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Doctoral Thesis

Study on underwater actuators using piezoelectric
element thickness vibrations

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Contents

1	Introduction	1
1.1	Research Background	1
1.1.1	Miniature Underwater Propulsion Systems	1
1.1.2	Ultrasonic Underwater Actuator	3
1.2	Purpose and Thesis Outline	5
1.2.1	Purpose	5
1.2.2	Thsis Outline	6
2	Miniature Underwater Actuator using a PZT Disk Transducer	12
2.1	Piezo Transducer	13
2.2	Ultrasonic Actuator	14
2.3	Admittance Characteristics	16
2.4	Vibration Velocity Distribution	18
2.4.1	Measurement System	18
2.4.2	Results of Measurements	20
2.5	Sound Pressure	25
2.5.1	Measurement System	25
2.5.2	Results of Measurements	27
2.6	Propulsion Force	28
2.6.1	Measurement System	28
2.6.2	Results of Measurements	28
2.7	No Load Speed	30
2.7.1	Ultrasonic Underwater Actuator	30
2.7.2	No Load Speed Measurement	30
2.8	Acoustic radiation force	33
2.8.1	Calculation of Acoustic Propulsion Force	36
2.9	Conclusion	37

3	Miniature Underwater Actuator using (Bi, Na, Ba)(Ti, Mn)O₃ Disk-shaped Transducer	42
3.1	Miniature Underwater Actuator using BNBTM Transducer . .	43
3.2	Design of underwater actuator	43
3.3	Admittance Characteristics of BNBTM Transducer	46
3.4	Vibration Amplitude Distribution of BNBTM Transducer . . .	47
3.5	Propulsion force characteristics	49
3.6	No-load speed of the BNBTM underwater actuator	49
3.7	Conclusion	54
4	Miniature Underwater Actuator using a PZT Ring Transducer	56
4.1	Ring Ultrasonic Transducer	58
4.1.1	Admittance Characteristics	60
4.1.2	Vibration Velocity Distribution	60
4.2	Prototype Underwater Actuator	62
4.2.1	Admittance characteristics	64
4.2.2	Vibration Velocity Distribution	64
4.2.3	Zero-Speed Propulsion	66
4.2.4	No-load Speed	66
4.3	Conclusion	69
5	Equivalent Circuit of Disk Transducer	71
5.1	Longitudinal vibration	72
5.1.1	Equivalent Circuit	77
5.1.2	Admittance Characteristics without External Load . .	79
5.1.3	Mason Equivalent Circuit	85
5.1.4	Admittance Characteristics with Exterbal Load	88
5.1.5	Vibration Velocity Distribution	92
5.1.6	Conclusion	94
5.2	Radial Vibration	96
5.2.1	Vibration Distribution	99
5.2.2	Conclusion	103
5.3	Thickness Vibration	104
5.3.1	Speed of Sound v_r and v_z	107
5.3.2	Equivalent Circuit	108
5.3.3	Admittance Characteristics without External Load . .	114
5.3.4	Mason Equivalent Circuit	119
5.3.5	Conclusion	123
5.4	Conclusion	123

6	CONCLUSIONS AND FUTURE WORK	127
6.1	Conclusions	127
6.2	Future Work	129

Chapter 1

Introduction

1.1 Research Background

1.1.1 Miniature Underwater Propulsion Systems

Miniature Underwater Propulsion Systems is an important part of Miniature underwater robotics. In recent years, with the development of micro-electronic devices, material science, artificial intelligence and bionics, micro underwater propulsion systems have made significant progress in design, application and performance. Their main application areas include environmental monitoring, scientific research, military reconnaissance and search and rescue operations. [1–3, 16, 17]

The main types of propulsion technology include:

(1) Propeller propulsion

Propeller propulsion system is the most traditional and mature propulsion method, which is widely used in micro underwater robots. The miniature propeller propeller generates thrust through rotating blades to push the robot forward in the water.

Advantage: simple design, low manufacturing cost, mature technology, suitable for micro-robots to run stably for a long time.

Challenges: Propellers are large and less efficient at low speeds, and they are not suitable for tasks that require quiet operation due to the noise generated by mechanical rotation.

(2) Jet Propulsion [11, 12]

Jet propulsion systems generate thrust by inhaling and ejecting a stream of water at high speed. This technology is also widely used in micro-underwater robots for fine handling and movement in confined spaces.

Advantages: Compared to propeller propulsion, jet propulsion systems are more flexible in operation, have more stable thrust, and can control

motion in multiple directions.

Challenges: Relatively low energy efficiency and high energy consumption, especially during prolonged operation.

(3) Bionic Propulsion [4–6]

Bionic propulsion systems mimic the movement of aquatic animals, such as fins, octopus tentacles, or jellyfish swings, to propel the micro-robot forward. This type of propulsion has received increasing attention in recent years, and has shown potential especially in micro- and nanoscale underwater robots.

Advantages: Bionic propulsion systems reduce environmental disturbance during underwater motion and are flexible in tight environments. Extremely low noise levels make them ideal for quiet operation tasks.

Challenges: Complex designs, often involving flexible materials and complex control systems, and therefore high manufacturing costs.

(4) Electromagnetic fluid propulsion [7–10]

Magnetohydrodynamic Propulsion propels a robot forward by applying an electric current to a magnetic field, causing ions in water or a conductive fluid to be subjected to the Lorentz force.

Advantages: no mechanical moving parts, quiet operation, suitable for use in hidden or noise-sensitive environments.

Challenges: lower efficiency, especially in small systems, stronger magnetic fields required to generate sufficient thrust, higher energy consumption.

(5) Ultrasonic Propulsion [13–15]

Ultrasonic propulsion systems use ultrasonic vibrations to generate tiny pressure waves to propel flow in the water column, which in turn drives a micro-underwater robot. This technology mainly generates thrust through cavitation effect or acoustic wave-driven liquid movement.

Advantages: no mechanical moving parts, reducing maintenance requirements and noise pollution, suitable for high-precision operations.

Challenges: Smaller thrust force, can only be applied to very small-scale underwater equipment at this stage.

Combining the characteristics of the above propulsion systems, ultrasonic propulsion is a simple structure because it is driven only by the vibration of the vibrator. Although reducing the size of the vibrator makes the vibration amplitude smaller, reducing the size of the vibrator also increases its resonance frequency, thus as long as there is a high enough resonance frequency, the same thrust can be obtained. Therefore, ultrasonic propulsion is more suitable for miniaturisation than bionic propulsion, propeller propulsion or even jet propulsion. Moreover, ultrasonic propulsion is directly driven by electricity, which is less susceptible to external interference than electromagnetic fluid propulsion.

In conclusion, ultrasonic propulsion has achieved a good balance between miniaturisation, stability and driving power. Therefore, ultrasonic propulsion is expected to be used in the narrow environment of industrial survey and maintenance, as well as medical target therapy, minimally invasive surgery. In order to realise the application of ultrasonic underwater propulsion system, this study focuses on the ultrasonic underwater propulsion system, and improves the design and theoretical system of ultrasonic underwater propulsion system.

