator equation for the cooperators in the group n is as follows,

$$\frac{dn_c}{dt} = n_c(1 - n_c)(P_{n_c} - P_{n_d}) = n_c(1 - n_c)\{c_n(g + \rho f - x_n)\}, \tag{2.6}$$

where the average payoff of the cooperators in the group n, $P_{n_c} = c_n(b_n - x_n) + d_{nb}(-g)$ and the average payoff of the defectors in the group n, $P_{n_d} = c_n(b_n - g - \rho f) + d_{nb}(-g)$.

2.2.3 The first role sanction system

In the first role sanction system, the fine is the same as the defector sanction system, f, and a defection in the chain of the linear division of labour is always found with probability 1, because the final product or service does not appear if defection occurs, and players can know that defection occurs without monitoring a defector. Therefore, the finding probability is one. The player in the first role always gets punished for the defection, no matter which role defected. For example, this sanction system is executed to prevent illegal dumping in Japan (Nakamaru et al., 2018).

For the first role sanction system for the generalized ith player the payoff matrix is the same as the baseline except that for group 1. If anyone defects, the first role gets punished, and sanction f appears in the first group's payoff matrix (Table 2.3a). The payoff matrix for a player in the group 1 in the first role sanction system is in Table 3a, and for a player in the group $i (2 \leq i \leq n)$ is in Table 2.3b.

The replicator equation for a cooperator in the group 1 is as follows;

$$\frac{d1_c}{dt} = 1_c(1 - 1_c)(P_{1_c} - P_{1-d}) = 1_c(1 - 1_c)\{c_1(g + f) - x_1\}, \tag{2.7}$$

Table 2.3a: The payoff matrix in the first role sanction system for group 1

Cases	All being cooperator except the 1st	A defector after the 1st
1st player cooperator	$b_1 - x_1$	$b_1 - x_1 - g - f$
1st player defector	$b_1 - g - f$	$b_1 - g - f$

Table 2.3b: The payoff matrix in the first role sanction system for a player in the group $2 \leq i \leq n$

Cases	All being cooperator except the ith	A defector before ith	A defector after ith
ith player cooperator	$b_i - x_i$	$-g$	$b_i - x_i - g$
ith player defector	$b_i - g$	$-g$	$b_i - g$

Table 2.4: Parameters

n	The number of groups, ($n \in \mathbf{N}$ and $n \geq 2,$ here $\mathbf{N}$ is the set of natural numbers)
i	Index of groups, $1 \leq i \leq n$
b_i	The benefit given by the ith group player to the $i+1$th group player
x_i	The cost of cooperation for the ith group player
i_c	The frequency of cooperators in the ith group
i_d	The frequency of defectors in the ith group
g	A damage for all players once one defector appears in the line
f	The amount of punishment
ρ	The probability of catching a defector
c_i	The probability of having all cooperators in the line except the ith group
d_{ib}	The probability of having a defector in the line before the ith group
d_{ia}	The probability of having a defector in the line after the ith group

where the average payoff of the cooperators in the group 1, $P_{1_c} = b_1 - x_1 + d_{1a}(-f - g)$ and the average payoff of the defectors in the group 1, $P_{1_d} = b_1 - g - f$.

The replicator equation for the cooperators in the groups $1 < i < n$ and group n are the same as the baseline model. The replicator equation for the cooperators in the group i is (when $1 < i < n$) as follows,

$$\frac{di_c}{dt} = i_c(1 - i_c)(P_{i_c} - P_{i_d}) = i_c(1 - i_c)\{c_i g - x_i(1 - d_{ib})\}, \tag{2.8}$$

where the average payoff of the cooperators in the group i is, $P_{i_c} = c_i(b_i - x_i) + d_{ib}(-g) + d_{ia}(b_i - x_i - g)$ and the average payoff of the defectors in the group i is, $P_{i_d} = c_i(b_i - x_i - g) + d_{ib}(-g) + d_{ia}(b_i - x_i - g)$.

The replicator equation for the cooperators in the group n is as follows,

$$\frac{dn_c}{dt} = n_c(1 - n_c)(P_{n_c} - P_{n_d}) = n_c(1 - n_c)\{c_n(g - x_n)\}, \tag{2.9}$$

where the average payoff of the cooperators in the group n, $P_{n_c} = c_n(b_n - x_n) + d_{nb}(-g)$ and the average payoff of the defectors in the group n, $P_{n_d} = c_n(b_n - x_n - g) + d_{nb}(-g)$.

Table 2.4 shows the parameter list in our model.

2.3 Results

2.3.1 The summary of results in all three systems

We find four sorts of equilibrium in all three systems in $n \geq 3$ (see Appendix A). One is the all cooperation equilibrium which we represent as $[1_c, 2_c, ..., n_c] = [1, 1, ..., 1]$. The second one is the first group defection equilibirum which we represent as $[0, *, ..., *]$ where "$*$" is any value between 0 and 1. In equilibrium, i_c is neutral ($i \geq 2$) because the game stops after the player in the group 1 chooses defection and the players in the later roles gets the same payoff regardless of the behaviour. As a result, i_c neutrally converges to any value between 0 and 1 ($i > 1$). The third one is the cooperation-defection mixed equilibrium which is represented as $[1_c, 2_c, ..., (j-1)_c, j_c, (j+1)_c, ..., n_c] = [1, 1, ..., 1, 0, *..., *]$, where j is between 2 and $n-1$. The fourth is $[1_c, 2_c, ..., (n-1)_c, n_c] = [1, 1, ..., 1, 0]$ which is named the last group defection equilibrium. Appendix A proves that this system only has four types of equilibrium points. We analyse the local stability of these four equilibria with the Jacobian matrix (see Appendix B). Table 2.5 presents the summary of the analyses.

Table 2.5 shows that the first group defection equilibrium is one stable equilibrium in the baseline. It is also locally stable in the first role sanction system when $c_1(g+f) < x_1$. It is stable in the defector sanction system, if $c_1 g + \rho f < x_1$.

The cooperation-defection mixed equilibrium is unstable in both the baseline and the first role sanction system. It is stable in the defector sanction system when $\rho f < x_j - c_j g$ and $\max\{x_i\}_{i=1}^{j-1} < \rho f$, where a player in the group j defects and everyone cooperates in the groups before the group j.

The last group defection equilibrium is unstable in the baseline and the first role sanction system. It is stable in the defector sanction system when $\max\{x_i\}_{i=1}^{n-1} < \rho f$ and $x_n > g + \rho f$.

The all cooperation equilibrium is locally stable if $g > \max\{x_i\}_{i=1}^{n}$ in the baseline system. This means that players in all groups are changed to cooperators in equilibrium when g, the loss caused by defectors, is higher than the cost of cooperator in the all cooperation equilibrium. However, we consider the PD situation in the baseline system, and then we can assume that $g < x_i$. This indicates that all cooperation equilibrium is unstable in the baseline. It is stable in the defector sanction system when $\rho f + g > \max\{x_i\}_{i=1}^{n}$, which is held even though $g < x_i$. Therefore, if ρf is large enough, the defector sanction system can promote the evolution of cooperation.

The all cooperation equilibrium is stable in the first role sanction system when $f + g > x_1$ and $g > \max\{x_i\}_{i=2}^{n}$. This indicates that the first role sanction system promotes the evolution of cooperation more than the baseline system. If we consider the assumption, $g < x_i$, which satisfies the condition of

Table 2.5: Local stability conditions for the general model

Equilibrium	Baseline	Defector sanction	First role sanction
$[0, *, .., *]$	Always stable	$\rho f < x_1 - c_1 g$	$c_1(g+f) < x_1$
$[1, ..., 1, 0, *, ..., *]$	Always unstable	$\max\{x_i\}_{i=1}^{j-1} < \rho f < x_j - c_j g$	Always unstable
$[1, ..., 1, 0]$	Always unstable	$\max\{x_i\}_{i=1}^{n-1} < \rho f$ & $x_n > g + \rho f$	Always unstable
$[1, ..., 1]$	$g > \max\{x_i\}_{i=1}^{n}$	$\rho f + g > \max\{x_i\}_{i=1}^{n}$	$f + g > x_1$ & $g > \max\{x_i\}_{i=2}^{n}$
j is the first defector in $[1, ..., 1, 0, *, ..., *]$			

the PD game, the all cooperation equilibrium is considered stable if the condition that $x_1 - f < g < x_1$ is possible in the first role sanction system.

Appendix B and Table 2.5 suggest that the benefit given by the ith group player to the $i+1$ th group player, b_i, is cancelled out and does not influence the local stability of each equilibrium point.

To understand the dynamics well, we will discuss three special cases as well as other cases about the cost of cooperation in the following sections.

2.3.2 Three special cases

The cost of cooperation is the same for all the groups

The simplest assumption is that x_i is the same for all the $1 \leq i \leq n$; $x_1 = ... = x_n = x$. After exploring the local stability conditions for each of the equilibrium in each of the three systems, we can summarize the results as Table 2.6, which represents that the first group defection equilibrium is a stable equilibrium in the baseline. It is also locally stable in the first role sanction system when $c_1(g+f) < x$. In the defector sanction system, it is locally stable when $\rho f < x - c_1 g$.

The cooperation-defection mixed equilibrium and the last group defection equilibrium are unstable in all the systems. Because $\rho f > x$ and $x > x - c_j g > \rho f$ are contradictory when j is the first defector. This indicates that once cooperation in group 1 starts, the all cooperation equilibrium will be stable. It is intuitive because all players face the same cost $-g$ once a player chooses defection and the cost of cooperation x is the same in all the groups. Therefore, the players in the later groups will follow the cooperators in the first group. It is meaningful to punish a defector in the earliest group to promote cooperation. Therefore, the first role sanction system works.

Our analysis suggests that the all cooperation equilibrium is stable in the baseline system and in the first role sanction system only when $g > x$. As we assume that $g < x$ which meets the PD game, the equilibrium is not stable. It is stable when $\rho f + g > x$ for the defector sanction system.

Table 2.6: Local stability conditions when the cooperation cost is the same for all the groups

Equilibrium	Baseline	Defector sanction	First role sanction
$[0, *..., *]$	Always stable	$\rho f < x - c_1 g$	$c_1(g + f) < x$
$[1, ..., 1, 0, *, ..., *]$	Always unstable	Always unstable	Always unstable
$[1, ..., 1, 0]$	Always unstable	Always unstable	Always unstable
$[1, ..., 1]$	$g > x$	$\rho f + g > x$	$g > x$

The cost of cooperation is lower in higher i

Here, we consider the special case where the cost in a downstream group decreases in the linear division of labour, $x_1 > x_2 > ..... > x_n$.

After exploring the local stability conditions for each of the equilibrium in each of the three systems, we can summarize the results as Table 2.7, which represents that the first group defection equilibrium is a stable equilibrium in the baseline. It is also stable in the first role sanction system when $c_1(g+f) < x_1$. In the defector sanction system, it is locally stable when $\rho f < x_1 - c_1 g$.

The cooperation-defection mixed equilibrium and the last group defection equilibrium are unstable in all the system. Because, if $i < j$, then $\rho f > x_i$ and $x_j > x_j - c_j g > \rho f$ are contradictory when j is the first defector.

Our analysis suggests that the all cooperation equilibrium is stable in the baseline system only when $g > x_1$ because $x_1 > x_2 > ..... > x_n$. When the condition of the PD game is applied, the equilibrium is not stable. It is stable when $\rho f + g > x_1$ for the defector sanction systems (figure 2.2(a)), and is stable when $f + g > x_1$ and $g > x_2$ for the first role sanction system (figure 2.2(b)). Figures 2.2(a) and 2.2(b) shows that the same sanction f as the first role sanction system cannot create the evolution of cooperation in the defector sanction system, because of the low finding probability of the defector.

By comparison between Tables 2.6 and 2.7, it is shown that the outcomes of this case (Table 2.7) is the same as Table 2.6 which is the result when the cooperation cost is the same for all the group.

Surprisingly enough, figures 2.2(a) and 2.3 show the defector sanction system promotes cooperation even though cooperators are rare in the beginning in $\rho f > x_1$. In the region where $\rho f + g > x_1$ and $\rho f < x_1$, the system is bistable, where the sanction needs to be higher to create all cooperation with lower $i_c(0)$, and even low sanction can create all cooperation with higher $i_c(0)$. Figure 2.2(a) also shows that the dynamics is independent of the initial condition and goes to all defection in the region of $\rho f + g < x_1$. Therefore, if the probability of finding and catching a defector is too low and ρf is very low, the defector sanction system never promotes cooperation. If the sanction, ρf, is large enough to be effective, the defector sanction system promotes cooperation even though the initial frequency of cooperators is low.

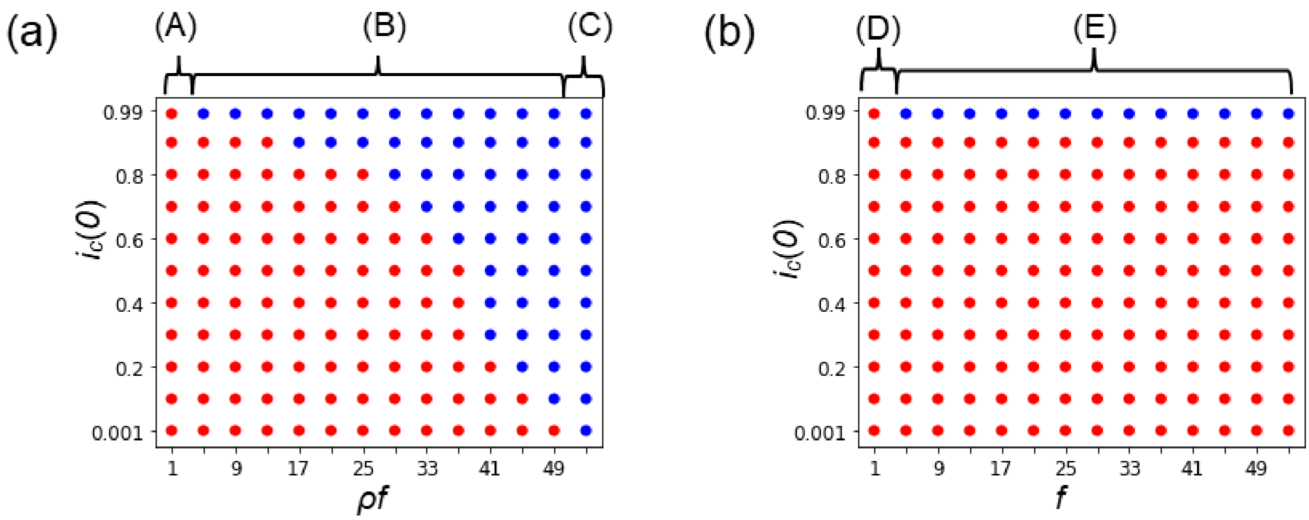

Figure 2.2: Initial frequency dependency in (a) the defector sanction system and (b)the first role sanction system with $n = 10$ when the cooperation cost decreases downstream. The horizontal axis is for ρf in (a) and for f in (b). The vertical is for the initial frequency of cooperators in group i, $i_c(0)$, when i is an integer between 1 and 10. Blue dots shows when the dynamics evolves into all cooperation and red dots shows when the dynamics evolves into the first group defection equilibrium. The parameters are: $g = 48$, $x_1 = 50$, $\rho = 0.001$ and $x_{i-1} - x_i = 5$ for all is. The numerical analyses show that c_1 is almost zero in the first group defection equilibrium. (A) in (a) means $\rho f < x_1 - g = 2$; (B), $x_1 - g = 2 < \rho f < x_1 - c_1 g = 50$; (C), $x_1 = 50 < \rho f$. (D) in (b) presents $f < x_1 - g = 2$; (E), $x_1 - g = 2 < f$ and $g > x_2$.

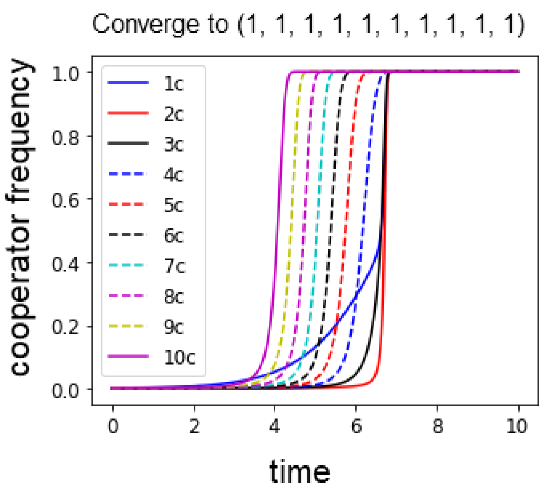

Figure 2.3: Dynamics of the system with $n = 10$ when cooperation cost decreases downstream in the defector sanction system with $\rho f = 51$. Even when the initial frequency of cooperators are rare, the evolution of cooperation happens if $\rho f > x_1$. The parameters are: $i_c(0) = 0.001$, $g = 48$, $\rho = 0.001$, $x_1 = 50$ and $x_{i-1} - x_i = 5$ (for all is).

Figure 2.2(b) shows clearly that the first role sanction system creates cooperation only when the initial frequencies of cooperators in all the groups are very high. When the $i_c(0)$ comes near 0.95, cooperation only evolves with very high sanction f. When $i_c(0)$ is low, the system goes to first group defection.

Table 2.7: Local stability conditions when the cooperation cost decreases in the downstream

Equilibrium	Baseline	Defector sanction	First role sanction
$[0, *, .., *]$	Always stable	$\rho f < x_1 - c_1 g$	$c_1(g+f) < x_1$
$[1, ..., 1, 0, *, ..., *]$	Always unstable	Always unstable	Always unstable
$[1, ..., 1, 0]$	Always unstable	Always unstable	Always unstable
$[1, ..., 1]$	$g > x_1$	$\rho f + g > x_1$	$f + g > x_1$ & $g > x_2$

The numerical analyses indicate that the parameter c_1 becomes almost zero so that this parameter does not influence the simulation outcomes in the first group defection equilibrium (figure 2.2).

The cost of cooperation is higher in higher i

Now we consider the model when the cost of cooperation rises in the downstream of the linear chain. Here we assume that, $x_i > x_{i-1}$ for all is. The local stability conditions for each of the four equilibria are shown in table 2.8.

The main difference between the condition where the cost of cooperation increases in downstream groups and the condition where the cost of cooperation decreases in downstream groups is that the defector sanction system makes the cooperation-defection mixed equilibrium as well as the last group defection equilibrium locally stable when the cost of cooperation increases in downstream groups (Table 2.8).

Figure 2.4(a) shows that the dynamics in the defector sanction system where all four of the equilibria are present when $x_{n-1} < x_n - g$; in the region of $\rho f < x_1$ the dynamics goes to the first group defection equilibrium, even with very high initial frequency of cooperators in all groups. When $x_{j-1} < \rho f < x_j$, and j is neither one nor the terminal, all players cooperate till the group $j-1$ before the group j choosing full defection; the cooperation-defection mixed equilibrium is locally stable shown in figure 2.5. When $x_{n-1} < \rho f$ and $x_n > g + \rho f$, the last group defection equilibrium is locally stable. When $\rho f + g > x_n$, all players in all groups go to cooperation even though their initial frequencies of cooperators are low.

When $x_{n-1} > x_n - g$ and $x_n - g > x_1$, figure 2.4(b) shows the bistable region where both the cooperation-defection mixed equilibrium and the all cooperation equilibrium are locally stable when $x_n - g < \rho f < x_j < x_{n-1}$ where j is the first defector $(1 < j < n-1)$. However, the last group defection equilibrium is not locally stable, because the condition for $x_{n-1} < \rho f$ and $x_n > \rho f + g$ do not hold.

Figure 2.4(c) shows us the outcomes when $x_n - g < \rho f < x_1$, we find the bistability of the all cooperation and the first group defection equilibrium. When $x_n - g < x_1 < x_{j-1} < \rho f < x_j < x_{n-1}$ there exists the bistability of the cooperation-defection mixed equilibrium and the all cooperation equilibrium. When $x_n - g < x_{n-1} < \rho f$ there only exists the all cooperation equilibrium. The last

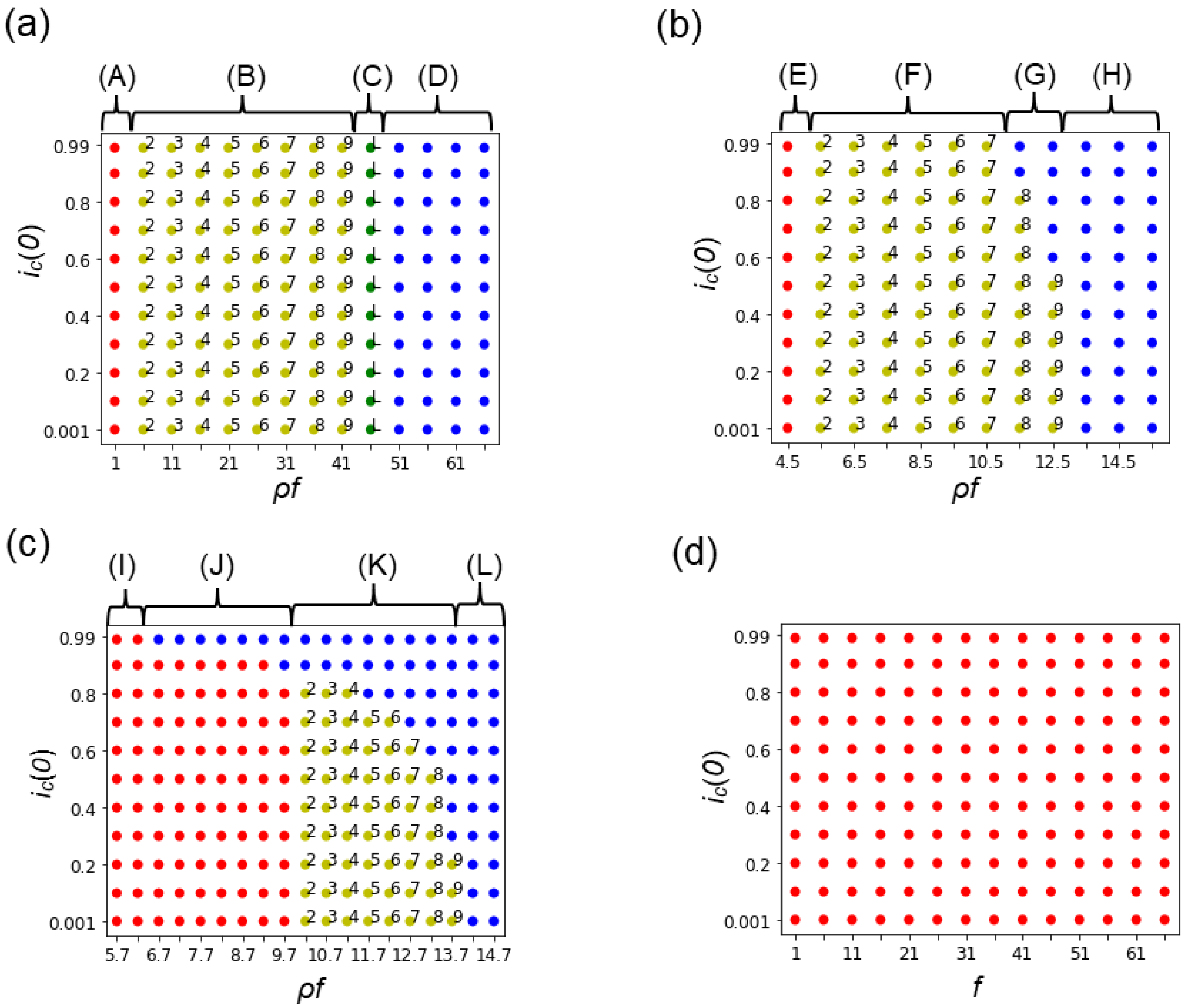

Figure 2.4: Initial frequency dependency in the defector sanction system in (a), (b) and (c), and in (d) the first role sanction system with $n = 10$ when the cooperation cost increases downstream. The horizontal axis is for ρf in (a), (b) and (c), and for f in (d). The vertical axis is for $i_c(0)$ when $i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. The parameters are: $g = 3$, $x_1 = 5$, and $\rho = 0.001$ for (a), (b) and (d). In (a) and (d) $x_i - x_{i-1} = 5$ for all is. In (b) $x_i - x_{i-1} = 1$ for all is. In (c) $g = 8$, $x_1 = 10$, $\rho = 0.001$, and $x_i - x_{i-1} = 0.5$. Blue dots show when the dynamics evolves into all cooperation and red dots shows when the dynamics evolves into the first group defection equilibrium. The yellow dots show when the dynamics evolves into a cooperation-defection mixed equilibrium. The number (j) outside each yellow dot shows all players in the group j are first defectors in the cooperation-defection mixed equilibrium. As $x_{j-1} < \rho f < x_j$ is the condition for the cooperation-defection mixed equilibrium to be stable where j is the first defector. The green dots with the letter "L" show when the dynamics evolves into the last group defection equilibrium. The numerical analyses show that c_i is almost zero ($1 \leq i \leq n$) unless the dynamics converge to all cooperation equilibrium. (A) in (a) means $\rho f < x_1 = 5$; (B), $x_{j-1} < \rho f < x_j - c_j g$; (C), $x_{n-1} < \rho f < x_n - g$; (D), $x_n - g = 47 < \rho f$. (E) in (b) means $\rho f < x_1 = 5$; (F), $x_{j-1} < \rho f < x_j - c_j g$; (G), $x_n - g < \rho f < x_{n-1} - c_{n-1} g$; (H), $x_n - g < x_{n-1} - c_{n-1} g < \rho f$. (I) in (c) means $\rho f < x_n - g < x_1 - c_1 g$; (J), $x_n - g < \rho f < x_1 - c_1 g$; (K), $x_n - g < x_{j-1} < \rho f < x_j - c_j g$; (L), $x_n - g < x_{n-1} - c_{n-1} g < \rho f$.