1.1.2 Ultrasonic Underwater Actuator

When a piezoelectric element, which can convert electrical energy into mechanical energy, is subjected to an external power source, an inverse piezoelectric phenomenon occurs, whereby the displacement is almost proportional to the external electric field. [18–20]. This phenomenon has led to the widely investigated compact ultrasonic underwater propulsion systems.

Based on piezoelectric actuators, Deng et al. have produced a cm-level miniature robotic cowfish that can be propelled underwater by the movement of its pectoral and tail fins. [21] Hu et al. proposed a small underwater actuator with flexible drive components in a piezoelectric macrofibre composite and its propulsion properties were evaluated. [22, 23] Zhao et al. designed a small semi-open robotic fish that can move underwater by using a piezoelectric cantilever beam. [24] Liu et al. A new underwater propulsion measure has been developed using piezoelectric actuators. [25] In the same year, a piezoelectric robot with a fish-like structure using piezoelectric elements was presented, inspiring research into small underwater propulsion systems. [26, 27] Furthermore, an ultrasonic motor with dual rotors was also investigated. [28] However, these underwater actuators with piezoelectric elements have complex mechanical structures, are heavy and have low energy conversion efficiency, thus the advantages of miniaturisation of piezoelectric elements have not been fully exploited.

The theory of acoustic flow and acoustic radiation forces due to non-linear acoustics has been widely investigated in the field of medicine. Non-linear acoustics shows that thrust is generated by the mechanical vibration of piezoelectric elements underwater. [29, 30] Then, a miniature underwater propulsion system combining the inverse piezoelectric phenomenon of piezoelectric elements with the acoustic radiation effect in water has attracted much attention. [31–34]

The main topics that our research team has completed prior to this are as follows:

Prototype underwater propulsion actuators using surface acoustic waves

and underwater propulsion actuators using bulk acoustic waves were fabricated and their propulsion characteristics were evaluated. [35, 36] In order to be able to increase the driving efficiency, it is necessary to obtain basic data on compact underwater propulsion systems using new piezoelectric elements. The structure of the actuator and the mechanical properties of the ultrasonic transducer were therefore evaluated. [37, 38]

As shown in Figures 1.1, 1.2 are the initial trial products when our research team carried out the topic related to ultrasonic underwater propulsion. Figure 1.1 shows the acoustic underwater actuator constructed by using the surface wave vibrator, which achieves a driving speed of 0.1 m/s at an input voltage of 9.61 MHz and 78 V_{rms}, while Figure 1.2 shows the acoustic underwater actuator constructed by using the thickness-vibrating bulk wave vibrator, which achieves a driving speed of 210 mm/s at an input voltage of 1.63 MHz and 70 V_{pp}.

Both bulk wave vibrator underwater propulsion and surface wave vibrator underwater propulsion are driven by mechanical vibration, the difference lies in the way the acoustic wave propagates. In the case of the thickness vibration mode of the bulk wave vibrator shown in Figure 1.2, the acoustic wave propagation direction is always parallel to the thickness direction, which means that the acoustic wave can be fully utilised in the underwater propulsion system.

The Rayleigh wave of a surface acoustic vibrator radiated into water is shown in Figure 1.1. When water is loaded onto the Rayleigh wave propagating surface of a surface acoustic wave (SAW) vibrator, the surface acoustic wave become leaky surface acoustic wave and efficiently radiate longitudinal waves into the water. If the speed of propagation in water is v_w and the speed of propagation of the Rayleigh wave is v_r , the radiation angle (Rayleigh angle) θ_r , is expressed as follows

$$\theta_r = \sin^{-1}\left(\frac{v_w}{v_r}\right) \quad (1.1)$$

In summary, surface waves propagate mainly along the surface of the vibrator, thus only partially leaked longitudinal waves can be applied to the water. In contrast, the bulk acoustic waves of the thickness vibration mode can be fully acted upon in the water. It can be seen that the thickness mode bulk acoustic wave vibrator is more suitable for the ultrasonic underwater propulsion system than the surface acoustic wave vibrator. Therefore, this study selects the underwater propulsion system based on the thickness vibration mode of the bulk wave piezoelectric vibrator as the research object.

Although the feasibility of ultrasonic underwater actuators has been verified and has many advantages over other types of small underwater propulsion systems, there is no perfect theoretical system to help its application, and the structural design of the actuator needs to be improved to enhance the underwater propulsion capability. Therefore, this study will start with underwater actuators for bulk acoustic wave vibrators with thickness vibrations, and then try to explore the theoretical system as well as a variety of actuator designs and evaluations.

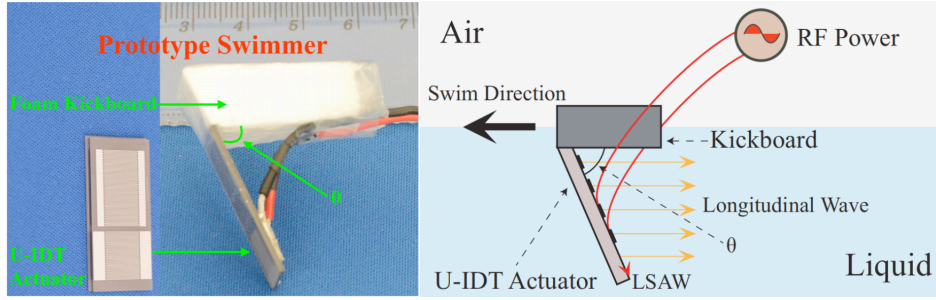


Figure 1.1: Actuator using surface acoustic wave underwater propulsion system(9.61MHz , $78V_{\text{rms}}$; 0.1m/s). [35]

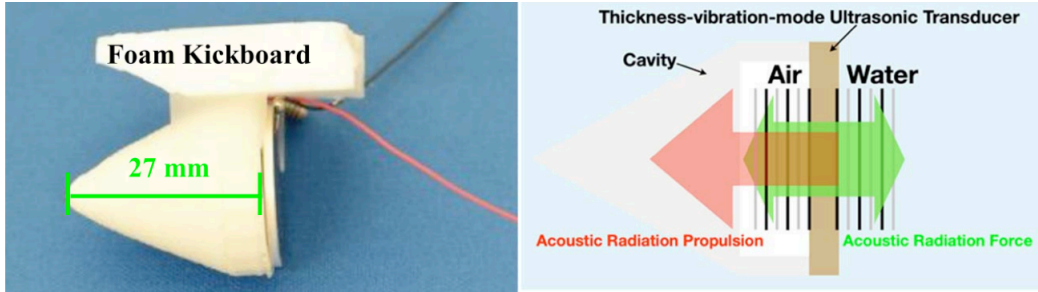


Figure 1.2: Actuator using bulk acoustic wave underwater propulsion system (1.63MHz , $70V_{\text{pp}}$; 210mm/s). [40]

1.2 Purpose and Thesis Outline

1.2.1 Purpose

One of the reasons why micro-robots based on ultrasonic vibrator underwater propulsion systems have shown promise but have not yet been applied is the problem that a model representing the relationship between drive voltage and propulsion force has not yet been completed. For the design of new

microrobots, it is necessary to quantitatively understand the individual characteristics of the ultrasonic vibrator's liquid propulsion system. Therefore, among the components of the new underwater propulsion system, the ultrasonic vibrator was subjected to experimental and analytical investigations, with the aim of clarifying the modelling problems and obtaining basic data for constructing the optimum system.

Specific topics for optimising the drive efficiency of ultrasonic underwater actuators:

1. In order to achieve reduced actuator size as well as lower impedance consumption, a disc-shaped vibrator is chosen as the preferred solution. Although the modelling of electrical-mechanical equivalent circuits of vibrators has been widely used, all of them consider only the vibration in a single direction to simplify the calculation. However, for the plate shaped vibrator with thickness vibration mode in this study, it has a complex vibration distribution, thus it is not possible to consider only a single direction of vibration. Therefore, in this study, an attempt will be made to solve the electrical-mechanical equivalent circuit model by considering two directional parameters in the longitudinal and radial directions.

2. In order to explore the propulsion principle of the ultrasonic underwater actuator, it is necessary to quantify its thrust force. In this study, the thrust characteristics are investigated by experimentally measuring the sound pressure to calculate the acoustic radiation force.

3. To take advantage of the miniaturisation of the ultrasonic underwater actuator. In this study, three different ultrasonic underwater actuators were investigated.

1.2.2 Thesis Outline

There are 6 chapters in this thesis. The contents of each chapter are mentioned in the table 1.1

In the chapter 1, The background of this study is presented, describing the advantages of ultrasonic underwater actuators for small underwater propulsion systems and the problems that still need to be solved in order to realise the application. There is also an introduction about the subject matter of this research.

In the chapter 2, Underwater actuators with bulk wave vibrators are specifically introduced by describing the principle, structure, admittance characteristics, vibration velocity distribution, acoustic pressure distribution, propulsion characteristics, and speed characteristics of underwater actuators including the PZT disk-shaped vibrator. The principle of propulsion is also discussed by calculating the acoustic radiation pressure.

Table 1.1: Contents of thesis

Thesis Outline
Chapter 1 Introduction
Chapter 2 Miniature Underwater Actuator using a PZT Disk Transducer
Chapter 3 Miniature Underwater Actuator using BNBTM Disk Transducer
Chapter 4 Miniature Underwater Actuator using a PZT Ring Transducer
Chapter 5 Equivalent Circuit of Disk Transducer
Chapter 6 Conclusions and Future Work

In the chapter 3, The ultrasonic underwater actuator was designed using a non-lead BNBTM disk-shaped vibrator for environmental protection, safety and excellent material properties compared to PZT. The design is evaluated in terms of admittance characteristics, vibration distribution, propulsion and speed characteristics.

In the chapter 4, because the vibration distribution of the disk-shaped vibrator exhibits a distribution of multiple peaks, this results in the portion at the trough not generating propulsion force and a reduction in drive efficiency. Moreover, from the structural design, the disk-shaped vibrator requires a hole on the outer side of the actuator. Therefore, the ring vibrator is designed to solve the above problems.

In the chapter 5, an attempt was made to solve the electrical-mechanical equivalent circuit model by considering two directional parameters in the longitudinal and radial directions.

In the chapter 6, Conclusions and future work are summarized.

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Chapter 2

Miniature Underwater Actuator using a PZT Disk Transducer

In this chapter, ultrasonic underwater actuators and various types of measurement platforms such as vibration velocity, Propulsion force characteristics, acoustic pressure will be described in detail. An ultrasonic underwater actuator will be designed using a vibrator with a thickness mode of vibration in the form of a disk-shaped vibrator with a diameter of approximately 10 mm made of PZT material. The actuator will be evaluated by measuring the admittance characteristics, vibration velocity distribution, propulsion force characteristics, and acoustic pressure. And an attempt is made to explore the tpropulsion force principle.

2.1 Piezo Transducer

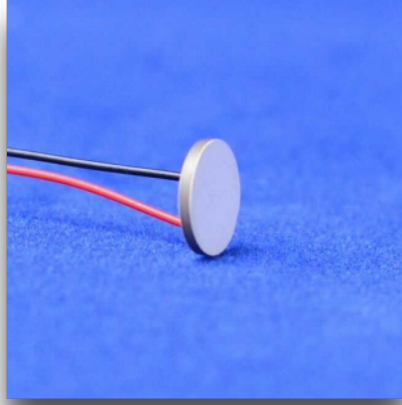


Figure 2.1: PZT Disk-shaped Transducer



Figure 2.2: Wire Connection Scheme for The Disk-shaped PZT Transducer

In order to achieve miniaturisation, reduce the size while increasing the resonance frequency and reducing the distance between electrodes (resistance reduction), the choice of a plate transducer is the preferred solution. As the project on underwater propulsion systems with thickness vibration piezoelectric elements is in its early stages, there is no complete design methodology. Therefore, it is convenient to conduct research using commercially available

PZT disk-shaped transducers in order to obtain basic data on underwater propulsion systems for thickness vibration piezoelectric elements. Therefore a disk-shaped transducer as shown in Figure 2.1 will be used to design the ultrasonic underwater actuator. The dimensions of this disk-shaped transducer are 10 mm in diameter and 1 mm in thickness. the material is the widely used $\text{Pb}(\text{Zr}, \text{Ti})\text{O}_3$ (PZT) piezoelectric material. The upper and lower circular surfaces of the transducer are covered with silver electrodes $8\mu\text{m}$ thick, and the electrodes on the upper surface extend to the back of the transducer, which allows the wires to be all arranged on the back of the transducer. The PZT was procured from FUJICeramics, and the various material parameters about PZTc204 are shown in Table 2.1.

The wire connection scheme for the disk-shaped PZT transducer is shown in Figure 2.2. The Fuji Ceramics disk-shaped PZT transducer used in this study is widely used in ultrasonic nebulisers. It has a wire connection as shown in the Figure 2.2, with the negative electrode extending to the back surface, allowing both positive and negative wires to be attached to the same surface, facilitating the design of ultrasonic underwater propulsion actuators. This wire connection method is the result of cost and performance considerations by the company and is not the result of an optimal design of the ultrasonic underwater propulsion system.