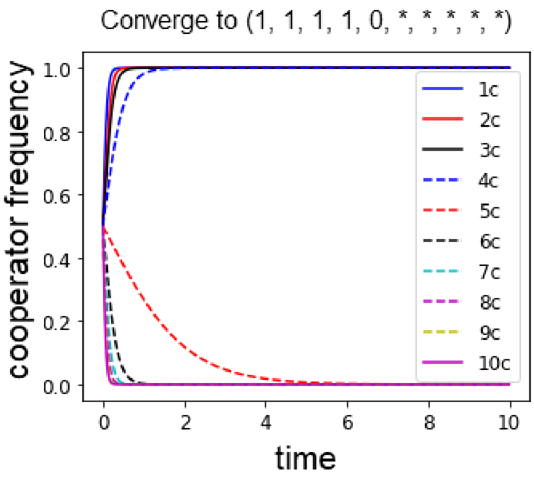

Figure 2.5: Time evolution in the defector sanction system with $n = 10$ when cooperation cost increases downstream. The horizontal axis is for time and the vertical one is for the frequency of cooperators in each group. $\rho f = 24$, $i_c(0) = 0.5$ shows the dynamics evolving into the cooperation-defection mixed equilibrium where all players are cooperators until the group 4 and all are defectors in the group 5 as $x_4 < \rho f < x_5$. The parameters are: $g = 3$, $x_1 = 5$, $\rho = 0.001$, and $x_i - x_{i-1} = 5$ for all is.

Table 2.8: Local stability conditions when the cooperation cost increases in downstream

Equilibrium	Baseline	Defector sanction	First role sanction
$[0, *, ..., *]$	Always stable	$\rho f < x_1 - c_1 g$	$c_1(g + f) < x_1$
$[1, ..., 1, 0, *, ..., *]$	Always unstable	$x_{j-1} < \rho f < x_j - c_j g$	Always Unstable
$[1, ..., 1, 0]$	Always unstable	$x_{n-1} < \rho f$ $\& x_n > g + \rho f$	Always unstable
$[1, ..., 1]$	$g > x_n$	$\rho f + g > x_n$	$g > x_n$
j is the first defector in $[1, ..., 1, 0, *, ..., *]$			

group defection equilibrium is not locally stable.

Figure 2.4(d) shows that first role sanction system can never create all cooperation, even with very high punishments f and along all the initial cooperation frequency.

In sum, the defector sanction system works as sanction and promotes the evolution of cooperation when $x_1 < x_2 < .. < x_n$ and ρf is large enough. The first role sanction system does not work and it is equivalent to the baseline system.

The numerical analyses indicate that c_j becomes almost zero so that this parameter does not influence the simulation outcomes when j is the first defector group (figure 2.4).

2.3.3 Other cases

We consider that the costs of the cooperation are given uniform randomly, do the numerical simulations in a parameter set, and then see if Table 2.5-2.8 can predict the dynamics. Figure 2.6(a) shows that the dynamics approximately converges to $[1_c, 2_c, 3_c, 4_c, 5_c, 6_c, 7_c, 8_c, 9_c, 10_c] = [0, 0.18, 0, 0, 0, 1, 1, 0, 1, 0.5]$

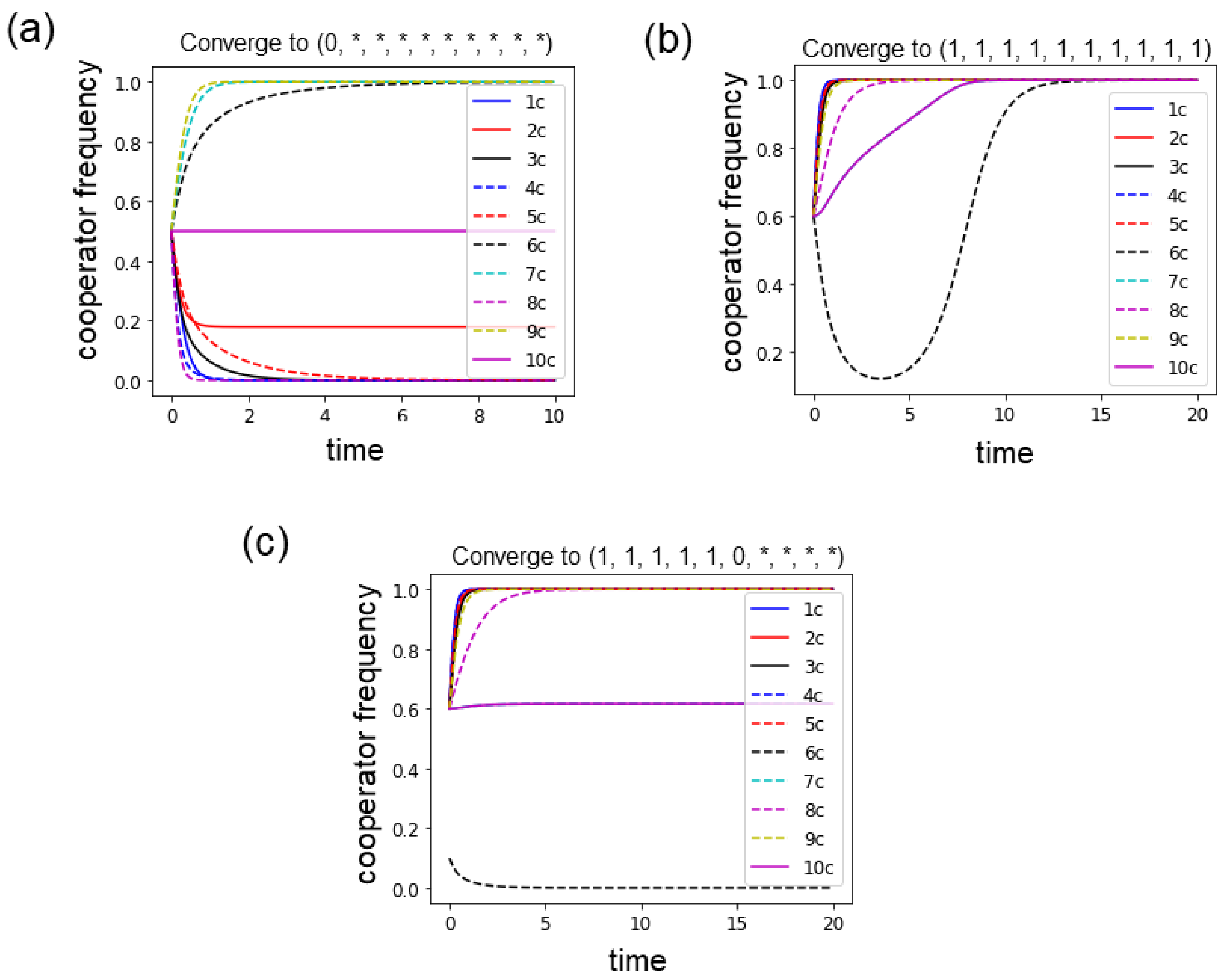

Figure 2.6: Time evolution with the costs of the cooperation given uniform randomly in the defector sanction system. In (a), $n = 10$, $g = 3$, $\rho f = 9$, $[x_1, x_2, ..., x_{10}] = [14, 20, 17, 20, 13, 6, 5, 19, 4, 9]$. In (b) and (c), $n = 10$, $g = 3, \rho f = 10$, $[x_1, x_2, ..., x_{10}] = [4, 5, 6, 4, 5, 12, 10, 9, 7, 10]$. The initial condition for (a) is $i_c = 0.5$ and (b) is $i_c = 0.6$ for all the groups, and that for (c) is $6_c = 0.1$ and $i_c = 0.6$ for other groups.

when $[x_1, x_2, ..., x_{10}] = [14, 20, 17, 20, 13, 6, 5, 19, 4, 9]$ in the defector sanction system when $g = 3$ and $\rho f = 9$. This convergence point seems to be a cooperation-defection mixed equilibrium. However, as $\rho f < x_1$ and c_1 converges to zero in the simulation, Table 2.5 predicts that it is the first group defection equilibrium. The simulation outcomes also show that, as 1_c converges to 0, it is the first group defection equilibrium. If we had only done the numerical analysis by computer simulations, we would have regarded this convergent point as a cooperation-defection mixed equilibrium. Therefore, the theoretical proofs help us understand the dynamics correctly.

When $[x_1, x_2, ..., x_{10}]$ is randomly assigned to [4, 5, 6, 4 ,5 ,12, 10, 9, 7, 10], we observe both the all cooperation equilibrium and the cooperation-defection mixed equilibrium are locally stable with different initial conditions. Here, $n = 10$, $g = 3, \rho f = 10$. We set the initial condition for figure 2.6(b), $i_c = 0.6$ for all the groups, hence the dynamics converges to the all cooperation equilibrium. Table 2.5

also predicts the all cooperation equilibrium is locally stable here as $\max\{x_i\}_{i=1}^{10} = 12 < \rho f + g = 13$. If we set the initial condition as $6_c = 0.1$ and $i_c = 0.6$ $(i \neq 6)$, the dynamics converge to the cooperation-defection mixed equilibrium where players in the group 6 are changed to defectors (figure 2.6(c)). This can be predicted by Table 2.5 in which $\max\{x_i\}_{i=1}^{5} = 6 < \rho f = 10 < x_6 - c_6 g = 10.86$ where c_6 converges to 0.379 in the simulation (Table 2.5).

2.4 Discussion and Conclusion

We took a system of linear division of labour where there are n roles $(n \geq 2)$. If a role gets subjected to defection by its defector, the labour stops there, and the players associated with the later roles do not get a chance to play the roles. Each player in each group gets subjected to the same loss once a player defects. We analyse three systems; the baseline system and the two sanction systems namely the defector sanction system and the first role sanction system, to see their effect on the evolution of cooperation. After applying the replicator equation of asymmetric game, we find four equilibria, 1) where all the players in the first group are defectors, 2) where all the players in all the groups are cooperators, 3) where the players in the earlier groups are all cooperators and the players in the later groups except the first and the terminal group are defectors which is called the cooperation-defection mixed equilibrium, and 4) the last group defection equilibrium where all the players in the last group are defectors, and all players in other groups are cooperators.

Our findings are as follows: the benefit given by a cooperator in an upstream group to a player in a downstream group does not influence the evolutionary dynamics, but the cost of cooperation does. We compare two sanction system, the defector sanction system and the first role sanction system, with the baseline system. Then, we found that the defector sanction system promotes the evolution of cooperation unless the probability of finding a defector is very low. However, when it is too hard to monitor and detect a defector, the defector sanction system does not work as sanction anymore. The first role sanction system promotes cooperation when the cost of cooperation decreases in downstream groups. Otherwise, the first role sanction system is equivalent to the baseline system; it does not work as sanction. The other important point is that, in addition to the all cooperation and the first group defection equilibria, the cooperation-defection mixed equilibrium and the last group defection equilibrium can be locally stable when the cost of cooperation increases with higher i in the defector sanction system.

Even though our results can be applied to any n if $n \geq 2$. However, the results in $n = 2$ is different from those in $n \geq 3$. The crucial difference is that there are three equilibrium points in $n = 2$: $[1_c, 2_c] = [0, *], [1, 0], [1, 1]$. When the costs of the cooperation decrease downstream, the local stability condition in $n = 2$ is the same as in $n \geq 3$. When the costs of the cooperation increase downstream in the defector sanction system, in $n = 2$, [1,0] is locally stable if $x_1 < \rho f$ and $x_2 > \rho f + g$. [1,1] is locally stable if $\rho f + g > x_2$, and [0,*] is locally stable if $\rho f < x_1$. There is a bistable region in $n = 2$; if $\rho f < x_1 < x_2 < \rho f + g$ holds, [0,*] and [1,1] are locally stable. While, $[1, 1, ..., 1]$ and $[0, *, .., *]$ are bistable in $n > 2$, when $x_n - g < \rho f < x_1$ (figure 4(c)). However, when n is larger, it becomes harder to have a bistable region in which [1,1,...,1] and [0,*,..,*] are locally stable, because $x_n - g < \rho f < x_1 < \cdots < x_n$ should be held under the assumption that $g < x_i$.

Our study might remind us of Boyd and Richerson (1989). Because players are in a unidirectional cycle network, two neighbours play the PD game in order and it goes on repeatedly. The two strategies are as follows, 1) Upstream tit for tat (UTFT): where if the upstream player cooperates/defects with the focus player, then he cooperates/defects with the downstream player, and 2) Downstream tit for tat (DTFT): where the focus player cooperates/defects with the downstream player if the downstream player cooperated/defected with his own downstream player, in the previous cycle. Boyd and Richerson (1989) found that DTFT evolved more than UTFT. Structurally this study might look similar to ours. However, there are some critical differences between the two. One difference is the research purpose; Boyd and Richerson (1989) investigated the evolution of indirect reciprocity. The network structure of Boyd and Richerson (1989) is a repeated cycle, and players observe the payoff of all players in a cycle and imitate the strategy of a player with a higher payoff. Basically, in our study, UTFT is not possible in our study as if the player in the upstream group defects, the game stops there and the players from the focus group do not get to choose their strategy. DTFT is not possible as the strategy of a player is premeditated and does not depend on downstream groups' player's choosing. However, Boyd an Richerson (1989) gives us a hint to develop the study of the division of labour. In our future work, we apply some of their assumption into our framework, modify and develop our study.

We consider the special case that $b_i = x_{i-1}$, which means the benefit given by a cooperator in group $i - 1$ is same as the cost of cooperation paid by the cooperator; a player in the 1st group gives 10,000 yen to a player in the 2nd group. 10,000 yen is the cost of the player in the group 1 and the benefit to the player in the group 2. As $b_i - x_i = x_{i-1} - x_i$ should be positive, x_i decreases as the i increases. Therefore, the result of the analyses corresponds to Table 2.7. The total sum of the net benefit of all cooperators in all roles is $= (x_0 - x_1) + (x_1 - x_2) + ... + (x_{n-1} - x_n) = x_0 - x_n$. Therefore, the assumption that this situation can be interpreted as that each player in each role decides how much the

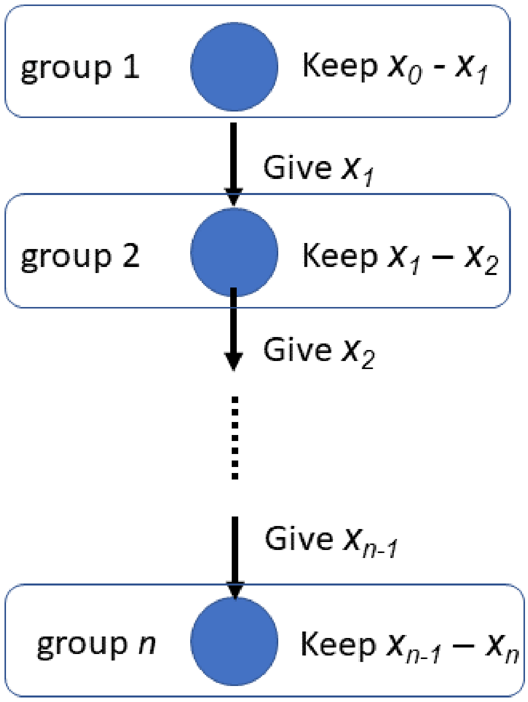

Figure 2.7: Allocation of the benefit among the players as an application of our model to government planning and spending. The net benefit of a player in group i is equal to the amount that the player keeps. This figure shows when all players in all groups are cooperators.

upstream player keeps and distribute to the downstream players (figure 2.7). For example, a cooperator in group 1 keeps x_0–x_1 and gives x_1 to a player in group 2. If the player in the group 2 is a cooperator, the cooperator keeps x_1–x_2 and gives x_2 to a player in the group 3. This continues before a player chooses defection. This situation has some implications not only for the division of labour but also for government planning and spending or subcontract because the model can be assimilated with the flow of government spending or subcontract (see figure 2.7). For government planning and spending, cooperation in the division of labour is required (Davis, 2001). Government of all forms performs a very significant role in running modern country states. The roles of it are as elabourated as they can get and a single individual cannot perform all roles. Therefore, all the related tasks are divided among many ministries and the ministries also divide the tasks among their workers. For a certain work to be done at the root level, the fund should go through multiple agents as well as be planned by multiple agents. Therefore, cooperation of all the roles is key for success in that particular task. For example, if a governmental head wants to spend some money at the root level, he/she allocates the money to his/her subordinate, who also allocates a part of the money to his/her subordinate, and it goes to many levels of subordinates until reaching the goal. Every cooperator in the group i here gains the net benefit $x_{i-1} - x_i$ from cooperation (see figure 2.7). And if someone stops or does not cooperate because of corruption or other reasons, the money does not reach the goal and therefore the task fails. As a result, all players additionally get damage, $-g$. In return, the defector, of course, gains the money which was given to him but to every agent as a part of the government comes a bad reputation for the defection which can be set as $-g$. By creating cooperation among all the roles, we make sure that the players in the first role engaging in government spending do not choose defection. The reason is as follows; our results suggest that all cooperation equilibrium or first group defection equilibrium can be locally stable but the cooperation-defection mixed equilibrium is not stable in Table 2.7. This means if a player in the first group is a cooperator, players in all other groups can be cooperators.

We did not comment particularly on the impact of the net benefit $b_i - x_i$ or the distribution of the benefit for the cooperators in the system because the benefit does not influence the dynamics (see Tables 2.5-2.8). Nowak (2006) shows that the cooperation can evolve in the network structured population, where each node has k regular links and b is the benefit from a cooperator and c is a cost of cooperation, in $b/c > k$. However, in our work, the benefit from cooperation is cancelled out, and then we cannot summarize our result using the benefit b. In this point, our work shed a new light on the evolution of cooperation in the networks.

This work only focused on one case of the division of labour. There are other types of division of labour. Here we assumed that each player in each group obtain the payoff after the player plays the game with the player in the downstream group. While, in the other type of division of labour, each player can get the benefit after all tasks in the division of labour are completed. In our future work, we will investigate how different outcomes we will obtain by analyzing the various cases of the division of labour by the replicator equations of asymmetric games. In addition, we will consider other types of sanction: for example, mistakenly regarding cooperators as defectors and sanctioning them.

Chapter 3

Tree graph study

3.1 Introduction

The study of the evolution of cooperation in the division of labour on the tree graph network has significantly more applicability than the linear network of Nirjhor and Nakamaru (2023) which is shown in the previous chapter 2. This is because the division of labour on a tree graph is more common than the linear division of labour in our real world. For example, government in most countries have a system of hierarchy. It has a governmental head or premier, under whom there are several departments, each of which has a head of its own. Each department then breaks down into several sub-departments and it goes until the root level. Therefore, a government system can be considered as a finite tree graph network, which has an origin at the premier and a finite number of branching. Each of the nodes represents a government official. Most governmental action can be considered as a division of labour (Bezes and Le Lidec, 2016), which is ordered by a head and then passes through the downstream nodes and gets fulfilled at some terminal node. Therefore, for an order to be carried out cooperation is very important. Often a single order is carried down and executed by a single linear chain of command which is similar to the linear division of labour which was our previous study (Nirjhor and Nakamaru, 2023). However, to see a governmental system as a whole, a tree graph structure is suitable.

The minimum structure of the supply chain is linear, consisting of suppliers, manufacturers, distributors, retailers, and customers (e.g. Min and Zhou, 2002). However, network-focused models can depict a better image of supply chain than linear chain models (Henderson et al., 2002). The supply chain looks more like a tree than a linear pipeline or chain (Lambert and Cooper, 2000; Cooper et. al, 1997). Therefore, a tree network is capable of depicting the linear network of the supply chain, as well as more general cases. When considering a unidirectional tree network as a supply chain, from the perspective of a player in a terminal node, it is a linear network of the supply chain. However, a player situated in some earlier node can divide the goods along the process links (Min and Zhou, 2002) as well as the labour or responsibility required to improve upon those according to the need. In addition, the multilayered subcontract can be depicted by a tree network (Tam et. al, 2011).

In this chapter we study the evolution of cooperation in the division of labour on a general tree graph. No previous models capture all the aspects of a general tree graph in the supply chain (e.g. Min and Zhou, 2002), and our study challenges this problem by means of the evolutionary game theory.

3.2 Baseline Model and Results

3.2.1 Model Assumption

We take a model where the whole task is divided and assigned to the groups who are present in the nodes of a connected directed tree graph. The model structure is shown in figure 3.1. In this model a task is always passed from the upstream to the downstream, never from downstream to upstream, hence, this is unidirectional. There is a unique central node in this graph, and from there branching starts. **G** is the set of nodes in the tree graph (figure 3.1). Each of the nodes has a group of players, each group consists of cooperators and defectors, and the group population is infinite. p is the index of the original node which represents the premier group. In the beginning, the whole task, which can be a service or development of a product, is assigned to the original group, where a player is randomly selected, who gets a benefit b_p. The player receives the benefit for receiving the task and it can be considered as the value of the task (Nirjhor and Nakamaru, 2023). For example, in the multi-layered contract development system, the orderer pays the contract money, b_p, to the contractor, which pays the money to the subcontractors and it continues when the terminal sub-subcontractors receive the money and do their tasks. In the industrial waste disposal system it can be considered as the benefit from the product, from which the waste was produced (Nakamaru et. al, 2018).

If the chosen player in a group is a cooperator, they pay a cost of cooperation x_p to improve upon the task. Then they divide the task and pass the task to one or more downstream branches. Each player who is chosen randomly from each of the receiving downstream groups, and also receives a benefit, as each has received the task. table 3.1a shows the payoff of a cooperator in the premier group when all players chosen from all downstream groups are cooperators: $b_p - x_p$ (table 3.1a).