Table 2.1: Material Parameters of PZTc204 (FUJI CERAMICS)

Electro-mechanical coupling coefficient		Relative permittivity			Equivalent piezoelectric constant [$\times 10^{-12}\text{m/V}$]	
k_p	k_{33}	k_t	$\varepsilon_{11}^T/\varepsilon_0$	$\varepsilon_{33}^T/\varepsilon_0$	d_{31}	d_{33}
0.63	0.74	0.52	2240	2200	-190	435
Voltage output coefficient [$\times 10^{-3}\text{V} \cdot \text{m/N}$]		Elastic constant [$\times 10^{10}\text{N/m}^2$]		Frequency constant [$\text{m} \cdot \text{Hz}$]		
g_{31}	g_{33}	$1/s_{11}^E$	$1/s_{33}^E$	N_p	N_{33}	N_t
-9.7	22.0	7.2	5.7	2110	1450	2070
Density [$\times 10^3\text{kg/m}^3$]		Mechanical quality factor		Dielectric loss		Poisson's ratio
ρ		Q_m		$\tan\delta$		σ
7.65		520		0.0025		0.29

2.2 Ultrasonic Actuator

To elaborate the principle of acoustic propulsion system, the mechanism can be illustrated in terms of the bio-inspired and biomimetic engineering.

Recent efforts to design bionic swimmer technologies have also renewed focus on elucidating the physical principles of animal swimming. A central tenet is the idea that propulsion force is generated by pushing the surrounding water rearward. This paradigm requires the animal to generate the higher pressure near the body. The higher pressure gradient would act to push fluid downstream, propelling the animal forward as a consequence. [1–6] For example, if we swim with our hands slapping the water, our palms will feel the pressure given by the water, and with this pressure, we can swim forward. This propulsion mechanism can also be considered as a reciprocating vibration or acoustic wave near the baseline. The mechanical output power is proportional to the square of the vibration velocity in Equation. 2.1. The vibration amplitude and frequency of the human are centimeter scale and several Hz, the vibration velocity can be calculated in Equation 2.2,

$$P \propto v^2 \quad (2.1)$$

$$v = 2 * \pi * f * A. \quad (2.2)$$

where v , f and A are the vibration velocity, frequency and vibration amplitude. The vibration of bulk ultrasonic transducer, a reciprocating vibration, has the nanometer/micrometer vibration amplitude and the MHz/kHz frequency. Based on equation 2.2, it is known that even though the transducer is reduced in size, the resonance frequency is increased and the same vibration speed as the original size is obtained. Thus, a acoustic propulsion system with ultrasonic disc-shaped transducer may be suitable for underwater actuator with the smaller vibration amplitude and higher frequency.

If the ultrasonic transducer is placed directly into the water, both the upper and lower surfaces of the ultrasonic transducer generate acoustic radiation pressure, according to Newton’s third law, the water exerts the same force on both surfaces of the ultrasonic transducer, and the ultrasonic transducer will not move. If, as shown in Figure 2.3, the lower surface of the ultrasonic transducer is always in contact with the air, only the surface in contact with the water will receive the force, thus this prevents short-circuiting of the wires on the back of the ultrasonic transducer, but also produces a propulsion force perpendicular to the surface. This is the principle of the acoustic underwater propulsion system.

Therefore, the ultrasonic underwater actuator was designed as shown in Figure 2.4. (a) represents the cross-section deconvolution plane of the actuator. The disc transducer is placed on an inverted cone cavity, which not only keeps the plate transducer horizontal contributing to the vibration distribution on the measurement surface, but also allows it to be secured while

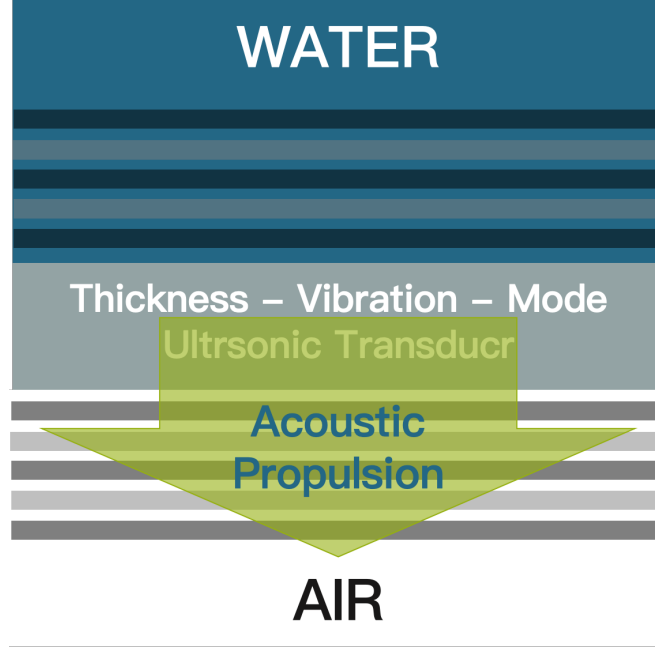


Figure 2.3: Principle of acoustic propulsion system.

minimising the contact area. And the fixation piece is printed and fabricated by a 3D printer and the material is TPU. the TPU has some elasticity and can clamp the circular plate transducer well. (b) is the photo of the actuator, the actual size is about 12 mm in diameter and 5 mm in height.

The fixed actuator is waterproofed by plugging the holes where the wires lead out with glue, and will be used in the measuring session.

2.3 Admittance Characteristics

After the PZT transducer was installed in the prototype actuator, its admittance was measured using a precision impedance analyzer in air and water. The results are shown in Figure 2.5. The resonance frequency of the PZT actuator in air and water was 2.13 and 2.11 MHz, respectively. The conductance of the PZT actuator in air and water was 85.8 and 49.9 mS at resonance frequency, respectively. It can be seen that the resonance frequency in water and air is almost constant, while the conductance at the resonance frequency is reduced by 35.9 mS.

Neglecting the loss of the induced impedance as well as the air loading, the vibrational loss of the PZT actuator at the resonance frequency is about the loss of $R_m = 11.7\Omega$ resistance at the resonance frequency shown in the

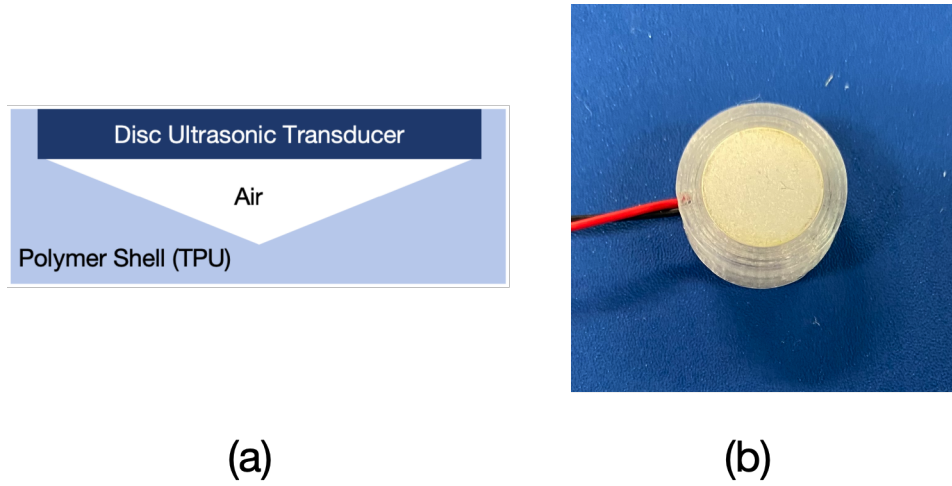


Figure 2.4: Actuator. (a): the cross-section deconvolution plane of the actuator;(b): the photo of the actuator

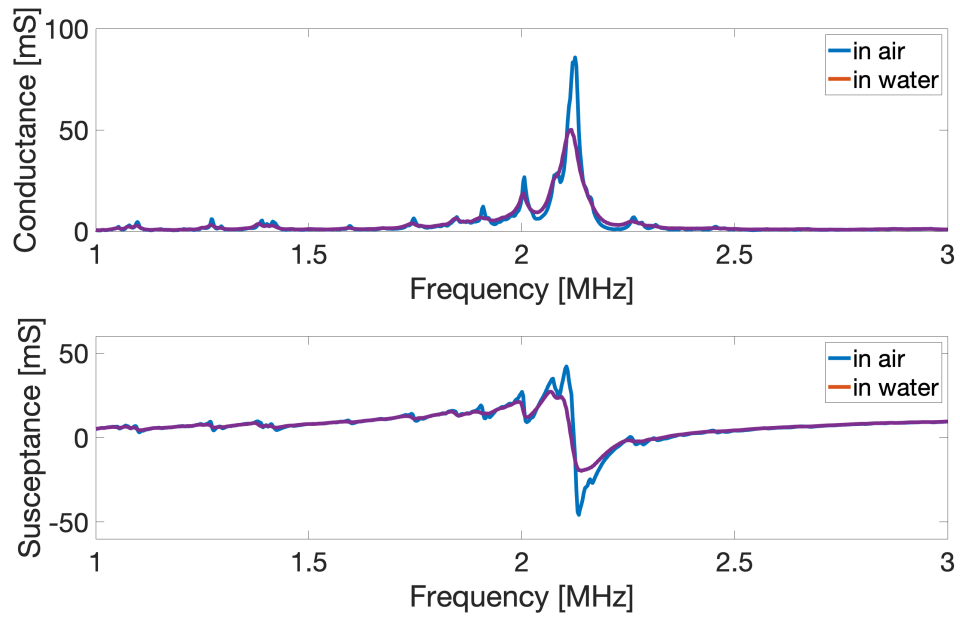


Figure 2.5: Admittance characteristics of the PZT actuator.

measurements of the admittance characteristics in air. The power on the resistance represented by the $R_{w-m} = 8.37\Omega$ difference between the resistance at the resonance frequency shown in the measurement results of the admittance characteristics in water and the resistance in air represents the output power of the PZT actuator in water. This means that the output efficiency of PZT actuator can be expressed as the following equation.

$$\eta = \frac{R_{w-m}}{R_{w-m} + R_m} = \frac{R_{w-m}}{R_w} = 42\%. \quad (2.3)$$

However, η is output efficiency, R_{w-a} is the output load expressed as the difference between the equivalent resistance in air and in water at the resonant frequency, R_w is equivalent resistance in water at resonance frequency.

2.4 Vibration Velocity Distribution

2.4.1 Measurement System

Exploring the vibration characteristics of thickness vibrations and calculating the propulsion force requires measurements of the vibrational velocity of the surface of an ultrasonic transducer. Measurements will be made using an actuator with the PZT ultrasonic transducer fixed in place. In addition, since the Doppler vibrometer (Polytec, VibraOne) in possession does not have an auto-focus and movement function, it is necessary to connect the function generator, oscilloscope, and moving stage to a computer and write an auto-scanning and focusing programme using matlab or Labview software to facilitate automatic reading of vibration velocities at the many measurement points.

Due to the construction of the ultrasonic actuator as shown in Figure 2.4, it is possible to keep the PZT transducer as horizontal as possible, which reduces the time spent on focusing during the measurement.

The measurement system for the vibration velocity in air is shown in Figure 2.6, which demonstrates the entire measurement system constructed. The Laser Doppler Vibrometer probe is mounted on a moving platform, which controls the probe and enables it to focus and move automatically. The red laser light emitted from the probe is the measurement point, and the laser light is kept as perpendicular as possible to the measurement plane. The ultrasonic transducer actuator to be measured is placed directly below the probe. The device shown here has a microscope lens attached to the probe, which makes the laser beam thinner, the minimum measurable interval smaller, and the focusing distance closer, about 3 cm.

Regarding the judgement of the measured signal, the Laser Doppler vibrometer shown in the figure comes with a detection port (RSSI OUT). When the probe receives a full signal (when the measurement error is minimal), the output voltage is about $2V_{0p}$, thus the strength of the signal can be judged by the voltage output from this port. In air because the measurement conditions are better, thus the signal strength threshold for reading data can be set to 90%, that is, the output voltage of the detection port is greater than or equal to $1.8V_{0p}$ when reading the measurement data, less than $1.8V_{0p}$ will take the null value or the default value.

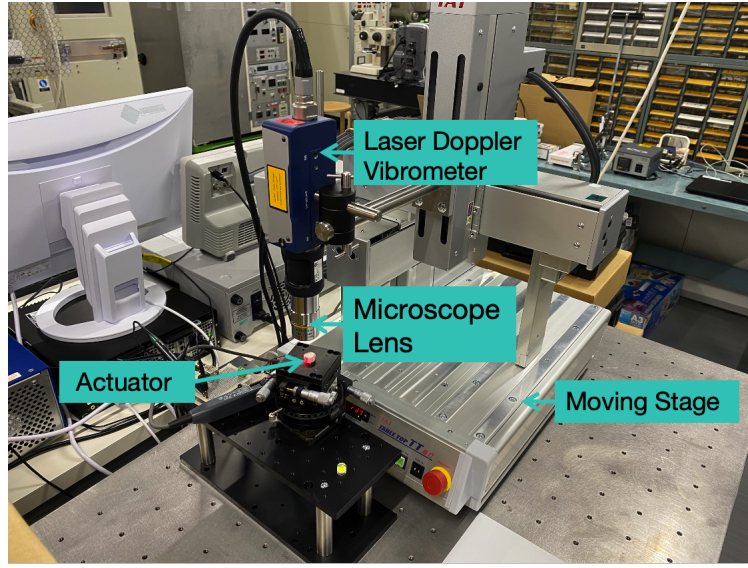


Figure 2.6: Vibration velocity measurement system in air.

The platform for measuring vibration velocity in water needs to be a little more complex, as shown in Figure 2.7. The PZT ultrasonic actuator needs to be placed in a vessel filled with water so that the water is submerged over the actuator.