If a defector is chosen from the premier group, the defector does not produce any benefit paying a cost of cooperation, and the division of labour does not start. As a result, the player from the premier

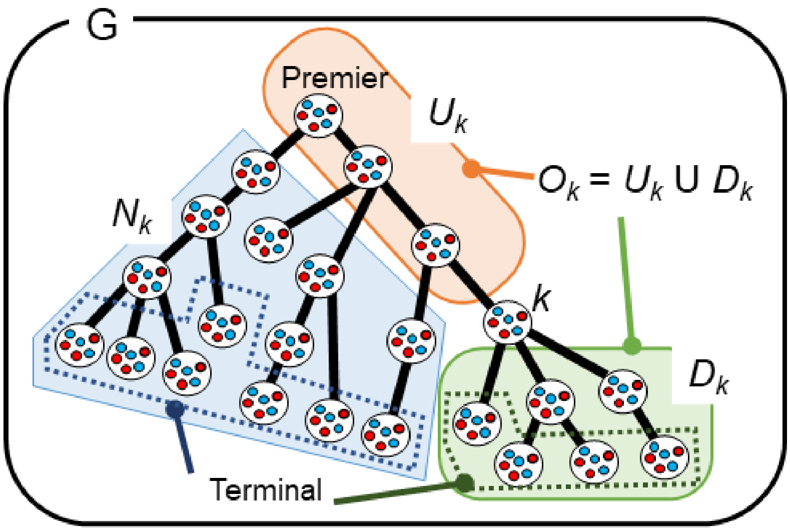

Figure 3.1: Division of labour in a downstream tree graph. k is the focal group, and based on it we divide the tree graph in several sets of nodes for the purpose of generalization. The upstream of k, U_k is a linear network from the premier, the downstream of k, D_k is a tree graph network. U_k and D_k makes the O_k. N_k is created with the groups that are not present in $O_k \cup k$.

group will suffer from the loss, g_p, which can be interpreted as the damage or the bad reputation from the incomplete tasks of the whole system (table 3.1a). Then, a player in the downstream group k ($k \neq p$) suffers from the loss, g_k, and it is assumed that $g_p = \sum g_j$, where js are the groups present in the immediate branching of the premier. We will explain the assumption of the loss caused by defection later.

Even though a cooperator is chosen from the premier group, the cooperator suffers from the loss, g_{op} if a defector is chosen in the downstream groups in D_p and the division of labour stops there (table 3.1a). To define D_p, we have to define O_k, U_k, D_k, and N_k with respect with group k at first (figure 3.1). O_k is the set of nodes that create k's connection with the premier and are present on the branches originating from k if k is not premier (figure 3.1). U_k is the set of the upstream nodes of O_k with respect to k and these nodes are in a linear division of labour with respect to k. D_k is the set of the downstream nodes of O_k with respect to k. Therefore, $O_k = U_k \cup D_k$ (figure 3.1). When a defector is chosen from group k, during that particular task, D_k's groups become neutral, as they do not have a choice. $N_k = \mathbf{G} - (O_k \cup \{k\})$ is the set of nodes that do not have the direct or indirect interaction with k. The $O_k \cup \{k\}$ can be considered as the community based on the subtask of k, as this is the part of the graph $\mathbf{G}$ which is connected with k (Fortunato and Castellano, 2007). On the other hand N_k is the part of $\mathbf{G}$ which is not connected with k.

If k is premier or p, both U_p and N_p do not exist, and D_p is equal to O_p. If k is terminal or t, D_t does not exist, U_t is equal to O_t, and $N_t = \mathbf{G} - (U_t \cup \{t\})$.

Table 3.1a: The payoff matrix in Baseline system for Premier

Cases	All being cooperator except the premier	A defector after the premier
Premier cooperator	$b_p - x_p$	$b_p - x_p - g_{op}$
Premier defector	$b_p - g_p$	$b_p - g_p$

Table 3.1b: The payoff matrix in the baseline system for $k \neq p$

Cases	All being cooperator in $\mathbf{O}_k$	A defector in U_k	A defector in D_k
k cooperator	$b_k - x_k - g_{nk}$	$-g_k - g_{nk}$	$b_k - x_k - g_{ok} - g_{nk}$
k defector	$b_k - g_k - g_{nk}$	$-g_k - g_{nk}$	$b_k - g_k - g_{nk}$

Table 3.1c: The payoff matrix in the baseline system for $n \neq p$ in terminal

Cases	All being cooperator in $\mathbf{O}_n$	A defector in U_n
n cooperator	$b_n - x_n - g_{nn}$	$-g_n - g_{nn}$
n defector	$b_n - g_n - g_{nn}$	$-g_n - g_{nn}$

We consider k is the index of our focus group, and this focus group can be any group in the graph. The value, b_k, is the benefit of the player chosen from the group k, given by the cooperator in the nearest upstream group u with paying a cost of cooperation x_u. The benefit can also be considered as the value of the task. If the player in group k is also a cooperator, he pays the cost of cooperation x_k to produce a new task or value, b_{kk}, and then passes it to his group's branch/branches if group k is not terminal. The net benefit of the cooperator in group k is $b_k - x_k$ if all other players are cooperators. If group k has two nearest downstream groups, for example, and they are named group A and group B. A cooperator in the group k gives a benefit to each player in two groups. There are two possible assumptions: the cooperator in group k will give a benefit b_A to a player in group A and b_B in a player in group B, where, (i) $b_{kk} = b_A + b_B$, or (ii) $b_{kk} = b_A = b_B$. If the benefit is a divisible good such as a product or money, it should be divided and (i) can be applied. If the benefit is an indivisible good such as a service, (ii) can be applied. Both of the cases can be covered in this model, as we shall see in the expansion of the model that the benefit itself shall disappear from the dynamics.

This continues until the terminals unless a defector is selected. If a defector is selected in group k, the defector just receives the benefit, b_k, from the cooperator in the upstream group and does not do a task paying the cost of cooperation, does not pass the task to his downstream. Hence, the division of labour stops there, that particular task is not completed and everyone in every group bears the loss, g_k.

Here, we explain our assumption about the losses caused by defectors. We assume that the loss to everyone if a defector is chosen in group k, is g_k, which is divided into the immediate branching of group k; $g_k = \sum_j g_j$ where js are the immediate branching of k. Each player suffers from the same

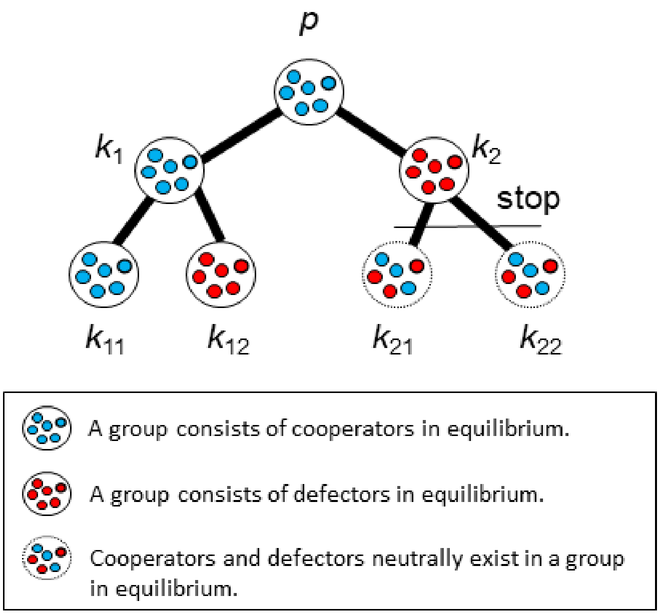

Figure 3.2: The image of a tree graph in equilibrium. The loss distribution in this system of 2-regular 2-branched directed tree graph is shown in table 2. The premier group's name is *premier* or p, and the other groups name except the premier group are k_is where is are 1, 2, 11, 12, 21, and 22. The groups p, k_1, and k_{11} are groups consisting of cooperators in equilibrium. The groups k_{12} and k_2 are groups consisting of defectors in equilibrium. As the division of labour stops at k_2, no interaction between groups occurs after k_2, and the groups k_{21} and k_{22} are groups where cooperators and defectors neutrally exist. The relative equilibria in this case from the perspective of each group are also enlisted in table 3.2.

losses caused by defectors in the whole system. However, from the viewpoint of group k, the losses a player in group k suffers from are classified into three types; the self-inflicted loss (γ_k), the potential loss (g_{ok}), and the loss caused by N_k (g_{nk}). The total loss to a player in group k is $\gamma_k + g_{ok} + g_{nk}$.

The self-inflicted loss γ_k is g_k when a defector is chosen in group k, and zero when a cooperator is chosen. When defection occurs in U_k, $\gamma_k = g_k$; we assume that when the downstream player suffers from the loss caused by the upstream defector, the loss of the downstream player is the same as what he would have suffered by his defection. This means, when a player in a group in U_k chooses defection, the product or service which would have been produced or done by the player in group k, was not produced or done, so the loss born by the player in group k is the same as g_k.

When the player in group k has already cooperated, some part of D_k, may cooperate and some part may not. Then, the cooperator in group k suffers from the loss by defection in D_k. This loss is called the potential loss to cooperators in the upstream groups, g_{ok}, which is defined as the combined loss through branching in D_k; $g_{ok} = \sum \delta_j(t) g_j$ where $j \in D_k$ and $\delta_j(t)$ is 1 when a chosen player in group js are defectors in D_k at time t, and zero otherwise. When the cooperator is in the group k and all the

Table 3.2: Losses to the players in the groups in figure 3.2

Group k	state	γ_k	g_{ok}	g_{nk}	relative equilibrium
Premier	Cooperation	0	$g_{k_{12}} + g_{k_2}$	0	mix_{D_p}
k_1	Cooperation	0	$g_{k_{12}}$	g_{k_2}	$mix_{D_{k_1}}$
k_{11}	Cooperation	0	0	$g_{k_{12}} + g_{k_2}$	$allc_{k_{11}}$
k_{12}	Defection	$g_{k_{12}}$	0	g_{k_2}	$mix_{U_{k_{12}} \cup \{k_{12}\}}$
k_2	Defection	g_{k_2}	0	$g_{k_{12}}$	$mix_{U_{k_2} \cup \{k_2\}}$
k_{21}	Neutral	$g_{k_{21}}$	0	$g_{k_{12}}+g_{k_{22}}$	$mix_{U_{k_{21}} \cup \{k_{21}\}}$
k_{22}	Neutral	$g_{k_{22}}$	0	$g_{k_{12}} + g_{k_{21}}$	$mix_{U_{k_{22}} \cup \{k_{22}\}}$

chosen players in the groups of k's immediate branching are defectors, then $g_{ok} = \sum g_j = g_k$. Otherwise, $g_{ok} < g_k$. This condition makes the choice of branching for a player impartial, as the distributed risk of defection is the same as or lower than defection by oneself.

We also assume that any player in group k suffers from the loss caused by N_k, g_{nk}. The value g_{nk} includes the losses caused by defectors in N_k. In addition, when there is a defection in a group $l \in U_K$, that also has branches in N_k, the loss g_l damages everyone in every group. A part of this loss comes to group k as the self-inflicted loss g_k, the rest of it flows in N_k and gives rise to the self-inflicted losses of players in groups those in $D_l \cap N_k$. To calculate this loss we take the summation of the self-inflicted losses of the terminals of $D_l \cap N_k$. In other words, g_{nk} also includes the self-inflicted losses of the groups which are in the terminal of N_k and had a defector in their upstream that intersects with U_k. In sum, the mathematical definition: $g_{nk} = \sum g_j + g_m$, where $j \in N_k$ and j are defectors when cooperators are selected in $U_k \cap U_j$, and $m \in N_k$ and m are terminals when a defector is selected in $U_k \cap U_m$.

Figure 3.2 and table 3.2 show the example of the losses in the 2-regular 2-branched directed tree graph. Here if the group index is k_j, the loss is shown as g_{k_j}. The relationship between losses is $g_p = g_{k_1} + g_{k_2}$, $g_{k_1} = g_{k_{11}} + g_{k_{12}}$, $g_{k_2} = g_{k_{21}} + g_{k_{22}}$. The self-inflicted loss of a cooperator in the premier group is zero. The potential loss that a cooperator in the premier group suffers from defection in the downstream groups, D_p, is g_{op}, which is the sum of the losses when defectors are selected after the premier group, $g_{op} = g_{k_{12}} + g_{k_2}$ in figure 3.2. g_{np} is zero because N_p is empty. In group k_{11}, the self-inflicted loss is zero, the potential loss is zero, and $g_{nk_{11}}$ is $g_{k_{12}} + g_{k_2}$ because defection occurs in both groups k_{12} and k_2 in $N_{k_{11}}$. Table 3.2 shows the three types of losses of other groups in figure 3.2.

Based on our assumption mentioned above, we can calculate the payoff of either a cooperator or a defector in group k (table 3.1); tables 3.1a, 3.1b and 3.1c are when $k = p$, k is neither p nor terminal, and $k = n$ is a terminal, respectively. The parameters are shown in table 3.3.

Table 3.3: Parameters

$\mathbf{G}$	Set of nodes in the tree graph
k	Index of nodes
p	Index of the first or original node, called the premier
O_k	Set of nodes which create k's connection with the premier and are present on the branches of k
N_k	$\mathbf{G} - (O_k \cup \{k\})$
U_k	Set of the upstream nodes of O_k with respect to k
D_k	Set of the downstream nodes of O_k with respect to k
c_{ok}	Probability of all the players in all the groups of $\mathbf{O}_k$
d_{uk}	Probability of a defector in U_k
d_{dk}	Probability of defector(s) in the downstream of group k in D_k
g_k	Loss to everybody if a defector is chosen in group k
γ_k	The self-inflicted loss in group k
g_{ok}	The potential loss in D_k
g_{nk}	The loss caused by N_k
x_k	Cooperation cost of a player in group k
b_k	Benefit of a player in group k
k_c	The frequency of cooperator in group k
k_d	The frequency of defector in group k
f	Amount of punishment
ρ	Probability of catching a defector, ρ is considerably low

3.2.2 Replicator equations for asymmetric games

If we assume that each player imitates the behaviour of others with a higher payoff in the same group, we can apply the replicator equations for asymmetric games. To calculate the expected payoff of players in the replicator equations of asymmetric games, three parameters of probability are defined; $c_{ok} = \Pi_{i \in O_k} i_c$ is the probability of all the selected players in all the groups of O_k being cooperators, where i_c is the frequency of cooperators, and i_d is the frequency of defectors in the group i. Here, $i_c + i_d = 1$. $d_{uk} = 1 - \Pi_{i \in U_k} i_c$ is the probability of a defector being selected in the groups of U_k. $d_{dk} = (1 - \Pi_{i \in D_k} i_c)(\Pi_{i \in U_k} i_c)$ is the probability of defector(s) being selected in the groups which are downstream of k, or in other words in D_k there is a defector or defectors. We find, $c_{ok} + d_{uk} + d_{dk} = 1$.

When k is the premier, if the player from k is a cooperator, and all the players who are selected from other groups are also cooperators, then the payoff of the player in k is $b_p - x_p$, as they receive the benefit b_p and pay the cost of cooperation x_p, to produce a product or do a task (table 3.1a). If there are one or more defectors in k's downstream then the payoff is $b_p - x_p - g_{op}$, as the combined loss due to defection g_{op} will also be born (table 3.1a). Therefore when the player from the premier group is a cooperator, then their expected payoff, Π_{cp}, is $c_{op}(b_p - x_p) + (1 - c_{op})(b_p - x_p - g_{op})$ (see table 3.1a). If the player from the premier group is a defector then their expected payoff, Π_{dp}, is $b_p - g_p$ (see table 3.1a) as he receives the benefit b_p but does not pay the cost of cooperation. However, due to his defection he needs to bear the loss g_p. As he is in the premier group, his defection leads to the linear division of labour being stopped. So, the latter groups' player's strategy does not have any effect on his payoff, when he is a defector. Therefore, the replicator equation of a cooperator in the premier group is,

$$\frac{dp_c}{dt} = p_c(1 - p_c)(\Pi_{cp} - \Pi_{dp}) = p_c(1 - p_c)(c_{op}g_{op} + g_p - g_{op} - x_p). \tag{3.1}$$

When k is neither the premier nor a terminal, if the player from k is a cooperator, and all the players who are selected from other groups in O_k are also cooperators, then the payoff of the player in k is $b_k - x_k - g_{nk}$, as they receive the benefit b_k, pay the cost of cooperation x_k, and also bear the loss g_{nk} due to the possible defections in N_k (table 3.1b). If there are one or more defectors in k's downstream then the payoff is $b_k - x_k - g_{ok} - g_{nk}$, as the combined loss due to defection g_{ok} will also be born (table 1b). If there is a defector in the U_k, then the task does not reach k, so in that case he only bears the loss g_k and their payoff becomes $-g_k - g_{nk}$. So, the expected payoff of a cooperator in group k, Π_{ck}, is $c_{ok}(b_k - x_k) - d_{uk}g_k + d_{dk}(b_k - x_k - g_{ok}) - g_{nk}$. If the player from k is a defector and all the chosen players in O_k are cooperators, or there are defectors to be chosen in D_k, then his payoff is $b_k - g_k - g_{nk}$ as they receive the benefit b_k but do not pay the cost of cooperation. However, due to their defection, they need to bear the loss g_k. If there is a defector in the U_k, their payoff is the same as being a cooperator, because they do not get a chance to play their strategy. So, when a player in group k is a defector, the expected payoff, Π_{dk}, is $c_{ok}(b_k - g_k) - d_{uk}g_k + d_{dk}(b_k - g_k) - g_{nk}$. Therefore, the replicator equation of a cooperator in group k is,

$$\frac{dk_c}{dt} = k_c(1 - k_c)(\Pi_{ck} - \Pi_{dk}) = k_c(1 - k_c)\{c_{ok}g_{ok} + (1 - d_{uk})(g_k - x_k - g_{ok})\}. \tag{3.2}$$

When n is a terminal group, the division of labour is effectively a linear network from the point of view of n. However, the aspect of possible defections in N_n needs to be considered. If the chosen player from n is a cooperator and players chosen from all the other groups in O_k are also cooperators, then their payoff is $b_n - x_n - g_{nn}$. If there is a defector chosen in any of the groups of O_n, then the task does not reach n, so their payoff in the terminal group becomes $-g_n - g_{nn}$, regardless of them being a cooperator or a defector. When all the players chosen from all the other groups in O_n are cooperators, however, the player chosen in n is a defector, then his payoff is $b_n - g_n - g_{nn}$, as he will have to bear the loss, g_n due to his own defection (table 3.1c).

Hence, a cooperator's expected payoff from a terminal group n, Π_{cn} is $c_{on}(b_n - x_n) - d_{un}g_n - g_{nn}$. When the player from the terminal n is a defector, his expected payoff, Π_{dn} is $c_{on}(b_n - g_n) - d_{un}g_n - g_{nn}$.

Therefore, the replicator equation of a cooperator in the terminal group is,

$$\frac{dn_c}{dt} = n_c(1 - n_c)(\Pi_{cn} - \Pi_{dn}) = n_c(1 - n_c)\{c_{on}(g_n - x_n)\}. \tag{3.3}$$

Here, the benefit b_k given by a cooperator of the upstream as well as the term g_{nk} which represents the loss caused by N_k are both cancelled in equations (3.1)-(3.3). Therefore, they do not have any effect on the dynamics. We will show that two values, b_k and g_{nk}, are also cancelled out when the sanction system is introduced in the equations (Section 3.2.2 and Appendices D and E).

3.2.3 Results

When we consider a system of 2-regular 2-branched directed tree graph such as figure 3.2 for example, there are 26 possible equilibrium points and we have to calculate the local stability of each of 26 equilibrium points. If we consider a larger system, the local stability of numerous possible equilibrium points should be calculated. Besides doing it, we propose a new method; the stability analysis of three sorts of equilibrium for each $O_k \cup \{k\}$ (figure 3.3).

The all-cooperation equilibrium is defined as that everyone in every group is a cooperator in $O_k \cup \{k\}$, which is hereafter called $allc_k$ (figure 3.3a). The premier group defection equilibrium is defined as that everyone in the premier group p is a defector, hereafter called $premierD$ (figure 3b). If all of the members in the premier group are defectors, then the division of labour does not start, and the latter group's players do not have a chance to play the game, so they remain neutral (represented with $*$).

The equilibria are as follows (figure 3.3(a) and (b)):

$$allc_k = [i_c = 1]_{i \in O_k \cup \{k\}},$$

$$premierD = [p_c = 0, i_c = *]_{i \in \mathbf{G} - \{p\}}.$$

Finally, there is a cooperator-defector mixed equilibrium, when some groups consist of all cooperators and some are all defectors, followed by the neutral groups in $O_k \cup \{k\}$. There are two types of mixed equilibrium, when considering from the point of view of k; (i) one defector group exists in $U_k \cup \{k\}$ (figure 3.3c), or (ii) there are only cooperator groups in $U_k \cup \{k\}$, and at least one defector group exists in D_k (figure 3d). This type of mixed equilibrium is hereafter called $mix_{U_k \cup \{k\}}$. This is represented as follows (figure 3.3(c)):

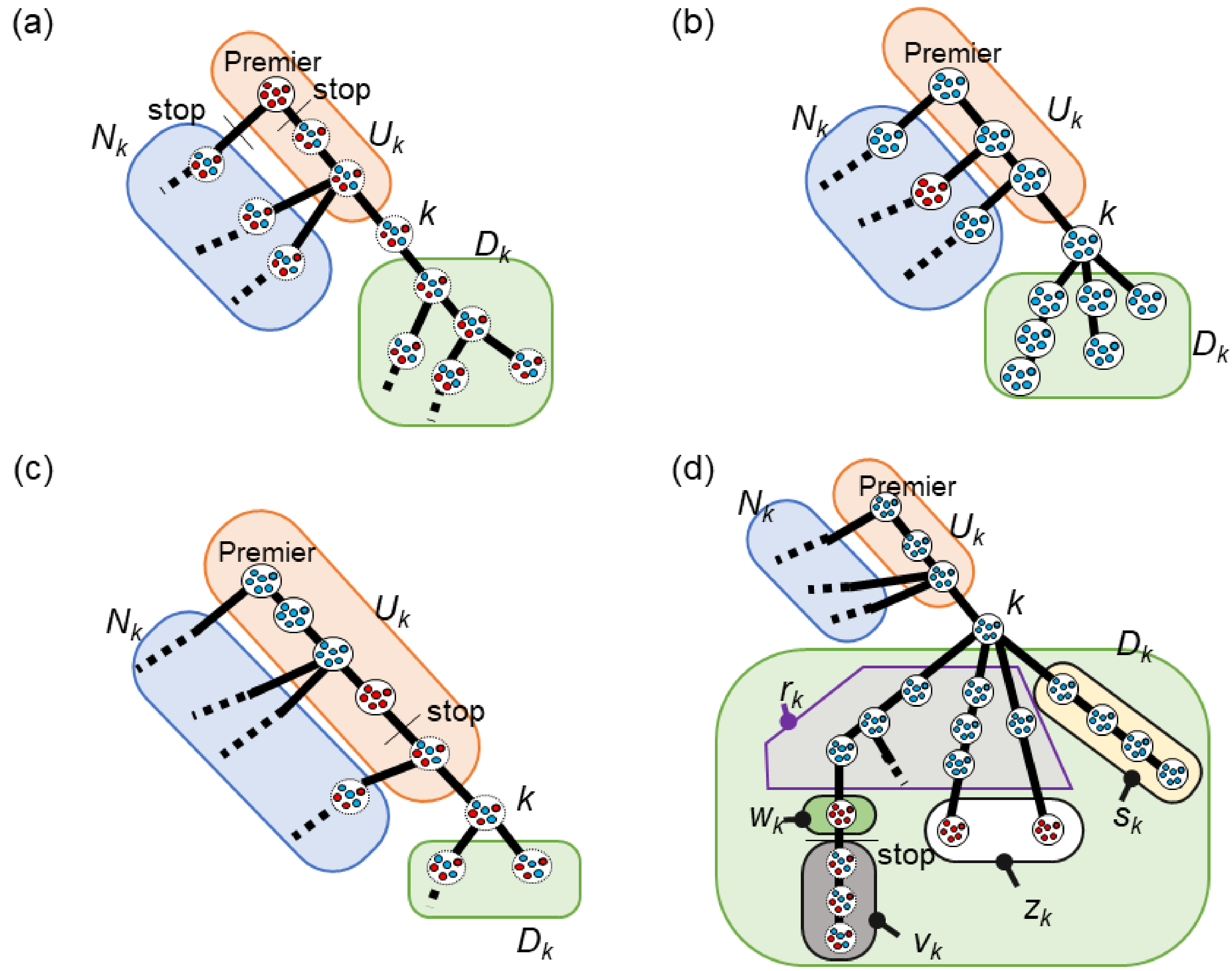

Figure 3.3: The equilibria in the system from the perspective of focal group k. (a) is showing the $allc_k$, (b) is showing the $premierD$, (c) is showing the $mix_{U_k \cup \{k\}}$, and (d) is showing the mix_{D_k}.

$$mix_{U_k \cup \{k\}} = [p_c = 1, i_{1c} = 1, i_{2c} = 0, i_{3c} = *]_{i_2 \in U_k \cup \{k\} - \{p\} \text{ and } i_1 \in U_{i_2} \text{ and } i_3 \in D_{i_2}}$$

Figure 3.3(c) shows that $U_k \cup \{k\}$ is similar to the linear division of labour (Nirjhor and Nakamaru, 2023); if someone chooses defection in a particular group, the task does not get passed to the next group, so the division of labour is stopped, and the later groups' strategy does not matter.