If the probe of the Laser Doppler vibrometer is not treated and measurements are taken on the water surface in the same way as in air, there will be two problems, one is that the laser light passes through the two mediums, will the readings still be reliable and the other is that the surface of the water will be turbulent due to the acoustic waves produced by the ultrasonic transducer, and these will affect the results of the measurements.

Thus the solution is to wrap a layer of transparent cling film around the probe and ensure that the laser can completely pass through the cling film. Then the probe is inserted into the water to measure the vibration velocity in the water. However, due to the small swimming floats and bubbles in the

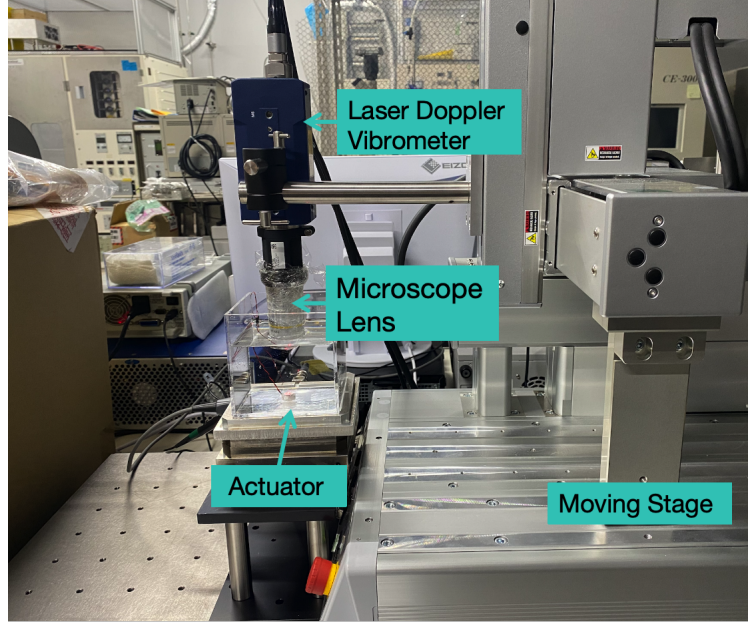


Figure 2.7: Vibration velocity measurement system in water.

water and other reasons, it is difficult to do with the same signal strength in the air, thus the signal strength threshold for obtaining measurement data should be appropriately lowered, about 80%(1.6 V_{0p}).

2.4.2 Results of Measurements

The vibration amplitude distribution of the surface of the PZT ultrasonic transducer in air is shown in Figure 2.8. The input voltage is at a frequency of 2.13 MHz, the resonance frequency of the ultrasonic actuator, and the peaks are 2 V_{pp} , 3 V_{pp} , 4 V_{pp} , and 5 V_{pp} . The interval between measurement points is 50 μm . The centre of the circle is roughly at the position (5,5).

It can be found that the vibration amplitude distribution of the thickness vibration is distinguished from the pure longitudinal vibration by the presence of about 11 peaks, and it's centrally symmetrical, and the peak at the centre is the largest, and decreases further away from the centre. As the input voltage increases, the vibration amplitude distribution becomes clearer. This shows that the thickness vibration is formed by the combination of radial vibration and longitudinal vibration.

In order to compare the values more visually, the vibration velocity distribution over the diameter has been measured as shown in Figure 2.9. The driving frequency is 2.13 MHz and the peak-to-peak values are 4 V_{pp} , 6 V_{pp} ,

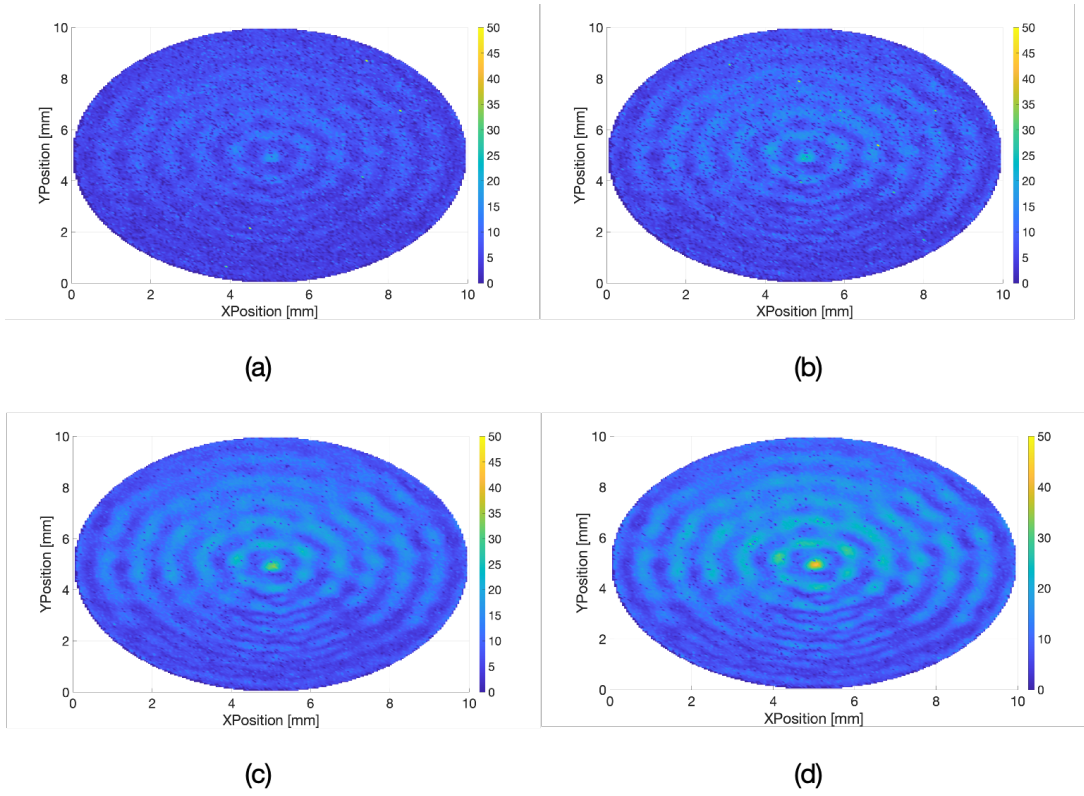


Figure 2.8: Vibration velocity Distribution on the surface in air. Driving frequency: 2.13 MHz; (a): $2V_{PP}$, (b): $3V_{PP}$, (c): $4V_{PP}$, (d): $5V_{PP}$

$8V_{pp}$ and $10V_{pp}$. The interval between measurement points is $50\mu m$. The (0,0) point indicates the centre of the circle on the surface of the PZT disk-shaped ultrasonic transducer. The vibration velocities at the centre of the circle is 280 mm/s, 430 mm/s, 580 mm/s, 775 mm/s, thus for every $2V_{pp}$ increase in the peak-to-peak value of the voltage, the vibration velocity at the centre increases by 150 mm/s, 150 mm/s, 195 mm/s, respectively.