The second type of mixed equilibrium, (ii), is called mix_{D_k}. Because of the presence of the branching in D_k, we generalize this equilibrium as follows (figure 3.3(d)):

$$mix_{D_k} = [u_{1c}, ..., u_{n1c}, k_c, s_{1c}, ..., s_{n2c}, r_{1c}, ..., r_{n3c}, w_{1c}, ..., w_{n4c}, v_{1c}, ..., v_{n5c}, z_{1c}, ..., z_{n6c}],$$

where, $u_{ic} = 1, k_c = 1, s_{ic} = 1, r_{ic} = 1, w_{ic} = 0, v_{ic} = *, z_{ic} = 0$ where $u_i \in U_k, s_i \in s_k, r_i \in r_k, w_i \in w_k,$

$v_i \in v_k$, and $z_i \in z_k$. We define $u_1 = p$. The mutually disjoint sets s_k, r_k, w_k, v_k and z_k are defined as follows,

$$s_k := \{s \in D_k | m \in O_s \cup \{s\} \Rightarrow m_c = 1\},$$

$$r_k := \{r \in D_k | m_1 \in U_r \cup \{r\} \Rightarrow m_{1c} = 1, \text{ and } \exists m_2 \in D_r, \text{ where } m_{2c} = 0\},$$

$$w_k := \{w \in D_k \text{ and not a terminal} | m \in U_w \Rightarrow m_c = 1, \text{ and } w_c = 0\},$$

$$v_k := \{v \in D_k | \exists m \in U_v \text{ where } m \in w_k\},$$

$$z_k := \{z \in D_k \text{ and a terminal} | m \in U_z \Rightarrow m_c = 1, \text{ and } z_c = 0\},$$

where, $|U_k| = n_1$, $|s_k| = n_2$, $|r_k| = n_3$, $|w_k| = n_4$, $|v_k| = n_5$ and $|z_k| = n_6$. Also,

$$s_k \cup r_k \cup w_k \cup v_k \cup z_k = D_k.$$

In simple terms, s_k is the set of the groups that are in D_k, and their respective trees of the division of labour have a fully cooperative population in all the groups. r_k is the set of the groups in D_k, which have fully cooperators, and the upstream groups of their respective trees of the division of labour have fully cooperators. However, at least one group has fully defectors in their downstream. w_k is the set of the groups that are in D_k, not terminals, and have a full defector population. v_k is the set of the groups that are in D_k and have a fully defective group in their upstream. The strategies of the members of these groups are neutral, which is represented by $*$ as explained before. z_k is the set of the terminal groups that are in D_k, and have a full defector population.

When k is the premier, p, the $mix_{U_p \cup \{p\}}$ equilibrium does not exist, because there is no U_p and the case of p being the defector group is included in the equilibrium $premierD$. When k is a terminal, we can consider $O_k \cup \{k\}$ as a linear division of labour (Nirjhor and Nakamaru, 2023), where mix_{D_k} does not exist.

If $k = p$, then $O_p \cup \{p\} = \mathbf{G}$, in other words all the groups are in it. If we consider a certain k, which is not the premier, eqs. (3.1)-(3.3) show that N_k does not have any effect in the dynamics, therefore, it is enough to only consider the stability of the equilibria across $O_k \cup \{k\}$.

The local stability of each equilibrium is analyzed (for calculations, refer to Appendix C). Because the equilibria in the general tree graph are hard to write individually, we consider an arbitrary group k and define the classes of the equilibria while focusing on that group. The results are mentioned in table C1. To determine the stability of the whole system, we need to consider each of the groups individually and obtain the locally stable conditions of the respective equilibria of those individual focus groups and finally combine their conditions to obtain the locally stable state of the whole system. We shall explain this in detail in the following section using a specific example.

To study a social dilemma situation in the baseline, we consider $g_i < x_i$ for all group i. Moreover, if g_i is high enough, it is natural that cooperation among all groups can evolve, and then we do not consider $g_i > x_i$. With this social dilemma condition we summarize the results in table 3.3, which shows that *premierD* is the only stable equilibrium in the Baseline, because $g_p - (1 - c_{op})g_{op} < x_p$ is always held (table C1).

3.3 Two sanction systems

We introduce two types of sanction systems following Nirjhor and Nakamaru (2023). One is called the defector sanction system, where the exact defector is caught and sanctioned with the amount f. The finding probability of the exact defector is ρ. The other is called the premier sanction system, where if the defection is present, whoever defects, always the player in the premier group is sanctioned with the amount f. We study the evolution of cooperation in the system without punishment named the baseline system and then compare its result with the two systems with sanction. The two sanction systems are also compared with each other, to find their effectiveness.

Appendices D and E show the replicator equations for asymmetric games and the results of the local stability analysis of the defector sanction system and the premier sanction system, respectively. Table 3.4 and C1 summarize the local stability condition for each of the equilibrium points.

We find that *premierD* is the only stable equilibrium in the premier sanction system as well as in the Baseline (table 3.4 and C1), although the condition is somehow less strict for *allc* to be stable because of the punishment, the same conclusion can be drawn here as well as the baseline. In the defector sanction system, however, all the equilibria are conditionally stable.

We would like to explain the equilibrium using figures 3.2 and 3.4(a). When a defector-sanction system is applied, figures 3.2 is the image of the equilibrium and figure 3.4(a) is the time-change of the frequencies of cooperators in each group. To obtain the local stability condition for this whole system, **G**, we do the local stability analysis for the equilibrium point with respect to k without considering the effect of N_k, using eq.(D1)-(D3). In the case of figures 3.2 and 3.4(a), where **G** converges to the mix_{D_p} equilibrium, we can obtain the local stability condition of the mix_{D_p} equilibrium in the whole system **G**. Additionally, the $allc_{k_{11}}$ equilibrium with respect to k_{11} should be locally stable, the $mix_{U_{ki} \cup \{k_i\}}$

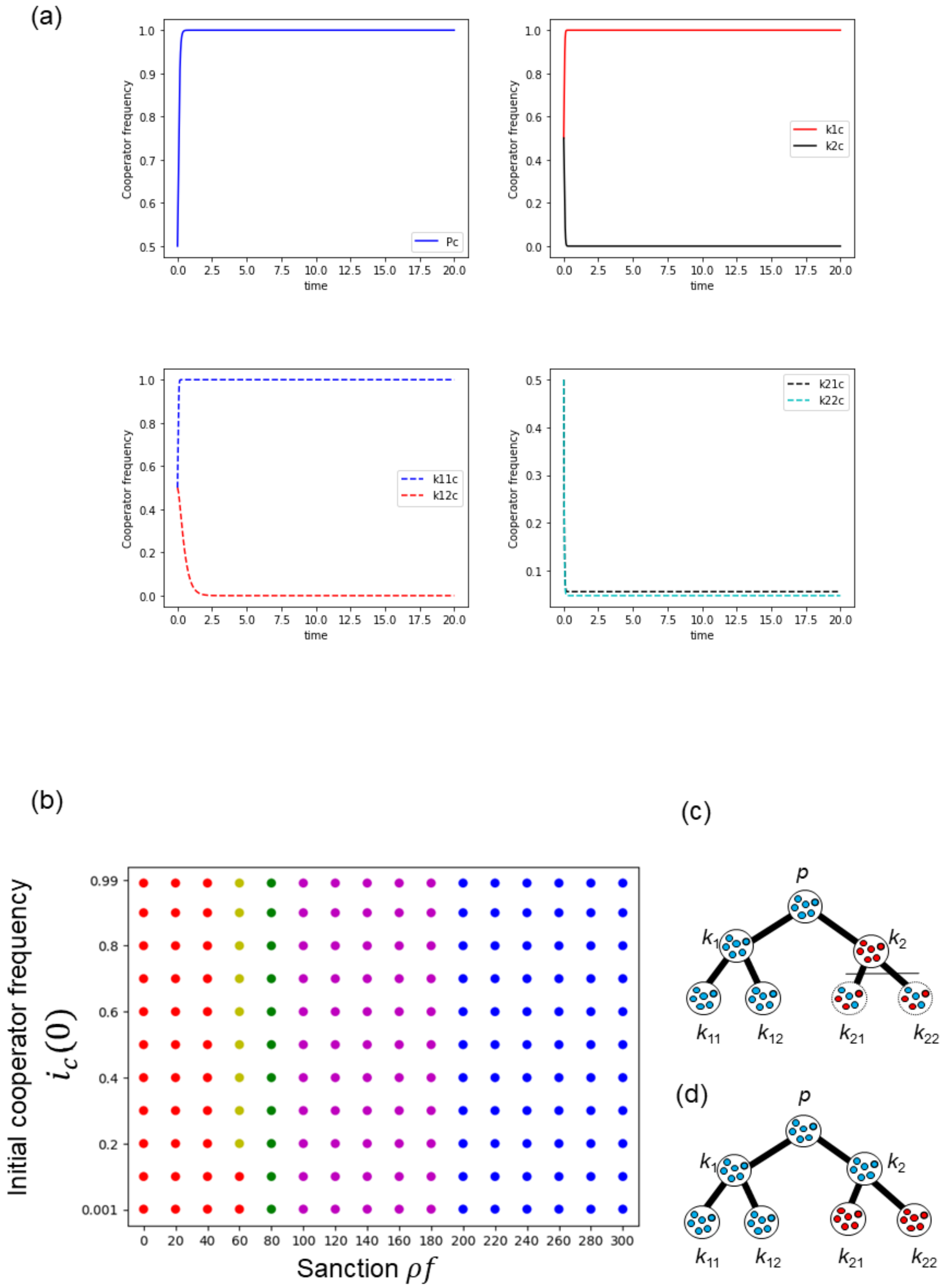

Figure 3.4: (a) presents the numerical simulation outcomes in a 2-regular 2-branched network as shown in figure 2 when the $i_c(0) = 0.5$ and $\rho f = 58$, and the evolutionary dynamics converges to one of states such as figure 3.2. In each graph of (a), the horizontal axis is for time and the vertical axis is for the frequency of cooperators in each group. The left-upper graph shows the dynamics in the premier group; the right-upper graph, group k_1 (red) and group k_2 (black); the left-lower, the group k_{11}(blue dashes) and group k_{12} (red dashes); the right-lower, the group k_{21}(black dashes) and group k_{22} (light blue dashes). (b) presents the effect of sanction, ρf in the defector sanction system, and the initial frequency of cooperators, $i_c(0)$, on the simulation outcomes. In (b), yellow, green, and magenta dots present three types of mixed equilibria, presented by figure 3.2, figure 3.4(c) and figure 3.4(d), respectively. The red and blue dots represent the *premierD* and *allc* equilibria, respectively. The parameters are: $g_p = 64, g_{k_1} = g_{k_2} = 32, g_{k_{11}} = g_{k_{12}} = g_{k_{21}} = g_{k_{22}} = 16, x_p = 65, x_{k_1} = 35, x_{k_2} = 99, x_{k_{11}} = 20, x_{k_{12}} = 77, x_{k_{21}} = 207, x_{k_{22}} = 215$.

Table 3.4: Local stability conditions when k is the focal group in a social dilemma

Equilibrium	Baseline	Defector sanction	Premier sanction
$premierD$	Always stable	$g_p - (1-c_{op})g_{op} + \rho f < x_p$	$g_p - (1-c_{op})g_{op} + c_{op}f < x_p$
$allc_k$	Always unstable	$g_i + \rho f > x_i$ where $i \in O_k \cup \{k\}$.	Always unstable
$mix_{U_k \cup \{k\}}$	Always unstable	$g_i - g_{oi} + \rho f > x_i$, $g_j - (1-c_{oj})g_{oj} + \rho f < x_j$, when k is not a terminal.	Always unstable
		$g_i - g_{oi} + \rho f > x_i$, and $g_j - (1-c_{oj})g_{oj} + \rho f < x_j$, when k is a terminal and $j \neq k$.	
		$g_i - g_{oi} + \rho f > x_i$, and $g_j + \rho f < x_j$, when k is a terminal and $j = k$.	
		where $j \in U_k \cup \{k\}$, j is the defector group and $i \in U_j$.	
mix_{D_k}	Always unstable	$g_{i_1} - g_{oi_1} + \rho f > x_{i_1}$, and $g_{i_2} + \rho f > x_{i_2}$, and $g_{i_3} - (1-c_{oi_3})g_{oi_3} + \rho f < x_{i_3}$ and $g_{i_4} + \rho f < x_{i_4}$	Always unstable
		here $i_1 \in U_k \cup \{k\} \cup r_k$, $i_2 \in s_k$, $i_3 \in w_k$, and $i_4 \in z_k$	

equilibrium with respect to k_i where i is 2, 12, 21, 22, and the $mix_{D_{k_1}}$ equilibrium with respect to k_1 should be locally stable (figures 3.2 and 3.4(a)). The stable equilibria from the perspective of different groups are included in table 3.2.

In figure 3.4(a), as $x_p - g_p + (1-c_{op})g_{op} = 49 < 58 = \rho f$, the premier group becomes full cooperator. As $x_{k_1} - g_{k_1} + (1-c_{ok_1})g_{ok_1} = 19 < 58 = \rho f$ and $x_{k_2} - g_{k_2} + (1-c_{ok_2})g_{ok_2} = 97.27 > 58 = \rho f$, k_1 becomes full cooperator, and k_2 becomes full defector, which in return makes k_{21} and k_{22} neutral. $x_{k_{11}} - g_{k_{11}} = 4 < 58 = \rho f$ and $x_{k_{12}} - g_{k_{12}} = 61 > 58 = \rho f$, which means k_{11} becomes full cooperator and k_{12} becomes full defector (table 3.4). For this reason, with respect to group k_{11}, the groups in $O_{k_{11}} \cup \{k_{11}\}$ converges to $allc_{k_{11}}$. With respect to k_{12}, the groups in $O_{k_{12}} \cup \{k_{12}\}$ converge to the $mix_{U_{k12} \cup \{k_{12}\}}$ equilibrium and are not influenced by the groups in $N_{k_{12}}$.

Figure 3.4(b) shows the outcomes of the numerical simulations when costs are determined randomly. When sanction is small, the simulations converges to the $premierD$ equilibrium (red); when sanction is large enough, the simulations converges to the $allc$ equilibrium (blue). Between $premierD$ and $allc$ equilibria, the dynamics converges to the mixed equilibrium point shown in figure 3.2 (yellow). When we change the value of sanction, we obtain other mixed equilibria presenting figures 3.4(c) and 3.4(d) shown with green and magenta dots, respectively.

To understand the dynamics more concretely we do some simplification and numerical analysis in the following section.

3.4 Numerical analysis

Firstly, we consider a special case; a κ-regular, μ-branched directed finite graph is assumed, for making numerical analysis. When js are the immediate branching of k, $g_k = \sum_j g_j$. In this case, we also consider the distribution of the loss is uniform in each branching. If k is at the μ_kth branch (where $0 \leq \mu_k \leq \mu$), we assume, $g_k = (\frac{1}{\kappa})^{\mu_k} g$.

For the 2-regular, 2-branched graph, the expected values of g_{op}, g_{ok_1}, and g_{ok_2}, $E[g_{op}]$, $E[g_{ok_1}]$ and $E[g_{ok_2}]$, are as follows (figure 3.2): $E[g_{op}] = g_{k_1}(1 - k_{1c}) + k_{1c}E[g_{ok_1}] + g_{k_2}(1 - k_{2c}) + k_{2c}E[g_{ok_2}]$, where $E[g_{ok_1}] = g_{k_{11}}(1 - k_{11c}) + g_{k_{12}}(1 - k_{12c})$, $E[g_{ok_2}] = g_{k_{21}}(1 - k_{21c}) + g_{k_{22}}(1 - k_{22c})$, and k_{22c}, for example, is the frequency of cooperators in group k_{22} (table 3.3).

Figure 3.5 shows the effect of ρf on the dynamics in the 2-regular 2-branched directed finite graph. For simplicity, we consider the cost of the cooperation for the groups which are present in the same level branching to be the same.

Figure 3.5(a) is the results of the numerical analysis of equations (D1)-(D3) in Appendix B when the cost of cooperation decreases downstream under the defector sanction system. Figure 3.5(a) shows that our numerical simulation results match our theoretical prediction (table 3.4). In figure 3.5(a), the simulation dynamics converges to the *premierD* equilibrium in $\rho f < 66$, which matches $\rho f < x_p$ where all groups are almost full of defectors, $c_{op} \approx 0$, and $g_{op} = g_p$, and the dynamics also converges to the *allc* equilibrium in $\rho f \geq 46$ starting from a very high initial value of $i_c(0)$, which matches $\rho f + g_i > x_i$ for all the group is. The initial frequencies determine which dynamics converge to the *premierD* or *allc* equilibria in ρf between 46 and 64.

Figure 3.5(b) shows the numerical simulation outcomes when the cost of the cooperation increases downstream. The simulation dynamics converges to the *premierD* in $\rho f \leq 94$ which matches $\rho f + g_p - (1 - c_{op})g_{op} < x_p$ where all groups are almost full of defectors, $c_{op} \approx 0$, and $g_{op} = g_p$ (table 3.4). When ρf is 92 in figure 3.5(b), there is a co-presence of green dots and red dots, which indicates the stability predicted by table 3.4; the initial frequency of cooperators in groups determines if the dynamics converge to either the mixed equilibrium mix_{D_p} or the *premierD*. When $\rho f \geq 94$, the simulation converges to the *allc* equilibrium starting from the almost full cooperator groups, which can be predicted by $g_i + \rho f > x_i$ for all group is (table 3.4). The red dots and the blue dots are co-present when ρf is 94 in figure 3.5(b), which matches the theoretical prediction; the bistability between the

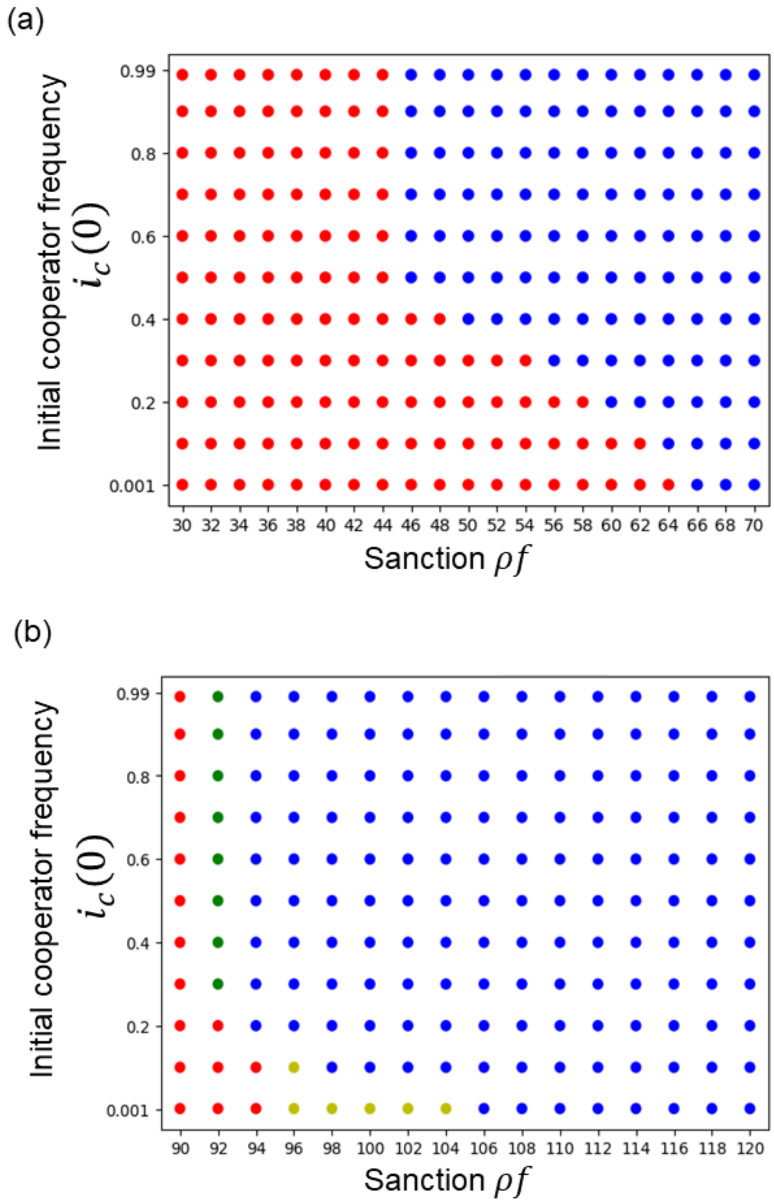

Figure 3.5: Evolutionary dynamics in the 2-regular 2-branched directed finite graph when (a) the cost of the cooperation decreases downstream, and (b) when the cost of the cooperation increases downstream. The blue dot represents when the system converges to *allc* equilibrium, or everyone in every group is a cooperator. The red dots represent the simulation dynamics converged to the *premierD* equilibrium, the yellow dot represents the mixed equilibrium when the premier group is full cooperator and the groups in the first branching or k_1 and k_2 are full defectors, the green dot represents the mixed equilibrium when the groups premier, k_1 and k_2 are full cooperators and the groups in the 2nd branching k_{11}, k_{12}, k_{21} and k_{22} are full defectors. (a) shows the bistability between the red-blue bistability, and (b) shows the red-blue bistability, red-green and yellow-blue co-stability under the same sanction. Both of the figures show that sanction promotes the evolution of cooperation. The stable existence of mixed equilibrium in (b) represents the stable co-existence of fully cooperator and fully defector groups in the same network when sanction is applied. The parameters in (a) are: $g_p = 64, g_{k_1} = g_{k_2} = 32, g_{k_{11}} = g_{k_{12}} = g_{k_{21}} = g_{k_{22}} = 16, x_p = 65, x_{k_1} = x_{k_2} = 63, x_{k_{11}} = x_{k_{12}} = x_{k_{21}} = x_{k_{22}} = 61$. The parameters in (b) are: $g_p = 92, g_{k_1} = g_{k_2} = 46, g_{k_{11}} = g_{k_{12}} = g_{k_{21}} = g_{k_{22}} = 23, x_p = 95, x_{k_1} = x_{k_2} = 105, x_{k_{11}} = x_{k_{12}} = x_{k_{21}} = x_{k_{22}} = 115$.