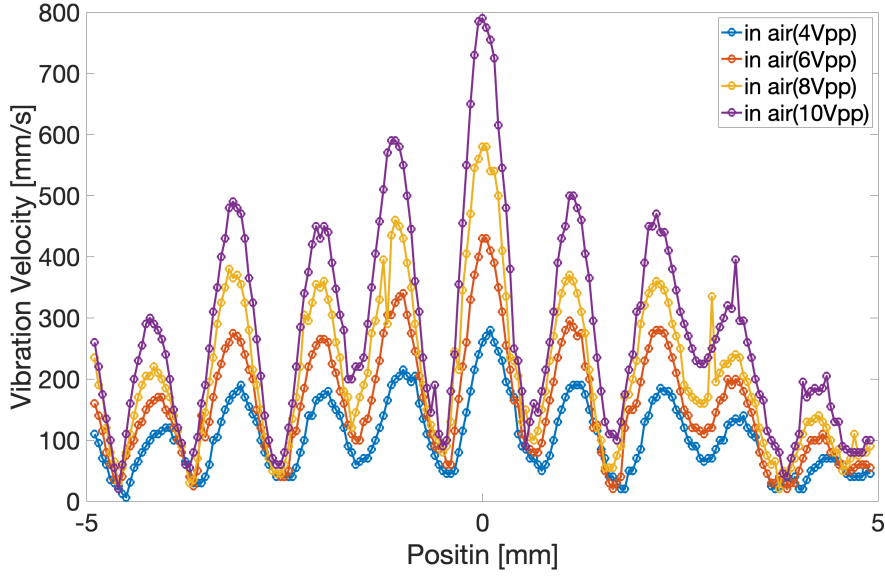


Figure 2.9: Vibration velocity Distribution on the diameter in air. Drive frequency: 2.13 MHz.

Combined with the above measurements of the vibration distribution in the air, it is possible to derive a distribution pattern for the amplitude of the thickness vibration, but the vibration velocity is time-dependent, thus it is not sufficient to determine that each peak is in the same phase, that is, it is not possible to determine whether the vibration velocity at each peak reaches its maximum value at the same time, or whether it reaches its maximum value alternately. Therefore, it is necessary to measure the phase difference between the drive voltage and the vibration velocity to solve the problem.

The phase of the 2.13 MHz, $10V_{pp}$ drive voltage is used as a reference, and the result of the phase difference with the vibration velocity is measured as shown in Figure 2.10. From Figure 2.10, it can be seen that the phase difference between the drive voltage and the vibration velocity at different positions varies around -100° , but the maximum phase difference differs from the minimum phase difference by only 60° . Therefore, it can be de-

terminated from the measurement results of the phase difference between the drive voltage and the vibration velocity that the vibration distribution of the thickness vibration is indeed as shown in Figure 2.9.

Theoretically, the drive current is in phase with the vibration velocity and numerically proportional to each other. In addition, the phase difference between the drive voltage and drive current is constant, thus the phase difference between drive voltage and vibration velocity is always kept at a constant phase difference.

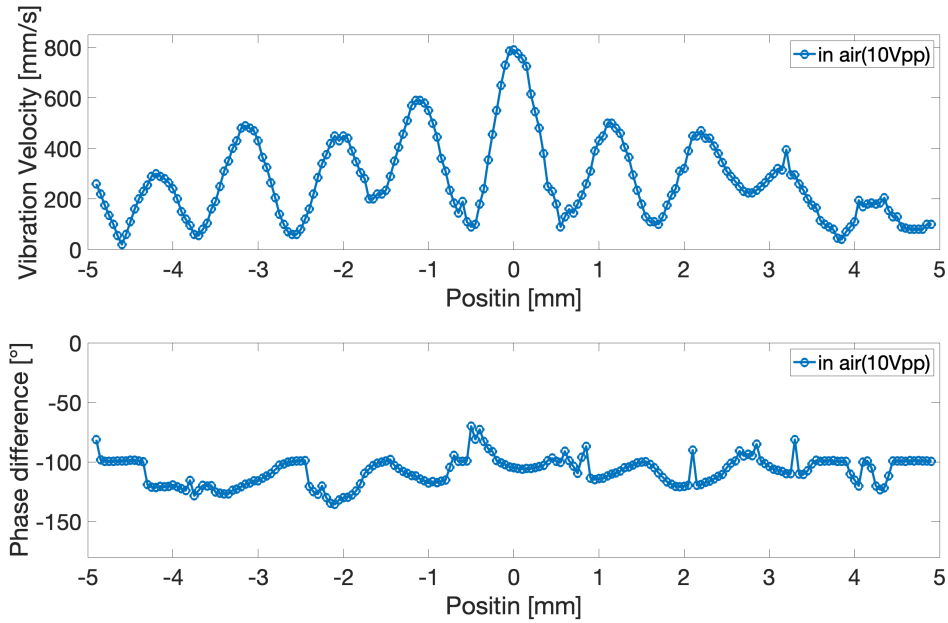


Figure 2.10: Vibration velocity Distribution and Phase Difference between drive voltage and vibration velocity. Drive Voltage: 2.13 MHz, 10V_{pp}.

Figure 2.11 shows the results of a burst-driven measurement of the vibration distribution of a thickness-vibration disc-shaped PZT vibrator, with reference to Nakazawa's master thesis [16]. The numbers on the vertical axis represent the period of vibration after the start of input.

The standing waves caused by radial vibration from the outer circumference of the PZT vibrator towards the centre as time progresses are clearly seen. It can also be seen that in the initial phase of the rise of vibration from the first to the fifth cycle, an almost uniform amplitude is obtained.

After a few transient cycles, a vibration velocity distribution for the thickness vibration similar to the measured vibration velocity distribution of the present study was obtained.

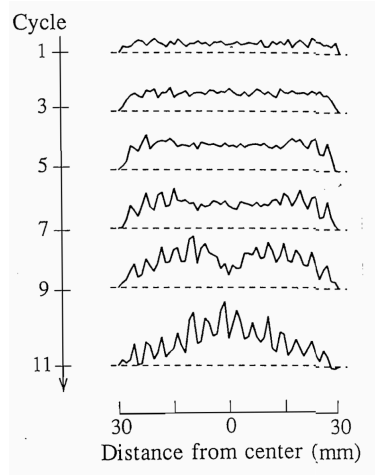


Figure 2.11: Velocity distribution of the vibrating surface during burst drive (5 waves at 570 kHz input). [16]