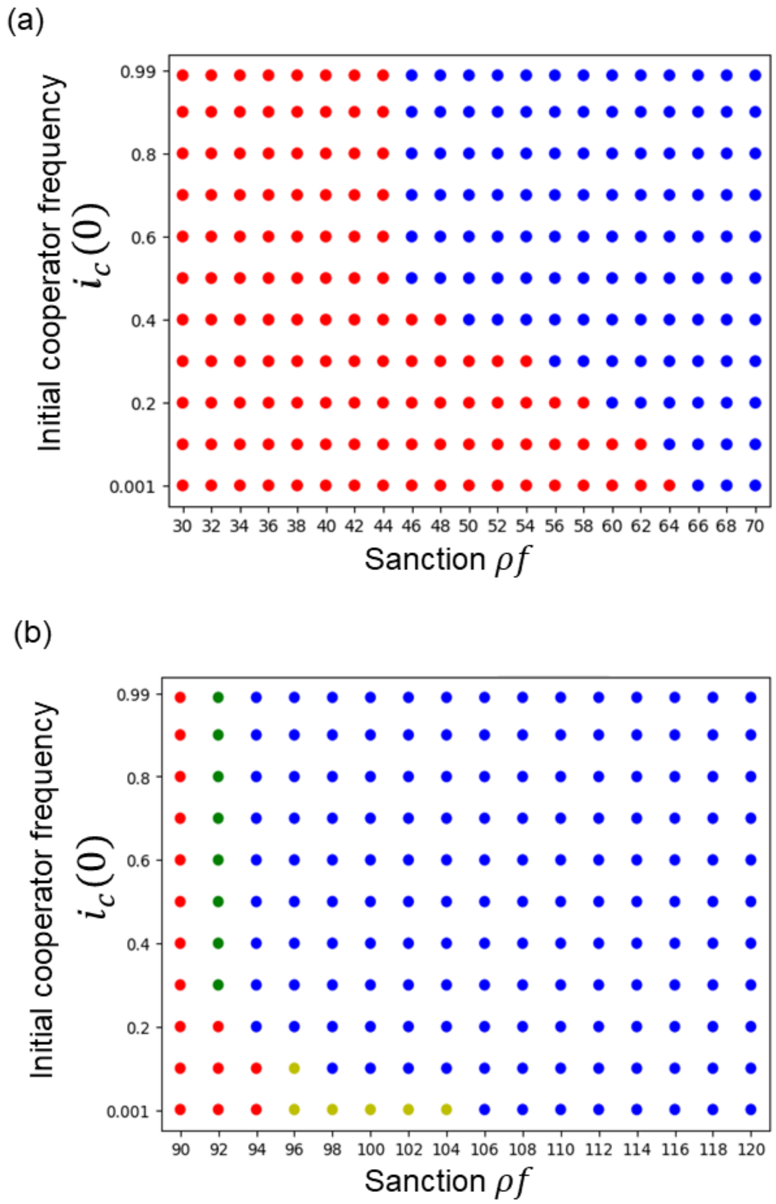

Figure 3.6: (a) presents the network structure which imitates the bureaucratic structure of the US Department of State, and one of equilibrium states. (b) presents the effect of sanction, ρf, and the initial frequency of cooperators on the numerical simulation outcomes. The red and blue dots represent the *premierD* and *allc* equilibria, respectively. The yellow dot presents the dynamics converges to the equilibrium shown in (a). Even though the cost is decreasing downstream, the simulation converges to the mixed equilibrium presented by (a) in (b). The parameters, which are given irrelevant to the bureaucratic structure, are: $x_p = 65, x_{k_1} = x_{k_2} = x_{k_3} = x_{k_4} = x_{k_5} = 63, x_{k_{11}} = x_{k_{12}} = x_{k_{13}} = 59, x_{k_{21}} = x_{k_{22}} = 61, g_p = 60, g_{k_1} = g_{k_2} = g_{k_3} = g_{k_4} = g_{k_5} = 12, g_{k_{11}} = g_{k_{12}} = g_{k_{13}} = 4, g_{k_{21}} = g_{k_{22}} = 6.$

equilibria *premierD* and *allc*. The numerical simulations can show the co-presence of blue and yellow dots when ρf is from 96 to 104 in figure 3.5(b).

We have only considered the evolutionary dynamics in the symmetric tree network. In reality, there are division of labour in asymmetric tree networks. The network in figure 3.6(a) is inspired by the simplest tree-like bureaucratic structure of the US Department of State shown in the 15th chapter of

American Government (2e – Second Edition) (2019) by Openstax and Lumen Learning. The network has one premier node as the secretary of state, then 23 nodes branched in the first level of branching, and 7 of them have branching of themselves in another level of branching. We consider the ratio of it and take a simpler network with a similar ratio for doing our numerical analysis. We take a network in which in the first level of branching, 5 branches come out of the premier node, then two of them have further branching, one has 2 branches and one has 3 branches in figure 3.6(a).

The numerical simulation outcomes is shown in figure 3.6(b) where costs decrease downstream. Table 3.4 indicates that the mixed equilibria can also be locally stable when the cost of the cooperation decreases downstream. This is because $g_i - g_{oi} + \rho f > x_i > x_j > g_j - (1 - c_{oj})g_{oj} + \rho f$ as well as $g_i - g_{oi} + \rho f > x_i > x_j > g_j + \rho f$ have no contradiction when $j \in D_i$ (table 3.4). Our numerical simulations can also show that the dynamics converge to the mixed equilibrium point (yellow dots in figure 3.6(b)). While, the mixed equilibrium point cannot be locally stable and no numerical simulations converged to the mixed equilibrium point in the linear division of labour when the cost decreases downstream (Nirjhor and Nakamaru, 2023), and therefore the existence of mixed equilibrium points when the cost decreases downstream is unique from this study.

3.5 The effect of the network size

There are two ways in which a given network can increase in size. One is when more levels are added downstream of it and the other is when more branches are added to a network in parallel to other nodes. We analyse the effects of both with mathematical analysis and then evaluate the results through numerical analysis. Appendix D shows mathematically that adding more levels downstream of the same network hinders the evolution of cooperation. This is evaluated through the comparison of numerical analysis in figures 3.5(b), 3.7(a) and 3.7(b). Figure 3.7(a) shows the evolutionary dynamics converges to equilibria of the network which has one less level downstream than the network of figure 3.5(b), in other words, 2-regular once-branched network (see figure F.1(a)), with the same cost of cooperation, and losses to the groups in each node as the figure 3.5(b). Figure 3.7(b) shows the evolutionary dynamics converges to equilibria of the network which has one more level downstream than the network of figure 3.5(b), in other words, 2-regular thrice branched network (see figure F.1(b)), with the same cost of cooperation, and losses to the groups in each node as the figure 3.5(b). For simplicity and consistency with the settings in figure 3.5(b), we considered that, as there is one more level of branching, the loss is uniformly divided once more and the cost is increased with the same

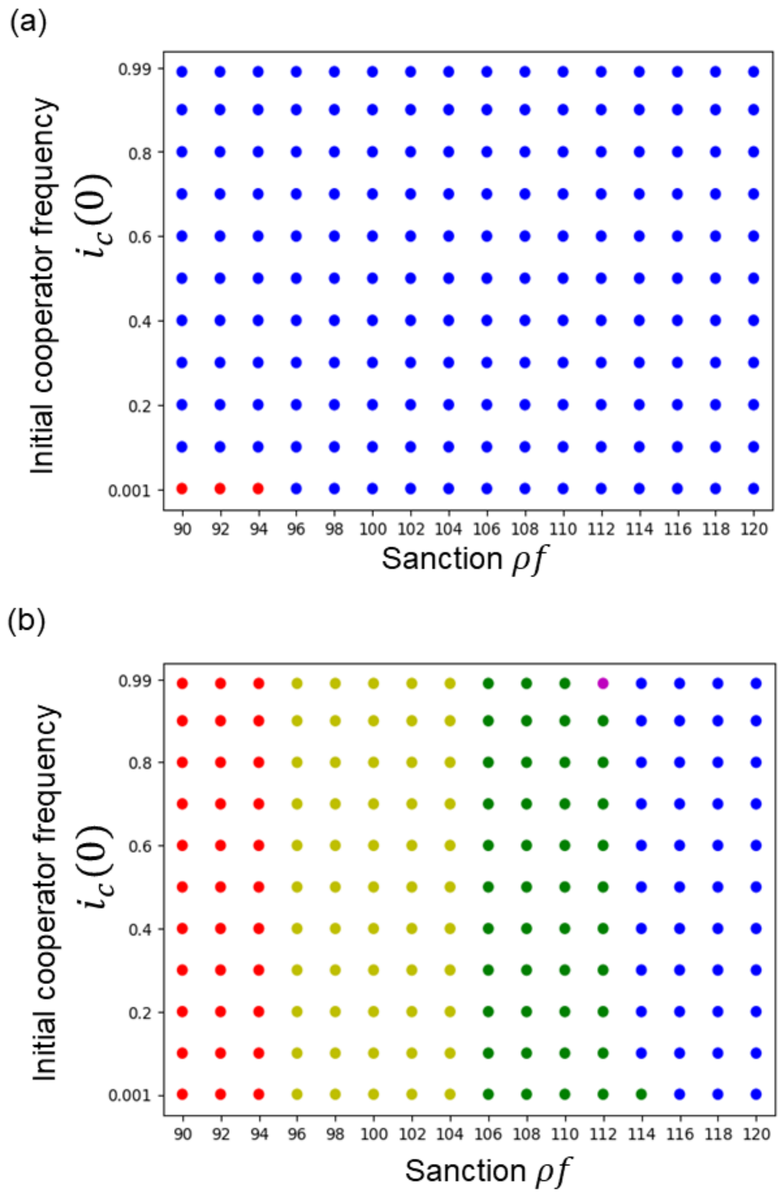

Figure 3.7: Showing the effect of adding or removing a level of branching in downstream of a network on the evolution of cooperation compared with figure 3.5(b). (a) shows the numerical analysis results in a two-regular once-branched network (see figure F.1(a)). (b) shows the numerical analysis results in a two-regular thrice-branched network (see figure F.1(b)). The red and blue dots represent the *premierD* and *allc* equilibria, respectively. The yellow, green, and magenta dots present the mixed equilibria when the first level of branching, second level of branching, and the third level of branching become the full defector groups, respectively. The parameters are in (a): $x_p = 95, x_{k_1} = x_{k_2} = 105, g_p = 92, g_{k_1} = g_{k_2} = 46$. The parameters in (b) are: $x_p = 95, x_{k_1} = x_{k_2} = 105, x_{k_{11}} = x_{k_{12}} = x_{k_{21}} = x_{k_{22}} = 115, x_{k_{111}} = x_{k_{112}} = x_{k_{121}} = x_{k_{122}} = x_{k_{211}} = x_{k_{212}} = x_{k_{221}} = x_{k_{222}} = 125, g_p = 92, g_{k_1} = g_{k_2} = 46, g_{k_{11}} = g_{k_{12}} = g_{k_{21}} = g_{k_{22}} = 23, g_{k_{111}} = g_{k_{112}} = g_{k_{121}} = g_{k_{122}} = g_{k_{211}} = g_{k_{212}} = g_{k_{221}} = g_{k_{222}} = 11.5$. See figure F.1(b) for the notations.

increment as the previous levels in the network for which figure 3.7(b) shows the convergence of the evolutionary dynamics.

When the initial frequency of the cooperators is very low such as 0.001, as $1 - c_{op} \simeq 1$ and $g_{op} = g_p$, the local stability condition for *premierD* in group p is that $\rho f < x_p$ regardless of figures 3.5(b), 3.7(a) and 3.7(b) (table 3.4). The local stability condition for $allc_k$ indicates that with adding more levels downstream cooperation is less evolved (see table 3.4). As a result, the bistable region or the region with mixed equilibria is wider in the network with more levels (see figures 3.5(b), 3.7(a) and 3.7(b)).

Appendix F also shows that the effect of adding a branch in parallel to the same network is uncertain.

3.6 Discussion and conclusion

We took a model of the division of labour in a finite tree graph and studied the effect of sanctions on it. There is a premier group and then the division of labour is branched from it. Each node of the tree graph has a group which has a role in the division of labour. The task flows from the upstream to the downstream and gets divided through the branching. If a player who is randomly selected from a group chooses defection, the division of labour stops there and everyone in every group needs to bear a loss according to their position. We compare the evolution of cooperation in the baseline system (which has no sanction) with the two sanction systems named the defector sanction system and the premier sanction system. We studied the general model and found three equilibria when the defector sanction system is applied in the social dilemma situation, 1) *premierD*, where all players in the whole premier group choose defection, 2) *allc* where everyone in every group is a cooperator, and 3) mixed equilibrium, where the premier group consists of only cooperators, some other groups also are full of cooperators, and somewhere in the network there are one or more group/groups who have a whole population of defectors. We did the local stability analysis of these equilibria. Then, for doing numerical analysis, we considered a special case and verified the results of the general case.

The previous theoretical studies show that the cooperation evolves in a network structured population when $b/c > k$, where each node representing a single player has k regular links and b is the benefit from a cooperator and c is a cost of cooperation (e.g. Nowak, 2006). However, in our study, the benefit to a player given by a cooperator from the upstream group is cancelled out in equations and does not have any effect on the evolution of cooperation. This means that our results cannot be summarized by the ratio of b and c. This result is the same as the linear division of labour (Nirjhor and Nakamaru, 2023). This is because both the cooperators and defectors in a group receive the benefit from the cooperation in the upstream groups regardless of their own strategy. This can be interpreted as the salary given to an employee in this sort of division of labour has no effect on the evolution of cooperation.

The loss via defection becomes distributed in the branches as $g_k = \sum_{j \text{ are immediate branching of } k} g_j$. The loss via defection g_k is subjective to the task assigned to the group k. Because of this setting a group k's evolutionary dynamics are only affected by the action of groups in $O_k \cup \{k\}$, as those are the groups directly associated with the action that is assigned to k. If there is a defection in the upstream of k, the loss to k is the same as if the task is not being completed by a player in group k. A defection downstream of k means a part of the task assigned to group k is not eventually fully completed, which in turn affects the payoff of k as g_{ok}. However, because of this setting the tasks assigned to groups present in N_k have no relation with the task of group k, and because of that, their defection does not affect the evolutionary dynamics of k. That is why g_{nk} is canceled out from the replicator equation for the dynamics of group k. This means that a group's decisions are influenced only by that part of the network with which the nodes have a direct hierarchical connection with that particular group. In simpler terms, a group is influenced by other groups which are either in its hierarchical upstream or downstream, not the groups which are branched separately from its upstream but belong to the same network. This result can be applied to the division of labour of the government, as it shows that a corrupt/honest sector can exist independently and in a government, even when other sectors of the government are honest/corrupt.

Another main point of this study is to show how to calculate the local stability of equilibrium point in a general tree network; there are numerous possible equilibrium points in a tree network. If we calculate the local stability condition of each of all possible equilibrium points, it is tough work. In our work, [1] we categorize various equilibrium points into four types in terms of a specific node k, [2] obtain the local stability of each of four types, [3] the combination of these four types presents a specific equilibrium point, and so [4] we can obtain the local stability of the specific equilibrium point by the combination of four types. This is the contribution of our work from the viewpoint of mathematical modelling.

In our study the defector sanction system prevails in both the evolution of cooperation and sustaining the co-existence of the fully cooperator and defector groups than the premier sanction system. However, this sanction system depends on the finding probability of the exact defector ρ. We do not assume that players have perception bias. However, in reality, there are perception bias. In Jiang et al. (2023), it is stated through a human experiment that even though the subjective perception of being sanctioned is often lesser than the actual threat of being sanctioned, the higher threat regardless makes the population choose cooperation more. In our future studies, we will introduce a new assumption about risk perception and investigate the effects of subjective risk perception of sanctions in the evolution of cooperation in the division of labour.

With comparison with the linear division of labour (Nirjhor and Nakamaru, 2023), we find that the mixed equilibria can be stable in the baseline as well as the premier sanction system, when we do not consider the social dilemma situation, in other words, $g_i < x_i$ for all the group is does not necessarily hold. In Nirjhor and Nakamaru (2023), the mixed equilibria were unstable in the baseline and the first role sanction system regardless of social dilemma or not. We theoretically find that the mixed equilibrium can be stable even when the cost is decreasing downstream in the defector sanction system and show it with the figure 6. In the linear division of labour of Nirjhor and Nakamaru (2023), the mixed equilibrium is never stable when the cost is decreasing downstream in the defector sanction system. In other words the coexistence of the full cooperator and full defector groups have more scopes to the stable in the tree graph network than the linear network.

We should mention the applicability of this study to the supply chain. Our study can be applied to the multilayered subcontract (e.g. Tam et al, 2011) which has the tree structure assumed in our study. There are various networks among roles and stakeholders in the supply chain. In Lambert and Cooper (2000), the generalized supply chain network was shown to be an uprooted tree-like, where there is a central body that can be considered as the stem of the tree from which branches spread in both directions of the root and the shoot. From one direction the branches merge upstream towards the central body showing many divisions of the labour merging into the completion of a single labour, and from there the branches split downstream showing the labour is being divided. Our study addresses the evolution of cooperation in the later part of the supply chain where the labour is being divided downstream. When each player is assumed to be located at each node of trees and to interact with the neighbors, the effect of the merge of networks or directed cycles on the evolution of cooperation has been investigated (e.g. Su et al., 2022). As we assume that a group is located at each node of trees, our model and results would be different from the previous studies. In our future research, we wish to address the problem of the evolution of cooperation in the former part of the supply chain as well, where the division of labour merges together upstream to complete a single labour, and then extend our study to the uprooted tree-like networks.

Chapter 4

Conclusion

4.1 Summary of previous chapters

We have studied the evolution of cooperation on the division of labour on two types of networks. Firstly we studied the linear division of labour with finite number of roles. Then we studied the division of labour on a unidirectional finite tree graph. In both of the studies we applied the sanction systems to see their effects in the evolution of cooperation.

In both of the studies we found that the benefit from others' cooperation does not have any effect on the evolution of cooperation. The benefit was initially included in the payoff matrix, however through the calculation of the replicator equation, the benefit was canceled out. The interpretation of this result is somewhat difficult, as most of the previous studies showed that the evolution of cooperation is often dependent on the benefit from others' cooperation. However, as in our models the benefit is given to cooperators and defectors of a particular group regardless of their action, only based on the previous group's player's cooperation, it has no effect on that particular group's evolution of cooperation. In many developing countries now defection in the office works is very high in the so-called governmental steady jobs, because of its guaranteed salary to defectors as well as cooperators, the salary can not work as an incentive towards cooperation. It has been discussed as a root cause of official corruption in Bangladesh, saying that "Numerous public officials, although not formally assigned any work, continue to receive salaries from the public purse" (McDevitt, 2015). In case of the supply chain, sometimes the benefit is given as a loan through the contract even before the work begins, so due to the contract once a player is given the work, shall also receive the benefit, regardless of him supplying or not in the short term, so the benefit can be guaranteed based on a player receiving the job (Li et al., 2016).

There can be many types of sanction system, we have considered these two types for two reasons, first, our study is inspired by Nakamaru et al. (2018), and there the first role sanction system and the defector sanction system were compared and it was shown that both of these are in practice in the division of labour of industrial dumping system in Japan. Second, the first role/premier sanction system can be considered as making the head of a division responsible for any defection in his/her subordinate. We found that the defector sanction system is effective in promoting the evolution of cooperation more than the first role or premier sanction system. Also, the defector sanction system promotes the stable coexistence of the fully cooperator and fully defector groups in both models. This shows that punishing the exact defector even when he/she is difficult to find rather than punishing the head of a system, or making responsible the so-called "top person" who has nothing to do with the defection, is a better way to promote the evolution of cooperation. As in our model for any defection, everyone suffers the loss g, we think that the type of sanction system where everyone suffers for an individual's defection can also be included through our model.

We have applied a novel method in simplifying both of the models, that is focusing on a single arbitrary group, dividing the whole graph into several sets based on that arbitrary group, and creating dynamical equations for that group. As the group is arbitrarily taken, the dynamics equations for most of the groups are similar except for the extreme cases such as premier and terminals. This method makes our task easier in analysing the finite-dimensional system compared to Nakamaru et al. (2018), where each of the interaction were considered with separate payoff matrices. In the tree graph network, we also apply similar idea in the local stability analysis of the equilibria, where there can be an immense number of distinct equilibria. However, through our method we define only four types of equilibria based on an arbitrary group and show that the combination of the local stability analysis of only those four types of equilibria can identify the local stability of the whole system. Nakamaru et al. (2018) did the stability analysis of the equilibria, however for considering the special case, there were several more parameters and thus it was difficult to interpret the results. Because of the simplicity of our model, it became easier to interpret. Particularly "the benefit does not affect the evolution of cooperation" was also true in Nakamaru et al. (2018), however, it was first discovered by our linear model. The result of our model regarding the stability of mixed equilibrium and last group equilibrium when the cost increases downstream, and the instability when the cost decreases downstream, helped us understand the stability of mixed equilibrium in Nakamaru et al. (2018).

In the tree graph study, we found that the loss g_{nk} from the not directly related part of the network to a group k, has no effect in the evolution of cooperation of that particular group k. This indicates that the full cooperator and full defector divisions in a network can co-exist independently. As an example, we think of the bureaucratic systems in a government, where although all the ministries have connections with the premier or the prime minister, a defection/cooperation in the education ministry does not affect the evolution of cooperation/defection in the health ministry. So a full defector education ministry can co-exist side by side with a full cooperator health ministry.

Although the linear division of labour is in fact a special case of the tree graph network, because of the presence of branching in the tree graph we find some differences in the results with the linear graph. First of which is that the stable coexistence of the cooperator and defector groups under the defector sanction system even when the cost of cooperation is decreasing downstream in the tree graph network, which was not possible in the linear graph network. Also, if we do not consider the social dilemma, meaning the loss through defection $g_i <$ the cost of cooperation x_i is not considered the coexistence of the cooperator and defector groups are possible in the tree graph network in the the baseline as well as in the premier sanction system, where in the linear network this coexistence was not possible regardless the social dilemma is considered or not.

To conclude, we have generalized the study of evolution of cooperation in the division of labour on unidirectional finite linear and tree graph networks under sanction systems.

4.2 Applications in the innovation science

We discussed about the vertical and horizontal division of labour in the introduction. In our study we focused on a combined network of vertical and horizontal division of labour. In the linear model explained in the 2nd chapter of this thesis, it is a vertical division of labour, and the 2nd model explains a division of labour which is generally vertical but possible expansion in the horizontal direction is also possible. Thus our study will be able to explain the evolution of cooperation in both vertical and horizontal division of labour.

This study broadens the applicability of the theoretical approach towards the evolution of cooperation in the division of labour. The linear supply chain comes into various lengths. As we have discussed in the introduction. Min and Zhou (2002) considered mainly 5 roles in the linear supply chain, suppliers, producers, distributors, retailers and customers. But Farooque et al., (2019) considered 7 roles in total by adding environment in the beginning and waste disposal in the end (figure 4.1(a)). The waste disposal system can be of various length as well, as Nakamaru et al.(2018) showed it first to be 5 and then for simplification 3 roles. The other parts of the linear supply chain can also be of various lengths. Therefore, to study the evolution of cooperation in the division of labour of linear supply

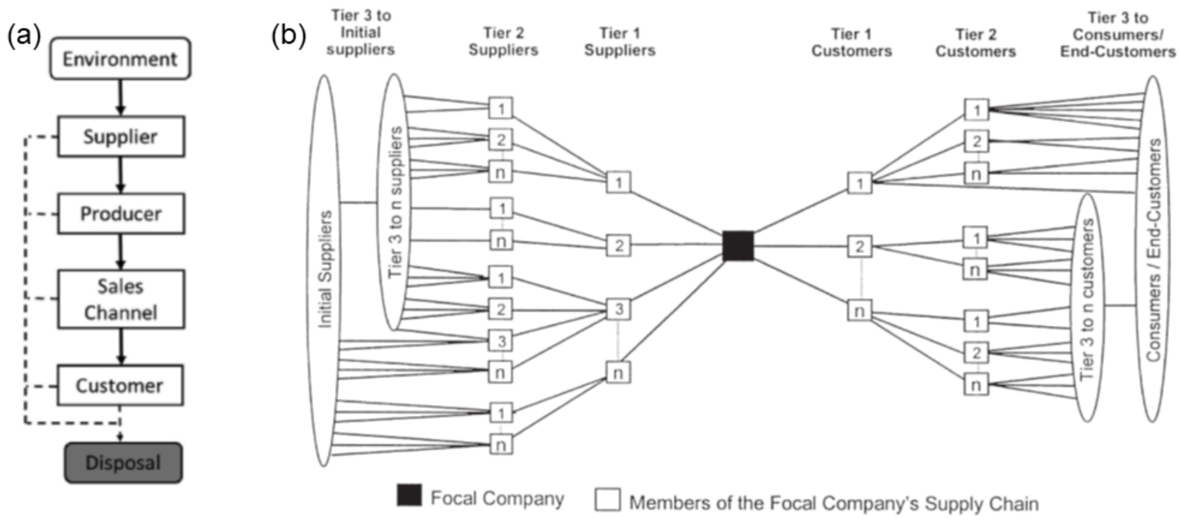

Figure 4.1: Structure of (a) the linear supply chain according to Farooque et al. (2019), and (b) the general supply chain according to Lambert and Cooper (2000).

chain a general finite role model is needed. Our model of the finite linear graph makes sure that any length of the linear supply chain can be modeled with this. However, there can be several suppliers and customers linked through the same producer as shown in the figure figure 4.1(b) (Lambert and Cooper, 2000), which gives the supply chain a form of an uprooted tree. Our study of the unidirectional one sided tree graph model can be applied to one side of this network, i.e. from producer to multiple tiers of many customers. Our model of the tree graph can also be applied to a network where there is a single group of suppliers in several tiers to the producer and from there to the multiple tiers of many customers. Therefore, our first model deals with the generalization of the length of the supply chain and the second model deals with the both the length and width of the supply chain. As we considered not a single player but a group of player in each of the nodes in our models, this can explain the evolution of cooperation among the competing farms for the same subtask in the supply chain, which is now the common practice in the global production network (Fengru and Guitang, 2019). The benefit in our model can be considered as the pre-agreed salary to the labourers in the supply chain, and the loss through defection can be considered as the reputation loss to the brand name of supply chain. To conclude, our models can explain the evolution of cooperation in the division of labour on the supply chain in a generalized fashion.

As shown in figure 4.2 a subcontract system has several divisions and a tree like structure (Tam et al., 2011). The main contractor needs to find a suitable sub-contractor for each of the subtasks, which creates a group of competing candidates for each of the jobs (concreting, steelwork, formwork, plumbing etc. according to figure 4.2). This essentially creates a network of groups for each of the subtasks. Thus our models, particularly the model of the division of labour on tree network can be

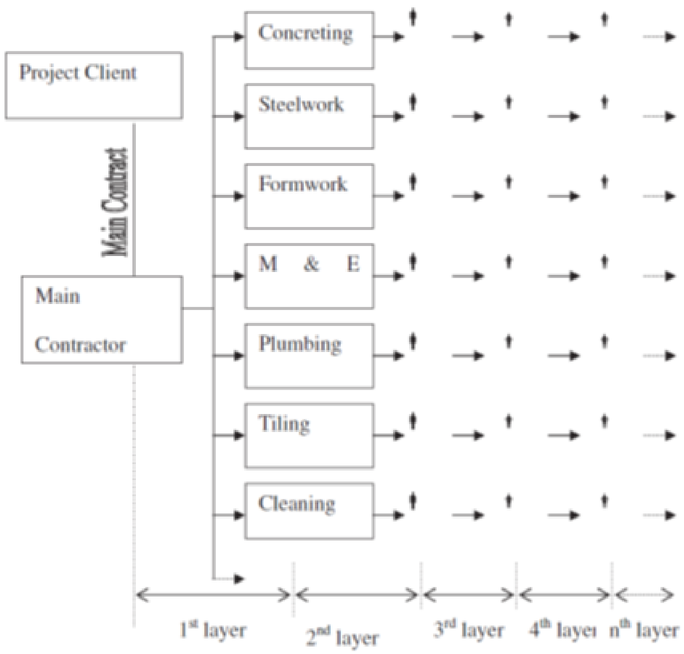

Figure 4.2: Structure of a multilayered subcontract system in a construction project according to Tam et al. (2011).

applied to understand the evolution of cooperation in the multilayered subcontract systems as well.

The government office works, planning and spending through the bureaucratic system is also a division of labour on the networks as shown in figure 4.3. In the government, a task is done by office workers through the division of labour consisting hierarchical order and subdivisions, thus possible branching. In each of the sub divisions of the bureaucracy there are many workers for the same task creating a group of workers (e.g. Eisenstadt, 1958; Hinings et. al, 1967; Bolin and Harenstam 2008). Our model's benefit can be considered as the pre-agreed salary to the workers, the cost of cooperation can be considered the amount of time and skill spent to perform a job and providing order to one's subordinates, the loss through defection can be considered as the bad reputation a department of bureaucracy gets when some job is not completed. The sanction can be considered as the punishment a worker gets if the work is not done properly. Our model depicts a type of red tape corruption, where the bureaucrat does not do his job even after getting the salary, it can be considered as defection in our model (Guriev, 2004). Our results regarding the evolution of cooperation in the division of labour on linear and tree network will pave the way to understand the evolution of cooperation in the bureaucratic systems.

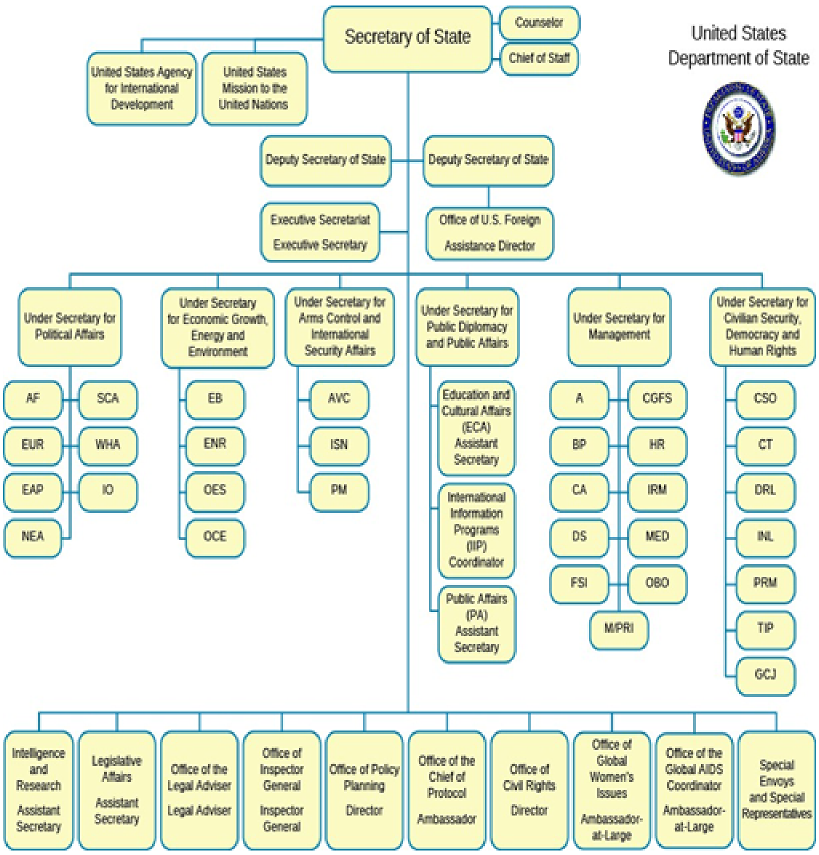

Figure 4.3: Tree graph structure of the division of labour of an American bureaucratic system. This figure shows the State Department according to Openstax and Lumen learning, 2019.

4.3 Future research

The division of labour can be a lot more complex networks such as the uprooted tree, bidirectional tree, cycle networks, parallel networks, and a combination of these networks (Ghiani et al., 2004). We shall focus on extending our study of the evolution of cooperation in the division of labour to these networks in our future works. Our present study will pave the path towards the simplification of these networks as well.

We did not consider the traditional police corruption in our model, which is defined as the defection of the sanction givers by taking a bribe from the defectors (Lee et al., 2019) for the sake of simplicity in our model. However, in reality, often the institutional sanction is coupled with police corruption. So we shall include police corruption in our existing models in future, and analyse the results.

The utilities we considered in this model are rigid form of objective universal utilities, however utilities are often subjective, changes their values based on the perspective of a player (Levy, 1992). In future, we shall expand our study based on subjective utilities as well.

We consider that if once defection happens, the division of labour stops. However, in many cases even after defection the division of labour does not stop. For example in the case of supply chain of manufacturing, a faulty product can be supplied in one or more stages, without being noticed, only being found after the final product is sold by the customer, which is a defection, and the whole manufacturing chain's reputation is in question after it happens. We are currently working on such a model.

In the automobile industry often the supply of the materials such as the screw, spring etc. is been done through division of labour. Ususally, when a product does not maintain the quality, the materials are not taken in, and by that the punishment is given, although for the maintainence of the flow, a chance is given before sanctioning. Suppliers work to update the quality of the material as long as it does not satisfy the automobile company, however, the benefit for the supplier does not change. They do it because of the guarantee given by the company that they will buy product from the supplier in the long term. For our future research, we shall consider the expectation of being able to sustain a business in our models.

In this study we have shown the effects of changes in network size on the evolution of cooperation. However, we have not studied the effects of change in network type in the evolution of cooperation in depth. We have compared the linear network, with the tree graph network and found some differences in the general model, as the changes in the scope of co-existence of cooperators and defectors. However, changing the type of connection among nodes by keeping the number of nodes the same is another way of comparison among networks. In our future study, we shall compare different types of network by keeping the number of nodes the same.

In our model, the information on the reputation of the players from different groups is not available, the players are randomly chosen from the groups with an infinite population of cooperators and defectors. In future, we shall consider including the reputation of groups and merging with the mechanism of indirect reciprocity we shall investigate the evolution of cooperation. Morozumi (2023) has considered the effects of reputation in the division of labour.

In our model the benefit is given regardless of choosing the strategy, thus it is working as a pre-agreed salary. We are currently working on a model where the benefit is not pre-agreed but is determined by one's action by a method of reinforcement learning called Q learning. Here one's action is chosen and thus payoff is given based on the Q value which depends on the whole system's previous actions. However, even in such cases, the benefit does not have effects on the dynamics (Liu, 2023).

The speed of evolution of cooperation in each group can be determined through the replicator equation. In this study we did not particularly comment on the speed of evolution of cooperation, however, in future, we shall focus on it, as in our society often evolution of cooperation is not only needed, but also needed fast.

References

Axelrod, R. (1986). An Evolutionary Approach to Norms. American Political Science Review, 80(4), 1095-1111. doi:10.1017/S0003055400185016

Axelrod, R., & Hamilton, W. D. (1981). The evolution of cooperation. Science, 211(4489), 1390–1396. https://doi.org/10.1126/science.7466396

Barley, S. R. (1996). Technicians in the workplace: Ethnographic evidence for bringing work into organizational studies. Administrative science quarterly, 404-441.

Baumgardner, J. R. (1988). The division of labour, local markets, and worker organization. Journal of Political Economy, 96(3), 509-527.

Bezes, P., & Le Lidec, P. (2016). The Politics of Organization. The New Divisions of labour in State Bureaucracies. Revue française de science politique, 66, 407-433. https://www.cairn-int.info/journal–2016-3-page-407.htm.

Bolin, M., & Härenstam, A. (2008). An empirical study of bureaucratic and post-bureaucratic characteristics in 90 workplaces. Economic and Industrial Democracy, 29(4), 541-564.

Boyd, R., Gintis, H., Bowles, S., & Richerson, P. J. (2003). The evolution of altruistic punishment. Proceedings of the National Academy of Sciences of the United States of America, 100(6), 3531–3535. https://doi.org/10.1073/pnas.0630443100

Boyd, R., & Richerson, P. J. (1989). The evolution of indirect reciprocity. Social Networks, 11(3), 213-236. https://doi.org/10.1016/0378-8733(89)90003-8

Boyd, R., & Richerson, P. J. (2009). Culture and the evolution of human cooperation. Philosophical transactions of the Royal Society of London. Series B, Biological sciences, 364(1533), 3281–3288. https://doi.org/10.1098/rstb.2009.0134

Chauvin, E. D., & Ozak, O. (2017). The Origins and Long-Run Consequences of the Division of Labour. Documento de Trabajo, 471, Instituto de Economía, Pontificia Universidad Católica de Chile.

Chen, X. & Szolnoki, A. & Perc, M. (2014). Probabilistic sharing solves the problem of costly punishment. New Journal of Physics. 16. 083016. 10.1088/1367-2630/16/8/083016.

Chen, X., Szolnoki, A., & Perc, M. (2015). Competition and cooperation among different punishing strategies in the spatial public goods game. Physical review. E, Statistical, nonlinear, and soft matter physics, 92(1), 012819. https://doi.org/10.1103/PhysRevE.92.012819

Chiang, Y. H. (2009). Subcontracting and its ramifications: A survey of the building industry in Hong Kong. International Journal of Project Management, 27(1), 80-88.

Cooper, M. C., Ellram, L. M., Gardner, J. T., & Hanks, A. M. (1997). Meshing multiple alliances. Journal of Business logistics, 18(1), 67.

Davis, L. S. (2003). The division of labour and the growth of government. Journal of Economic Dynamics and Control, 27, 1217–1235.

Dal Co, A., Brannon, C., & Ackermann, M. (2018). Division of labour in bacteria. eLife, 7, e38578. https://doi.org/10.7554/eLife.38578

Durkheim, É. (1893). De la division du travail social, 2. éd edn, F. Alcan, Paris.

Dudley S. A. (2015). Plant cooperation. AoB PLANTS, 7, plv113. https://doi.org/10.1093/aobpla/plv113

Eisenstadt, S. N. (1958). Bureaucracy and bureaucratization. Current Sociology, 7(2), 99-124.

Ember, C. R. (1978). Myths about hunter-gatherers. Ethnology, 17(4), 439-448.

Farooque, M., Zhang, A., Thürer, M., Qu, T., & Huisingh, D. (2019). Circular supply chain management: A definition and structured literature review. Journal of cleaner production, 228, 882-900.

Fengru, C., & Guitang, L. (2019). Global Value Chains and Production Networks: Case Studies of Siemens and Huawei, Academic Press.

Fortunato, S., & Castellano, C. (2007). Community structure in graphs. arXiv preprint arXiv:0712.2716.

Ghiani, G., Laporte, G., & Musmanno, R. (2004). Introduction to logistics systems planning and control. John Wiley & Sons.

Guriev, S. (2004). Red tape and corruption. Journal of development economics, 73(2), 489-504.

Hamilton W. D. (1964). The genetical evolution of social behaviour. I. Journal of theoretical biology, 7(1), 1–16. https://doi.org/10.1016/0022-5193(64)90038-4

Harari, Y. N. (2011). Sapiens: A Brief History of Humankind, Dvir Publishing House Ltd. (Israel) Harper.

Hauert, C., Traulsen, A., Brandt, H., Nowak, M. A., & Sigmund, K. (2007). Via freedom to coercion: the emergence of costly punishment. Science (New York, N.Y.), 316(5833), 1905–1907. https://doi.org/10.1126/science.1141588

Henderson, J., Dicken, P., Hess, M., Coe, N., & Yeung, H. W.-C. (2002). Global Production Networks and the Analysis of Economic Development. Review of International Political Economy, 9(3), 436–464. http://www.jstor.org/stable/4177430

Henrich, J., & Boyd, R. (2008). Division of labour, economic specialization, and the evolution of social stratification. Current anthropology, 49(4), 715-724. https://doi.org/10.1086/587889

Hinings, C. R., Pugh, D. S., Hickson, D. J., & Turner, C. (1967). An approach to the study of bureaucracy. Sociology, 1(1), 61-72.

Hofbauer, J., & Sigmund, K. (1998). Evolutionary Games and Population Dynamics. Cambridge: Cambridge University Press. doi:10.1017/CBO9781139173179

Iwasa, Y., & Yamaguchi, S. (2020). Task allocation in a cooperative society: specialized castes or age-dependent switching among ant workers. Scientific reports, 10(1), 3339. https://doi.org/10.1038/s41598-020-59920-5

Jiang, L. L., Chen, Z., Perc, M., Wang, Z., Kurths, J., & Moreno, Y. (2023). Deterrence through punishment can resolve collective risk dilemmas in carbon emission games. Chaos: An Interdisciplinary Journal of Nonlinear Science, 33(4).

Kollock, P. (1998). Social Dilemmas: The Anatomy of Cooperation. Review of Sociology, 24, 183-214. DOI:10.1146/ANNUREV.SOC.24.1.183

Krugman, P. R. (1981). Intraindustry Specialization and the Gains from Trade. Journal of Political Economy, 89(5), 959–973. http://www.jstor.org/stable/1830815

Kuhn, S. L., & Stiner, M. C. (2006). What's a mother to do? The division of labour among Neandertals and modern humans in Eurasia. Current Anthropology, 47(6), 953-980. https://doi.org/10.1086/507197

Lambert, D. M., & Cooper, M. C. (2000). Issues in supply chain management. Industrial marketing management, 29(1), 65-83. doi: 10.1016/S0019-8501(99)00113-3

Lee, J. H., Iwasa, Y., Dieckmann, U., & Sigmund, K. (2019). Social evolution leads to persistent corruption. Proceedings of the National Academy of Sciences, 116(27), 13276-13281.

Levy, J. S. (1992). An introduction to prospect theory. Political psychology, 171-186.

Li, Y., Zhen, X., Qi, X., & Cai, G. G. (2016). Penalty and financial assistance in a supply chain with supply disruption. Omega, 61, 167-181.

Liu, F. (2023). Master's thesis for graduation in Tokyo Institute of Technology.

McAvoy, A., & Hauert, C. (2015). Asymmetric Evolutionary Games. PLoS computational biology, 11(8), e1004349. https://doi.org/10.1371/journal.pcbi.1004349

McDevitt, A. (2015) Bangladesh: Overview of corruption and anti-corruption with a focus on the health sector. Bergen: U4 Anti-Corruption Resource Centre, Chr. Michelsen Institute (U4 Helpdesk Answer) https://www.u4.no/publications/bangladesh-overview-of-corruption-and-anti-corruption-with-a-focus-on-the-health-sector

Mersch, D. P. (2016). The social mirror for division of labour: what network topology and dynamics can teach us about organization of work in insect societies. Behavioral ecology and sociobiology, 70, 1087-1099.

Min, H., & Zhou, G. (2002). Supply chain modeling: past, present and future. Computers & Industrial Engineering, 43, 231-249. https://doi.org/10.1016/S0360-8352(02)00066-9

Morozumi, K., (2023). Project report for graduation in Tokyo Institute of Technology.

Nakahashi, W., & Feldman, M. W. (2014). Evolution of division of labour: Emergence of different activities among group members. Journal of Theoretical Biology, 348, 65-79. https://doi.org/10.1016/j.jtbi.2014.01.027

Nakamaru, M., & Iwasa, Y. (2005). The evolution of altruism by costly punishment in lattice-structured populations: Score-dependent viability versus score-dependent fertility. Evolutionary Ecology Research, 7(6), 853-870.

Nakamaru, M., & Iwasa, Y. (2006). The coevolution of altruism and punishment: role of the selfish punisher. Journal of theoretical biology, 240(3), 475–488. https://doi.org/10.1016/j.jtbi.2005.10.011

Nakamaru, M., & Kawata, M. (2006). Evolution of rumours that discriminate lying defectors. Evolutionary ecology research, 6, 261-283.

Nakamaru, M., Matsuda, H., & Iwasa, Y. (1997). The evolution of cooperation in a lattice-structured population. Journal of theoretical biology, 184(1), 65–81. https://doi.org/10.1006/jtbi.1996.0243

Nakamaru, M., Nogami, H., & Iwasa, Y. (1998). Score-dependent fertility model for the evolution of cooperation in a lattice. Journal of theoretical biology, 194(1), 101–124. https://doi.org/10.1006/jtbi.1998.0750

Nakamaru, M., Shimura, H., Kitakaji, Y., & Ohnuma, S. (2018). The effect of sanctions on the evolution of cooperation in the linear division of labour. Journal of theoretical biology, 437, 79–91. https://doi.org/10.1016/j.jtbi.2017.10.007

Nirjhor, M. S. A., & Nakamaru, M. (2023). The evolution of cooperation in the unidirectional linear division of labour of finite roles. R. Soc. open sci.10:220856. http://doi.org/10.1098/rsos.220856

Nirjhor, M. S. A., & Nakamaru, M. (2023). The evolution of cooperation in the unidirectional division of labour on a tree network. bioRxiv, 2023-06.

Nolan, P., & Lenski, G. E. (2011). Human societies: an introduction to macrosociology, 11th edition, Paradigm Publishers, Boulder.

Nowak M. A. (2006). Five rules for the evolution of cooperation. Science (New York, N.Y.), 314(5805), 1560–1563. https://doi.org/10.1126/science.1133755

Nowak, M. A., & May, R. M. (1992). Evolutionary games and spatial chaos. Nature, 359(6398), 826–829. https://doi.org/10.1038/359826a0

Nowak, M. A., & Sigmund, K. (1993). A strategy of win-stay, lose-shift that outperforms tit-for-tat in the Prisoner's Dilemma game. Nature, 364(6432), 56–58. https://doi.org/10.1038/364056a0

Nowak, M. A., & Sigmund, K. (1998). Evolution of indirect reciprocity by image scoring. Nature, 393(6685), 573–577. https://doi.org/10.1038/31225

Nowak, M. A., & Sigmund, K. (2005). Evolution of indirect reciprocity. Nature, 437(7063), 1291–1298. https://doi.org/10.1038/nature04131

Ohtsuki, H., Hauert, C., Lieberman, E., & Nowak, M. A. (2006). A simple rule for the evolution of cooperation on graphs and social networks. Nature, 441(7092), 502–505. https://doi.org/10.1038/nature04605

Ohtsuki, H., & Iwasa, Y. (2004). How should we define goodness?–reputation dynamics in indirect reciprocity. Journal of theoretical biology, 231(1), 107–120. https://doi.org/10.1016/j.jtbi.2004.06.005

OpenStax and Lumen Learning, American Government (2e − Second Edition) (2019), Chapter-15. https://pressbooks.online.ucf.edu/americangovernment2e/chapter/understanding-bureaucracies-and-their-types/

Pacheco, J. M., Traulsen, A., Ohtsuki, H., & Nowak, M. A. (2008). Repeated games and direct reciprocity under active linking. Journal of theoretical biology, 250(4), 723–731. https://doi.org/10.1016/j.jtbi.2007.10.040

Perc, M., Jordan, J. J., Rand, D. G., Wang, Z., Boccaletti, S., & Szolnoki, A. (2017). Statistical physics of human cooperation. Physics Reports, 687, 1-51. https://doi.org/10.1016/j.physrep.2017.05.004

Powers, S. T., & Lehmann, L. (2014). An evolutionary model explaining the Neolithic transition from egalitarianism to leadership and despotism. Proceedings. Biological sciences, 281(1791), 20141349.
https://doi.org/10.1098/rspb.2014.1349

Press, W. H., & Dyson, F. J. (2012). Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent. Proceedings of the National Academy of Sciences of the United

States of America, 109(26), 10409–10413. https://doi.org/10.1073/pnas.1206569109

Qin, S., Zhang, G., Tian, H., Hu, W., & Zhang, X. (2020). Dynamics of asymmetric division of labour game with environmental feedback. Physica A: Statistical Mechanics and its Applications, 543, 123550. https://doi.org/10.1016/j.physa.2019.123550

Rand, D. G., Armao, J. J., 4th, Nakamaru, M., & Ohtsuki, H. (2010). Anti-social punishment can prevent the co-evolution of punishment and cooperation. Journal of theoretical biology, 265(4), 624–632.
https://doi.org/10.1016/j.jtbi.2010.06.010

Rand, D. G., & Nowak, M. A. (2013). Human cooperation. Trends in cognitive sciences, 17(8), 413–425. https://doi.org/10.1016/j.tics.2013.06.003

Roithmayr, D., Isakov, A., & Rand, D. (2015). Should Law Keep Pace with Society? Relative Update Rates Determine the Co-Evolution of Institutional Punishment and Citizen Contributions to Public Goods. Games, 6(2), 124–149. doi:10.3390/g6020124

Roper, S., & Grimes, S. (2005). Wireless valley, silicon wadi and digital island—Helsinki, Tel Aviv and Dublin and the ICT global production network. Geoforum, 36(3), 297-313.

Santos, F. C., Santos, M. D., & Pacheco, J. M. (2008). Social diversity promotes the emergence of cooperation in public goods games. Nature, 454(7201), 213–216. https://doi.org/10.1038/nature06940

Sasaki, T., Uchida, S., & Chen, X. (2015). Voluntary rewards mediate the evolution of pool punishment for maintaining public goods in large populations. Scientific reports, 5, 8917.
https://doi.org/10.1038/srep08917

Sessa, P.G., Bogunovic, I., Kamgarpour, M., & Krause, A. (2020). Learning to Play Sequential Games versus Unknown Opponents. ArXiv, abs/2007.05271.

Sharma, N., Yagoobi, S., & Traulsen, A. (2023). Self-loops in Evolutionary Graph Theory: Friends or Foes?. bioRxiv, 2023-03.

Shimao, H., & Nakamaru, M. (2013). Strict or graduated punishment? Effect of punishment strictness on the evolution of cooperation in continuous public goods games. PloS one, 8(3), e59894.
https://doi.org/10.1371/journal.pone.0059894

Sigmund, K., Hauert, C., & Nowak, M. A. (2001). Reward and punishment. Proceedings of the National Academy of Sciences of the United States of America, 98(19), 10757–10762.
https://doi.org/10.1073/pnas.161155698

Sigmund, K., De Silva, H., Traulsen, A., & Hauert, C. (2010). Social learning promotes institutions for governing the commons. Nature, 466(7308), 861–863. https://doi.org/10.1038/nature09203

Smith, J. M. (1977). Parental investment: A prospective analysis. Animal Behaviour, 25(1), 1–9. https://doi.org/10.1016/0003-3472(77)90062-8

Sober, E., Wilson, D.S. (1999). Unto Others: The Evolution and Psychology of Unselfish Behavior. Cambridge, MA: Harvard University Press.

Szolnoki, A., & Szabó, G. (2007). Cooperation enhanced by inhomogeneous activity of teaching for evolutionary Prisoner's Dilemma games. Europhysics Letters, 77(3), 30004.

Su, Q., Allen, B., & Plotkin, J. B. (2022). Evolution of cooperation with asymmetric social interactions. Proceedings of the National Academy of Sciences of the United States of America, 119(1), e2113468118. https://doi.org/10.1073/pnas.2113468118

Sugden, R. (1986). The Economics of Rights, Co-Operation and Welfare, Oxford and New York: Basil Blackwell.

Svensson, J. (2006). Eight Questions About Corruption. Gospodarka Narodowa. The Polish Journal of Economics, 210(9), 77-106. https://doi.org/10.33119/GN/101466

Tam, V. W., Shen, L. Y., & Kong, J. S. (2011). Impacts of multi-layer chain subcontracting on project management performance. International Journal of Project Management, 29(1), 108-116.

Traulsen, A., & Nowak, M. A. (2006). Evolution of cooperation by multilevel selection. Proceedings of the National Academy of Sciences of the United States of America, 103(29), 10952–10955. https://doi.org/10.1073/pnas.0602530103

Tucker, A. W. "The Mathematics of Tucker: A Sampler." The Two-Year College Mathematics Journal, vol. 14, no. 3, 1983, pp. 228–32. JSTOR, https://doi.org/10.2307/3027092.

Turchin, P. (2016). Ultra Society: how 10,000 years of war made humans the greatest cooperators on earth, Beresta Books, LCC, Connecticut.

Varian, H.R. (1994). Sequential contributions to public goods. Journal of Public Economics, 53, 165-186. https://doi.org/10.1016/0047-2727(94)90019-1

Wakano, J. Y., Nowak, M. A., & Hauert, C. (2009). Spatial dynamics of ecological public goods. Proceedings of the National Academy of Sciences of the United States of America, 106(19), 7910–7914. https://doi.org/10.1073/pnas.0812644106

Wang, Z., Li, Y., During, H. J., & Li, L. (2011). Do clonal plants show greater division of labour morphologically and physiologically at higher patch contrasts?. PloS one, 6(9), e25401. https://doi.org/10.1371/journal.pone.0025401

Yuan, H., & Meng, X. (2022). Replicator dynamics of division of labour games with delayed payoffs in infinite populations. Chaos, Solitons & Fractals, 158, 112058.

Zhang, C., Li, Q., Zhu, Y., & Zhang, J. (2018). Dynamics of task allocation based on game theory in multi-agent systems. IEEE Transactions on Circuits and Systems II: Express Briefs, 66(6), 1068-1072. DOI: 10.1109/TCSII.2018.2873006

Zhang, C., Li, Q., Zhu, Y., Han, J., & Zhang, J. (2020). Evolutionary dynamics in division of labour games on cycle networks. European Journal of Control, 53, 1-9. https://doi.org/10.1016/j.ejcon.2019.11.002

Appendix

Appendix A

Proof of the equilibrium:

Assumption, $x_i > 0$, $x_i \neq g$ for all i's. From the model setting we also get when, $i_c = 0$, $j_c = *$, if, $j > i$. In an equilibrium, eq. (2.1), (2.2) and (2.3) are zero.

We know, $\frac{dn_c}{dt} = 0 \Rightarrow n_c = 0$, or $n_c = 1$, or $c_n = 0 \cdots (\alpha)$

When $n_c = 0$ in (α), $c_1 = 0$. Therefore $1_c = 0$, or 1 (as, $x_1 \neq 0$).

When $n_c = 1$ in (α), $(n-1)_c(1-(n-1)_c)\{c_{n-1}g - x_{n-1}(1-d_{(n-1)b})\} = 0$. $(n-1)_c \neq 0$ (as $n_c = 1$, and all players in the later group can not be cooperators if all players in the former group are defectors in our model). $\{c_{n-1}g - x_{n-1}(1-d_{(n-1)b})\} = 0 \Rightarrow g = x_{n-1}$ (as $c_{n-1} = 1 - d_{(n-1)b}$). However, it is a contradiction. Therefore, only $(n-1)_c = 1$ is possible. Following the same logic upstream, we get $1_c = 1$.

When $c_n = 0$ in (α), there exists $k = \{1, ..., n-1\}$ such that $k_c = 0$. When $k \neq 1$, $c_1 = 0$. Therefore $1_c = 0$, or 1.

Therefore all the cases in (α) lead to $1_c = 0$, or 1 when in equilibrium. When $1_c = 0$, the equilibrium is the first group defection equilibrium.

When $1_c = 1$, $1 - d_{2b} = 1 \cdots (\beta)$

Therefore $\frac{d2_c}{dt} = 2_c(1-2_c)\{c_2 g - x_2\} = 0$. By repeating the process from (α) we get $2_c = 0$, or 1. When $2_c = 0$ the equilibrium is a cooperation-defection mixed equilibrium. When $2_c = 1$, the process shall be repeated from (β) to get the equation for 3_c. By repeating this we finally get when $[1_c, ..., (n-1)_c] = [1, ..., 1]$, $n_c = 0$, or 1. When $n_c = 0$ it is the last group defection equilibrium, and when $n_c = 1$ it is all cooperation equilibrium.

Appendix B

We analysed the local stability of the equilibria in each of the three systems for the general model.

For the baseline system, all eigenvalues of the Jacobian matrix for the first group defection equilibrium are $c_1 g - x_1$ and 0. Therefore, the first group defection equilibrium is stable, as $c_1 g - x_1 < 0$. The cooperation-defection mixed equilibrium and the last group defecting equilibrium are unstable, as the eigenvalue x_1 is positive and it is always an eigenvalue for both of these as the first defector $j \geq 2$. All eigenvalues of the Jacobian matrix for the all cooperation equilibrium are $x_i - g$, $(1 \leq i \leq n\}$. Therefore, the all cooperation equilibrium is stable in the baseline system if $\max\{x_i\}_{1 \leq i \leq n} < g$, for all the $1 \leq i \leq n$. We consider the Prisoner's dilemma situation in the baseline model, therefore, $x_i > g$. This indicates that all cooperation equilibrium is unstable in the baseline.

For the defector sanction system, all eigenvalues of the Jacobian matrix for the first group defection equilibrium is $c_1 g + \rho f - x_1$ and 0. Therefore, the first group defection equilibrium is stable in the defector sanction system if $c_1 g + \rho f < x_1$. The cooperation-defection mixed equilibrium in the defector sanction system is stable if $\max\{x_i\}_{i<j} < \rho f$ and $x_j - c_j g > \rho f$ when j is the first defector. The last group defection equilibrium is stable when $\max\{x_i\}_{i=1}^{n-1} < \rho f$ and $x_n > \rho f + g$. All eigenvalues of the Jacobian matrix for the all cooperation equilibrium are $x_i - g - \rho f (1 \leq i \leq n)$. Therefore, the all cooperation equilibrium is stable in the defector sanction system if $g + \rho f > \max\{x_i\}_{1 \leq i \leq n}$.

For the first role sanction system, All eigenvalues of Jacobian matrix for the first group defection equilibrium are $c_1(g + f) - x_1$ and 0. Therefore, the first group defection equilibrium is conditionally stable. For the equilibrium to be a mixed equilibrium, x_1 is always an eigenvalue when the first defector $j \geq 2$. As a result, the cooperation-defection mixed equilibrium and the last group defection equilibrium are unstable, as the eigenvalue x_1 is positive. All eigenvalues of Jacobian matrix for the all cooperation equilibrium is $x_i - g (2 \leq i \leq n)$ and $x_1 - g - f$. Therefore, the all cooperation equilibrium is stable in the 1st role sanction system if $g + f > x_1$ and $g > \max\{x_i\}_{2 \leq i \leq n}$.

Appendix C The local stability analysis of the baseline model

On the basis of Eqs. 3.1-3.3 in the main text, we make the time differential equations for $O_k \cup \{k\}$. Then, we calculate Jacobian matrix from the differential equations for analyzing the local stability of the equilibria based on $O_k \cup \{k\}$. All Jacobian matrices are lower triangular matrices here, so the eigenvalues are just the main diagonal entries.

The Jacobian matrix for $premierD$ is $J(premierD)$. If ξ is a main diagonal component of $J(premierD)$ then,

$$\xi = \begin{cases} g_p - x_p - (1 - c_{op})g_{op}, & \text{if } p \text{ is the premier group} \\ 0, & \text{otherwise.} \end{cases}$$

When $g_p - (1 - c_{op})g_{op} < x_p$, $PremierD$ is locally stable. As $g_p, x_p, g_{op} > 0$, $0 \leq c_{op} \leq 1$, $g_{op} \leq g_p$, and g_p is always lower than x_p in the social dilemma. Therefore, $PremierD$ is always locally stable.

The Jacobian matrix for $allc_k$ is $J(allc_k)$. If ξ is a main diagonal component of $J(allc_k)$ then

$$\xi = \begin{cases} x_i - g_i, & \text{if } is \text{ are the groups in } O_k \\ 0, & \text{otherwise.} \end{cases}$$

As the baseline model has the social dilemma situation, $x_i > g_i$. Then, $allc_k$ is not locally stable.

The Jacobian matrix for $mix_{U_k \cup \{k\}}$ is $J(mix_{U_k \cup \{k\}})$. If ξ is a main diagonal component of $J(mix_{U_k \cup \{k\}})$ and k is not a terminal, or k is a terminal however, k is not the defector group, then

$$\xi = \begin{cases} x_i - g_i + g_{oi}, & \text{if } is \text{ are the groups in } O_k \text{ that are full cooperators} \\ g_j - x_j - (1 - c_{oj})g_{oj}, & \text{if } j \text{ is the group in } O_k \text{ that is full defector} \\ 0, & \text{otherwise.} \end{cases}$$

If ξ is a main diagonal component of $J(mix_{U_k \cup \{k\}})$ and k is a terminal and k is the defector group, then

$$\xi = \begin{cases} x_i - g_i + g_{oi}, & \text{if } is \text{ are the groups in } O_k \text{ that are full cooperators} \\ g_j - x_j, & \text{if } j \text{ is the group } k \text{ that is full defector} \end{cases}$$

As the baseline model has the social dilemma situation, $x_i > g_i - g_{oi}$. Therefore, the $mix_{U_k \cup \{k\}}$ is not stable.

Table C1: Local stability conditions when k is the focal group

Equilibrium	Baseline	Defector sanction	Premier sanction
$premierD$	$g_p - (1 - c_{op})g_{op} < x_p$	$g_p - (1 - c_{op})g_{op} + \rho f < x_p$	$g_p - (1 - c_{op})g_{op} + c_{op}f < x_p$
$allc_k$	$g_i > x_i$ where $i \in O_k \cup \{k\}$	$g_i + \rho f > x_i$ where $i \in O_k \cup \{k\}$	$g_p + f > x_p$ and $g_i > x_i$ where $i \in O_k \cup \{k\} - \{p\}$
$mix_{U_k \cup \{k\}}$	$g_i - g_{oi} > x_i$, and $g_j - (1 - c_{oj})g_{oj} < x_j$, when k is not a terminal.	$g_i - g_{oi} + \rho f > x_i$, and $g_j - (1 - c_{oj})g_{oj} + \rho f < x_j$, when k is not a terminal.	$g_i - g_{oi} > x_i$, and $g_j - (1 - c_{oj})g_{oj} < x_j$, when k is not a terminal.
	$g_i - g_{oi} > x_i$, and $g_j - (1 - c_{oj})g_{oj} < x_j$, when k is a terminal and $j \neq k$.	$g_i - g_{oi} + \rho f > x_i$, and $g_j - (1 - c_{oj})g_{oj} + \rho f < x_j$, when k is a terminal and $j \neq k$.	$g_i - g_{oi} > x_i$, and $g_j - (1 - c_{oj})g_{oj} < x_j$, when k is a terminal and $j \neq k$.
	$g_i - g_{oi} > x_i$, and $g_j < x_j$, when k is a terminal and $j = k$.	$g_i - g_{oi} + \rho f > x_i$, and $g_j + \rho f < x_j$, when k is a terminal and $j = k$.	$g_i - g_{oi} > x_i$, and $g_j < x_j$, when k is a terminal and $j = k$.
	where $j \in U_k \cup \{k\}$, j is the defector group and $i \in U_j$.		
mix_{D_k}	$g_{i_1} - g_{oi_1} > x_{i_1}$, $g_{i_2} > x_{i_2}$, $g_{i_3} - (1 - c_{oi_3})g_{oi_3} < x_{i_3}$, and $g_{i_4} < x_{i_4}$	$g_{i_1} - g_{oi_1} + \rho f > x_{i_1}$, $g_{i_2} + \rho f > x_{i_2}$, $g_{i_3} - (1 - c_{oi_3})g_{oi_3} + \rho f < x_{i_3}$, and $g_{i_4} + \rho f < x_{i_4}$	$g_{i_1} - g_{oi_1} > x_{i_1}$, $g_{i_2} > x_{i_2}$, $g_{i_3} - (1 - c_{oi_3})g_{oi_3} < x_{i_3}$, and $g_{i_4} < x_{i_4}$
	here $i_1 \in U_k \cup \{k\} \cup r_k$, $i_2 \in s_k$, $i_3 \in w_k$, and $i_4 \in z_k$		

The Jacobian matrix for mix_{D_k} is $J(mix_{D_k})$. If ξ is a main diagonal component of $J(mix_{D_k})$ then

$$\xi = \begin{cases} x_{i_1} - g_{i_1} + g_{oi_1}, & \text{if } i_1\text{s are the groups in } U_k \cup \{k\} \cup r_k \\ x_{i_2} - g_{i_2}, & \text{if } i_2\text{s are the groups in } s_k \\ g_{i_3} - x_{i_3} - (1 - c_{oi_3})g_{oi_3}, & \text{if } i_3\text{s are the groups in } w_k \\ g_{i_4} - x_{i_4}, & \text{if } i_4\text{s are the groups in } z_k \\ 0, & \text{otherwise.} \end{cases}$$

This equilibrium is always unstable as the baseline model has the social dilemma situation, $x_{i_1} > g_{i_1} - g_{oi_1}$, because for $J(mix_{D_k})$, $U_k \cup \{k\}$ is never an empty set. table C1 summarizes the local stability condition of four equilibrium points in the baseline model.

Appendix D The defector sanction system

The payoff matrices of the defector sanction system are shown in tables D1a, D1b, and D1c. The payoffs are the same as the baseline except while choosing defection, the players are subjected to the sanction ρf. Premier's payoff in the defector sanction system when he is a cooperator, Π_{cp} is $b_p - x_p - g_{op} + c_{op}g_{op}$. Premier's payoff in the defector sanction system when he is a defector, Π_{dp}, is $b_p - g_p - \rho f$. Therefore, the replicator equation is,

$$\frac{dp_c}{dt} = p_c(1-p_c)(\Pi_{cp} - \Pi_{dp}) = p_c(1-p_c)(c_{op}g_{op} + g_p + \rho f - g_{op} - x_p). \tag{D1}$$

Any player k's payoff when k is neither premier nor a terminal, and k is a cooperator, Π_{ck}, is $c_{ok}(b_k - x_k) - d_{uk}g_k + d_{dk}(b_k - x_k - g_{ok}) - g_{nk}$. When k is a defector the payoff, Π_{dk}, is $c_{ok}(b_k - g_k - \rho f) - d_{uk}g_k + d_{dk}(b_k - g_k - \rho f) - g_{nk}$. Therefore, the replicator equation is,

$$\frac{dk_c}{dt} = k_c(1-k_c)(\Pi_{ck} - \Pi_{dk}) = k_c(1-k_c)\{c_{ok}g_{ok} + (1-d_{uk})(g_k + \rho f - x_k - g_{ok})\}. \tag{D2}$$

Any player n's payoff when n is a terminal, and n is a cooperator, Π_{cn}, is $c_{on}(b_n - x_n) - d_{un}g_n - g_{nn}$. When n is a defector the payoff, Π_{dn}, is, $c_{on}(b_n - g_n - \rho f) - d_{un}g_n - g_{nn}$. Therefore, the replicator equation is,

$$\frac{dn_c}{dt} = n_c(1-n_c)(\Pi_{cn} - \Pi_{dn}) = n_c(1-n_c)\{c_{on}(g_n + \rho f - x_n)\}. \tag{D3}$$

Here, the benefit b_k given by a cooperator of the upstream as well as the term g_{nk} which represents the relation of ks dynamics with N_k are both canceled in the replicator equation. Therefore, they do not have any effect on the dynamics.

On the basis of Eqs. D1-D3, we make the time differential equations for $O_k \cup \{k\}$. Then, we calculate Jacobian matrix from the differential equations for analyzing the local stability of the equilibria based on $O_k \cup \{k\}$.

The Jacobian matrix for *premierD* is $J(premierD)$. If ξ is a main diagonal component of $J(premierD)$ then,

$$\xi = \begin{cases} g_p - x_p - (1-c_{op})g_{op} + \rho f, & \text{if } p \text{ is the premier group} \\ 0, & \text{otherwise.} \end{cases}$$

When $g_p - (1-c_{op})g_{op} + \rho f < x_p$, $PremierD$ is locally stable.

In the defector sanction system the Jacobian matrix for $allc_k$ is $J(allc_k)$. If ξ is a main diagonal

Table D1a: The payoff matrix in Defector sanction system for Premier

Cases	All being cooperator except the premier	A defector after the premier
Premier cooperator	$b_p - x_p$	$b_p - x_p - g_{op}$
Premier defector	$b_p - g_p - \rho f$	$b_p - g_p - \rho f$

Table D1b: The payoff matrix in the defector sanction system for $k \neq p$

Cases	All being cooperator in $\mathbf{O}_k$	A defector in U_k	A defector in D_k
k cooperator	$b_k - x_k - g_{nk}$	$-g_k - g_{nk}$	$b_k - x_k - g_{ok} - g_{nk}$
k defector	$b_k - g_k - g_{nk} - \rho f$	$-g_k - g_{nk}$	$b_k - g_k - g_{nk} - \rho f$

Table D1c: The payoff matrix in the Defector sanction system for n in terminal

Cases	All being cooperator in $\mathbf{O}_n$	A defector in U_n
n cooperator	$b_n - x_n - g_{nn}$	$-g_n - g_{nn}$
n defector	$b_n - g_n - g_{nn} - \rho f$	$-g_n - g_{nn}$

component of $J(allc_k)$ then

$$\xi = \begin{cases} x_i - g_i - \rho f, & \text{if } is \text{ are the groups in } O_k \\ 0, & \text{otherwise.} \end{cases}$$

When $x_i < g_i + \rho f$, for all is, $allc_k$ is stable.

The Jacobian matrix for $mix_{U_k \cup \{k\}}$ is $J(mix_{U_k \cup \{k\}})$. If ξ is a main diagonal component of $J(mix_{U_k \cup \{k\}})$ and k is not a terminal, or k is a terminal however, k is not the defector group, then

$$\xi = \begin{cases} x_i - g_i + g_{oi} - \rho f, & \text{if } is \text{ are the groups in } O_k \text{ that are full cooperators} \\ g_j - x_j - (1 - c_{oj})g_{oj} + \rho f, & \text{if } j \text{ is the group in } O_k \text{ that is full defector} \\ 0, & \text{otherwise.} \end{cases}$$

When $x_i < g_i - g_{oi} + \rho f$ for all is, and $g_j - (1 - c_{oj})g_{oj} + \rho f < x_j$, then here the $mix_{U_k \cup \{k\}}$ is stable.

If ξ is a main diagonal component of $J(mix_{U_k \cup \{k\}})$ and k is a terminal and k is the defector group, then

$$\xi = \begin{cases} x_i - g_i + g_{oi} - \rho f, & \text{if } is \text{ are the groups in } O_k \text{ that are full cooperators} \\ g_j - x_j + \rho f, & \text{if } j \text{ is the group } k \text{ that is full defector} \end{cases}$$

The Jacobian matrix for mix_{D_k} is $J(mix_{D_k})$. If ξ is a main diagonal component of $J(mix_{D_k})$ then

$$\xi = \begin{cases} x_{i_1} - g_{i_1} + g_{oi_1} - \rho f, & \text{if } i_1\text{s are the groups in } U_k \cup \{k\} \cup r_k \\ x_{i_2} - g_{i_2} - \rho f, & \text{if } i_2\text{s are the groups in } s_k \\ g_{i_3} - x_{i_3} - (1 - c_{oi_3})g_{oi_3} + \rho f, & \text{if } i_3\text{s are the groups in } w_k \\ g_{i_4} - x_{i_4} + \rho f, & \text{if } i_4\text{s are the groups in } z_k \\ 0, & \text{otherwise.} \end{cases}$$

When $g_{i_1} - g_{oi_1} + \rho f > x_{i_1}$ and $(g_{i_3} - (1 - c_{oi_3})g_{oi_3} + \rho f < x_{i_3}$ or $g_{i_4} + \rho f < x_{i_4})$ and/or $g_{i_2} + \rho f > x_{i_2}$, then the mix_{D_k} is stable. Table C1 summarizes the local stability condition of four equilibrium points in the defector sanction system.

Appendix E The premier sanction system

The payoff matrices of the premier sanction system are shown in table E1a, E1b, and E1c. The payoffs are the same as the baseline except while there is a defection in downstream the player in the premier group is subjected to the sanction f. Premier's payoff in the premier sanction system when he is a cooperator, Π_{cp}, is $b_p - x_p - (1 - c_{op})(g_{op} + f)$. Premier's payoff in the premier sanction system when he is a defector, Π_{dp}, is $b_p - g_p - f$. Therefore, the replicator equation is,

$$\frac{dp_c}{dt} = p_c(1 - p_c)(\Pi_{cp} - \Pi_{dp}) = p_c(1 - p_c)(c_{op}(g_{op} + f) + g_p - g_{op} - x_p). \tag{E1}$$

Any player k's payoff when k is neither premier nor a terminal, and k is a cooperator, Π_{ck}, is $c_{ok}(b_k - x_k) - d_{uk}g_k + d_{dk}(b_k - x_k - g_{ok}) - g_{nk}$. When k is a defector the payoff Π_{dk}, is $c_{ok}(b_k - g_k) - d_{uk}g_k + d_{dk}(b_k - g_k) - g_{nk}$. Therefore, the replicator equation is,

$$\frac{dk_c}{dt} = k_c(1 - k_c)(\Pi_{ck} - \Pi_{dk}) = k_c(1 - k_c)\{c_{ok}g_{ok} + (1 - d_{uk})(g_k - x_k - g_{ok})\}. \tag{E2}$$

Any player n's payoff when n is a terminal, and n is a cooperator, Π_{cn}, is $c_{on}(b_n - x_n) - d_{un}g_n - g_{nn}$. When n is a defector the payoff, Π_{dn}, is $c_{on}(b_n - g_n) - d_{un}g_n - g_{nn}$. Therefore, the replicator equation is,

$$\frac{dn_c}{dt} = n_c(1 - n_c)(\Pi_{cn} - \Pi_{dn}) = n_c(1 - n_c)\{c_{on}(g_n - x_n)\}. \tag{E3}$$

Here, the benefit b_k given by a cooperator of the upstream as well as the term g_{nk} which represents the relation of ks dynamics with N_k are both cancelled in the replicator equation. Therefore, they do not have any effect on the dynamics.

On the basis of Eqs. E1-E3, we make the time differential equations for $O_k \cup \{k\}$. Then, we calculate Jacobian matrix from the differential equations for analyzing the local stability of the equilibria based on $O_k \cup \{k\}$. The Jacobian matrix for *premierD* is $J(premierD)$. If ξ is a main diagonal component of $J(premierD)$ then,

$$\xi = \begin{cases} g_p - x_p - (1 - c_{op})g_{op} + c_{op}f, & \text{if } p \text{ is the premier group} \\ 0, & \text{otherwise.} \end{cases}$$

When $g_p - (1 - c_{op})g_{op} + c_{op}f < x_p$, *PremierD* is locally stable.

In the premier sanction system the Jacobian matrix for $allc_k$ is $J(allc_k)$. If ξ is a main diagonal

Table E1a: The payoff matrix in Premier sanction system for Premier

Cases	All being cooperator except the premier	A defector after the premier
Premier cooperator	$b_p - x_p$	$b_p - x_p - g_{op} - f$
Premier defector	$b_p - g_p - f$	$b_p - g_p - f$

Table E1b: The payoff matrix in the premier sanction system for $k \neq p$

Cases	All being cooperator in $\mathbf{O}_k$	A defector in U_k	A defector in D_k
k cooperator	$b_k - x_k - g_{nk}$	$-g_k - g_{nk}$	$b_k - x_k - g_{ok} - g_{nk}$
k defector	$b_k - g_k - g_{nk}$	$-g_k - g_{nk}$	$b_k - g_k - g_{nk}$

Table E1c: The payoff matrix in the Premier sanction system for n in terminal

Cases	All being cooperator in $\mathbf{O}_n$	A defector in U_n
n cooperator	$b_n - x_n - g_{nn}$	$-g_n - g_{nn}$
n defector	$b_n - g_n - g_{nn}$	$-g_n - g_{nn}$

component of $J(allc_k)$ then

$$\xi = \begin{cases} x_p - g_p - f, & \text{if } p \text{ is the premier group} \\ x_i - g_i, & \text{if } i\text{s are the groups in } O_k \\ 0, & \text{otherwise.} \end{cases}$$

As the model has the social dilemma situation, $x_i > g_i$. Then, $allc_k$ is not locally stable.

The Jacobian matrix for $mix_{U_k \cup \{k\}}$ is $J(mix_{U_k \cup \{k\}})$. If ξ is a main diagonal component of $J(mix_{U_k \cup \{k\}})$ and k is not a terminal, or k is a terminal however, k is not the defector group, then

$$\xi = \begin{cases} x_i - g_i + g_{oi}, & \text{if } i\text{s are the groups in } O_k \text{ that are full cooperators} \\ g_j - x_j - (1 - c_{oj})g_{oj}, & \text{if } j \text{ is the group in } O_k \text{ that is full defector} \\ 0, & \text{otherwise.} \end{cases}$$

If ξ is a main diagonal component of $J(mix_{U_k \cup \{k\}})$ and k is a terminal and k is the defector group, then

$$\xi = \xi = \begin{cases} x_i - g_i + g_{oi}, & \text{if } i\text{s are the groups in } O_k \text{ that are full cooperators} \\ g_j - x_j, & \text{if } j \text{ is the group } k \text{ that is full defector} \end{cases}$$

As the model has the social dilemma situation, $x_i > g_i - g_{oi}$. Therefore, the $mix_{U_k \cup \{k\}}$ is not stable.

The Jacobian matrix for mix_{D_k} is $J(mix_{D_k})$. If ξ is a main diagonal component of $J(mix_{D_k})$ then

$$\xi = \xi = \begin{cases} x_{i_1} - g_{i_1} + g_{oi_1}, & \text{if } i_1\text{s are the groups in } U_k \cup \{k\} \cup r_k \\ x_{i_2} - g_{i_2}, & \text{if } i_2\text{s are the groups in } s_k \\ g_{i_3} - x_{i_3} - (1 - c_{oi3}) g_{oi3}, & \text{if } i_3\text{s are the groups in } w_k \\ g_{i_4} - x_{i_4}, & \text{if } i_4\text{s are the groups in } z_k \\ 0, & \text{otherwise.} \end{cases}$$

This equilibrium is always unstable as the model has the social dilemma situation, $x_{i_1} > g_{i_1} - g_{oi_1}$, because for $J(mix_{D_k})$, $U_k \cup \{k\}$ is never an empty set. table C1 summarizes the local stability condition of four equilibrium points in the premier sanction system.

Appendix F Impact of added levels and branches in a network

The effect of the added levels on the evolution of cooperation

In order to see the effects of the added levels of branching downstream on the evolution of cooperation, we focus on the stability conditions of the equilibria (table 3.4 and C1). We find that, in the defector sanction system when a group i is not a terminal, whether it will be a full defector group in the equilibria depends on the condition $g_i - (1-c_{oi})g_{oi} + \rho f < x_i$. It stands for the premier group in the stability condition for $premierD$ equilibrium as well as for the other groups in the stability condition for the mixed equilibria. When we add another level of branching downstream of the same network, in the mentioned inequality, g_i and x_i do not change, however, c_{oi} and g_{oi} would change in group i. When levels are added downstream in the same network, c_{oi} decreases or stays the same by definition. So, we need to focus on g_{oi} to find out how it changes and that change will influence the local stability condition of $premierD$ and the mixed equilibria.

We take a two regular once branched tree network, where the premier node is given with p, and the two other nodes downstream are k_1 and k_2 (figure F.1(a)). The potential loss for the premier in the 1-level is $(g_{op})_1$. Then we add one more level of branching, to this exact network's downstream, with the branches of k_1 being k_{11}, and k_{12}, k_2 being k_{21}, and k_{22} as shown in figure 3.2. The potential loss for the premier in this network with two levels is $(g_{op})_2$. Now we take the difference between this two potential losses.

$$
\begin{aligned}
&(g_{op})_2 - (g_{op})_1 \\
&= (1-k_{1_c})(g_{k_1}) + k_{1_c}(g_{ok_1})_2 + (1-k_{2_c})(g_{k_2}) + k_{2_c}(g_{ok_2})_2 - [(1-k_{1_c})(g_{k_1}) + (1-k_{2_c})(g_{k_2})] \\
&= k_{1_c}(g_{ok_1})_2 + k_{2_c}(g_{ok_2})_2 \\
&\geq 0
\end{aligned}
$$

This means that the difference between $(g_{op})_2$ and $(g_{op})_1$ is always going to be non-negative as the potential losses do not exist in the terminal level groups. This is the idea behind the proof in the general case.

To prove this inequality in the general case, we take an arbitrary node in a tree graph network indexed k_{j_0} which is followed by n number of levels of branching downstream, meaning the groups in the nth level are the terminals (figure F.2). Keeping every parameter for every existing groups the

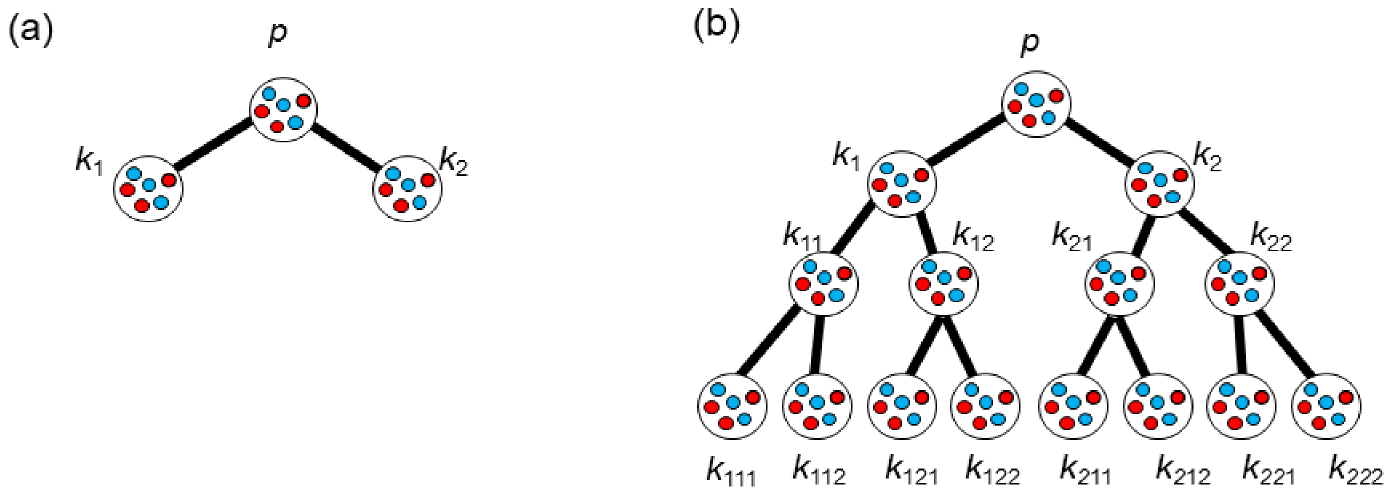

Figure F.1: (a) the two regular once-branched tree graph and (b) two regular thrice-branched tree graph network. By adding one more level downstream, (a) is transformed into the two regular twice-branched tree graph network (see figure 2), which is transformed into (b).

same, we add another level downstream of the same network, thus it becomes n levels network. The potential loss to k_{j_0} in the $n+1$ levels network is $(g_{ok_{j_0}})_{(n+1)}$, and to k_{j_0} in the n levels network is $(g_{ok_{j_0}})_{(n)}$. We will prove that $(g_{ok_{j_0}})_{(n+1)} - (g_{ok_{j_0}})_{(n)} \geq 0$ in the following. Through this proof, we find that the potential losses to each level of groups are greater than or equal to what they were before when a level is added to the same network. In other words, for a group i in any level other than the terminal, $(g_{oi})_{(n+1)} \geq (g_{oi})_{(n)}$. If a level is added in a n levels network, $(c_{oi})_{(n+1)} \leq (c_{oi})_{(n)}$ holds according to the definition. The local stability analysis where group i is not terminal indicates that the cooperation is less evolved in group i if $(1 - c_{oi})$ and g_{oi} is higher (table 3.4 and C1). Therefore, adding a level downstream in the same network hinders the evolution of cooperation.

Proof: Figure F.2 shows that the downstream branches of k_{j_0} includes from $k_{(j_0)1}$ to $k_{(j_0)m_1}$, which set is indexed in the summation with k_{j_1}. Therefore the first level of branching is indexed with j_1. This indexing goes on until the $(n+1)$th level, where the $k_{(j_n)1}$ to $k_{(j_n)m_{n+1}}$ are indexed with $k_{j_{n+1}}$. Here, $m_1, \cdot\ \cdot\ \cdot, m_{n+1} \in \mathcal{N}$ the set of natural numbers, which count the number of branches coming out of

$k_{j_0}, k_{j_1}, \cdots, k_{j_n}$ respectively. The proof is as follows;

$$
\begin{aligned}
& (g_{ok_{j_0}})_{(n+1)} - (g_{ok_{j_0}})_{(n)} \\
&= [(1-k_{(j_0)1})g_{k_{(j_0)1}} + k_{(j_0)1}(g_{ok_{(j_0)1}})_{(n+1)} + \cdots + (1-k_{(j_0)m_1})g_{k_{(j_0)m_1}} + k_{(j_0)m_1}(g_{ok_{(j_0)m_1}})_{(n+1)}] \\
&- [(1-k_{(j_0)1})g_{k_{(j_0)1}} + k_{(j_0)1}(g_{ok_{(j_0)1}})_{(n)} + \cdots + (1-k_{(j_0)m_1})g_{k_{(j_0)m_1}} + k_{(j_0)m_1}(g_{ok_{(j_0)m_1}})_{(n)}] \\
&= \sum_{j_1=(j_0)1}^{(j_0)m_1} k_{j_1c}\{(g_{ok_{j_1}})_{(n+1)} - (g_{ok_{j_1}})_{(n)}\} \\
&= \sum_{j_1=(j_0)1}^{(j_0)m_1} k_{j_1c}\{\sum_{j_2=(j_1)1}^{(j_1)m_2} k_{j_2c}\{(g_{ok_{j_2}})_{(n+1)} - (g_{ok_{j_2}})_{(n)}\}\} \\
& \cdot \\
& \cdot \\
& \cdot \\
&= \sum_{j_1=(j_0)1}^{(j_0)m_1} k_{j_1c}\{\sum_{j_2=(j_1)1}^{(j_1)m_2} k_{j_2c}\{\cdots \sum_{j_{n-1}=(j_{n-2})1}^{(j_{n-2})m_{n-1}} k_{(j_{n-1})_c}\{(g_{ok_{j_{n-1}}})_{(n+1)} - (g_{ok_{j_{n-1}}})_{(n)}\}\cdots\}\}
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{j_1=(j_0)1}^{(j_0)m_1} k_{j_1c}\{\sum_{j_2=(j_1)1}^{(j_1)m_2} k_{j_2c}\{\cdots \sum_{j_{n-1}=(j_{n-2})1}^{(j_{n-2})m_{n-1}} k_{(j_{n-1})_c}\{(1-k_{(j_{n-1})1_c})g_{k(j_{n-1})1} + k_{(j_{n-1})1_c}(g_{ok_{(j_{n-1})1}})_{(n+1)} + \cdots \\
&+ (1-k_{(j_{n-1})m_{nc}})g_{k(j_{n-1})m_n} + k_{(j_{n-1})m_{nc}}(g_{ok_{(j_{n-1})m_n}})_{(n+1)} - \{(1-k_{(j_{n-1})1_c})g_{k(j_{n-1})1} + \cdots \\
&+ (1-k_{(j_{n-1})m_{nc}})g_{k(j_{n-1})m_n}\}\}\cdots\}\} \\
&= \sum_{j_1=(j_0)1}^{(j_0)m_1} k_{j_1c}\{\sum_{j_2=(j_1)1}^{(j_1)m_2} k_{j_2c}\{\cdots \sum_{j_{n-1}=(j_{n-2})1}^{(j_{n-2})m_{n-1}} k_{(j_{n-1})_c}\{k_{(j_{n-1})1_c}(g_{ok_{(j_{n-1})1}})_{(n+1)} \\
&+ \cdots + k_{(j_{n-1})m_{nc}}(g_{ok_{(j_{n-1})m_n}})_{(n+1)}\}\cdots\}\} \\
&= \sum_{j_1=(j_0)1}^{(j_0)m_1} k_{j_1c}\{\sum_{j_2=(j_1)1}^{(j_1)m_2} k_{j_2c}\{\cdots \sum_{j_{n-1}=(j_{n-2})1}^{(j_{n-2})m_{n-1}} k_{(j_{n-1})_c}\{\sum_{j_n=(j_{n-1})1}^{(j_{n-1})m_n} k_{j_{nc}} g_{ok_{j_n}}\}\cdots\}\} \\
&\geq 0.
\end{aligned}
$$

The key point of this proof is to use $(g_{ok_{(j_{n-1})i}})_{(n)} = 0$ where i is $1, \cdots, m_n$, and $k_{(j_{n-1})i}$s are groups in the n-th level from group k_{j_0}, and are the terminals when the network has n or more number of levels.

The effect of the added branches on the evolution of cooperation

However, the same as the previous section cannot be said when a branch is added parallelly to a network. If we compare a one-level two-branched network (figure F.3(a)) with a one-level three-branched network (figure F.3(b)) from the perspective of the premier p, it becomes clear. Let the potential loss of the premier in the former be $(g_{op})_{(2)}$ and the latter be $(g_{op})_{(3)}$. The terminals of the former be k_1 and k_2, and in the latter k_3 is added in parallel to them.

$$
\begin{aligned}
&(g_{op})_{(3)} - (g_{op})_{(2)} \\
&= (1 - k_{1_c})(g_{k_1})_{(3)} + (1 - k_{2_c})(g_{k_2})_{(3)} + (1 - k_{3_c})(g_{k_3})_{(3)} - (1 - k_{1_c})(g_{k_1})_{(2)} - (1 - k_{2_c})(g_{k_2})_{(2)} \\
&= \alpha.
\end{aligned}
$$

The value α can take any real value. The only real constrain here is that $(g_{k_1})_{(3)}+(g_{k_2})_{(3)}+(g_{k_3})_{(3)} = g_p = (g_{k_1})_{(2)}+(g_{k_2})_{(2)}$, however, this condition is not enough to prove α being either positive or negative or zero. Therefore, the effect of the addition of a branch parallelly in a network is not determinable.

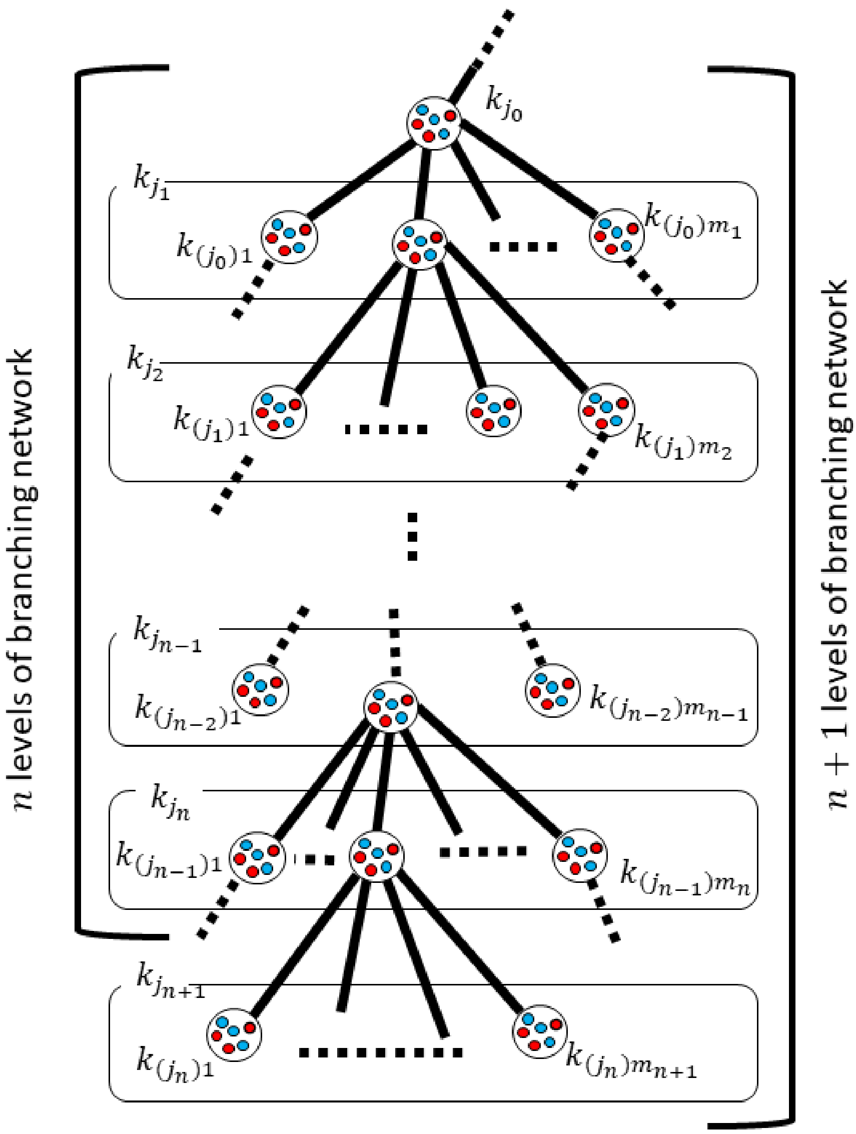

Figure F.2: The image of the network which is used in Appendix D to prove that adding levels of branching downstream of the same network, hinders the evolution of cooperation. It is A tree graph network of $n+1$ level starting from node k_{j_0}, which then branches into m_1 branches indexed as $k_{(j_0)1} \cdots k_{(j_0)m_1}$, and these branches are summarized in the summation with index k_{j_1}. This indexing continues until the $n+1$th level of branching in other words the terminals, and the terminals are indexed as $k_{(j_n)1} \cdots k_{(j_n)m_{n+1}}$.

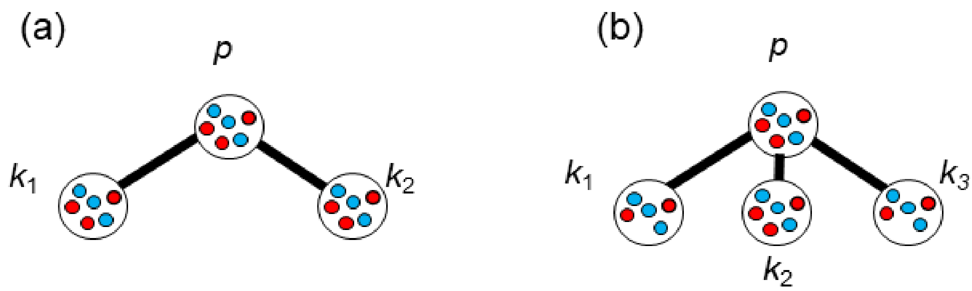

Figure F.3: (a) the two regular once-branched tree graph and (b) the three regular once-branched tree graph network. (a) is transformed into (b) by adding one more branch parallel to the first level of branching.