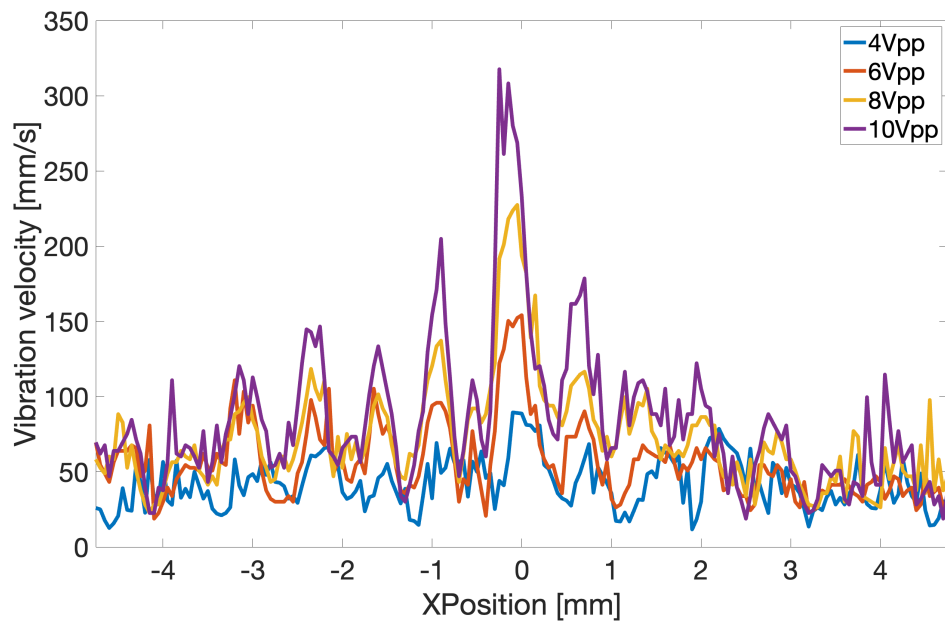


Figure 2.12: Vibration velocity Distribution on the diameter in water. Drive Voltage: 2.11 MHz.

Since the object of the study is an actuator in water, it is necessary to measure the vibration velocity distribution in water, and the measurements of the vibration velocity distribution over the diameter are shown in Figure 2.12. The driving voltage is $4V_{pp}$, $6V_{pp}$, $8V_{pp}$ and $10V_{pp}$, and the frequency is 2.11 MHz. The interval between measurement points is $50\mu\text{m}$. It should be noted that since the measurement is made in water, the propagation speed of light in air is about 1.33 times that in water, thus the raw data obtained from the measurement must be divided by the refractive index of water (1.33) in order to obtain the correct measurement results. (0, 0) is the centre of the circle on the surface of the ultrasonic transducer.

Due to the underwater measurement of the signal is not stable enough to obtain the measurement data signal strength threshold is reduced, resulting in the distribution of the measurement results is not as clear as the air which, but still can be seen about 11 peaks, and the peak at the centre of the circle is the largest, and the farther away from the centre of the circle, the smaller the peak.

When the driving voltage is $4V_{pp}$, $6V_{pp}$, $8V_{pp}$ and $10V_{pp}$, the vibration velocity at the centre of the circle is 89.5 mm/s, 154 mm/s, 227 mm/s, 313 mm/s. Every $2V_{pp}$ increase of the driving voltage, the vibration speed at the centre of the circle is increased by 74 mm/s, 73.3 mm/s, 85.53 mm/s respectively.

2.5 Sound Pressure

2.5.1 Measurement System

In order to explore whether the propulsion force of the underwater ultrasonic actuator is generated by the acoustic radiation pressure, not only the parameters of the vibration velocity but also the parameters of the acoustic pressure are needed in the theoretical calculation, therefore, a platform for measuring the acoustic pressure as shown in Figure 2.13 is built.

The basic structure is similar to that of the vibration velocity measurement platform, and the probe of the Laser Doppler vibrometer can be replaced by the acoustic pressure probe needle(ONDA-HNP). The moving stage controls the acoustic pressure probe to follow a specified path, and acquires the acoustic pressure measurements along that path. In order to measure the sound pressure in the near field of the ultrasonic transducer, the sound pressure probe needle should be as close as possible to the surface of the ultrasonic transducer, about 0.5 mm, before measurement.

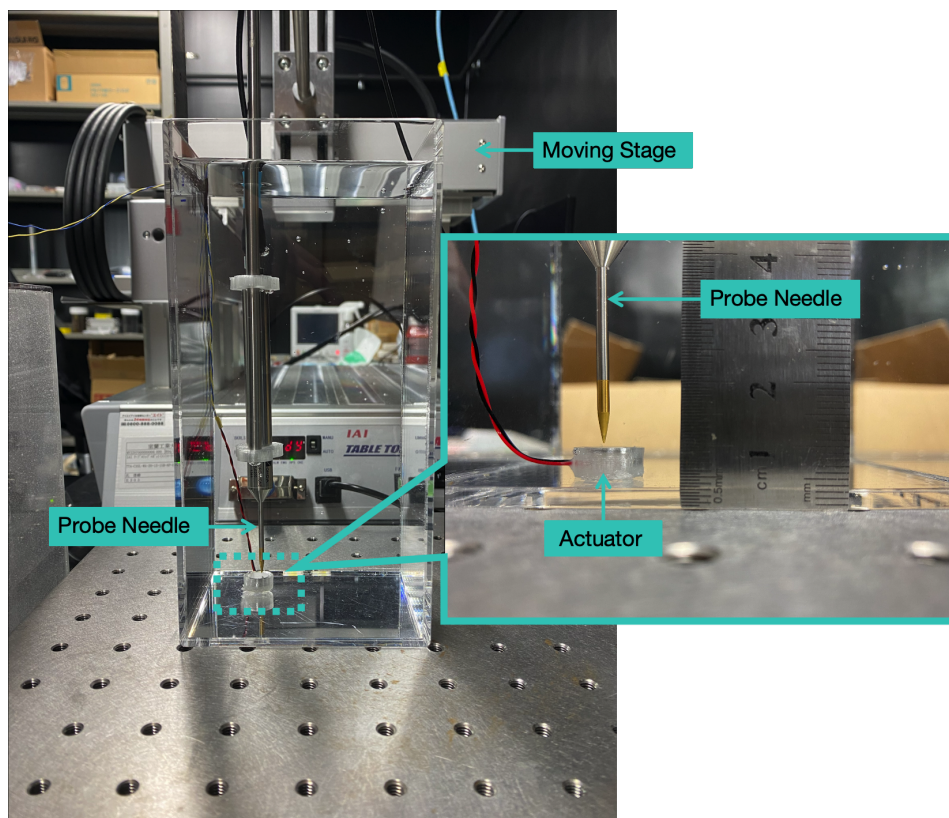


Figure 2.13: Sound Pressure Measurement System.

2.5.2 Results of Measurements

The sound pressure distribution over the diameter of the ultrasonic transducer is shown in Figure 2.14 when the driving voltage is $4V_{pp}$, $6V_{pp}$, $8V_{pp}$ and $10V_{pp}$, and the frequency is 2.11 MHz, respectively. The coordinates at (0,0) indicate the centre of the circle on the surface of the ultrasonic transducer. The interval between the measurement points is $100\mu\text{m}$. Since the sound pressure measurement is easily affected by the environment in the water and does not have numerical screening conditions, the existence of some outliers can not be avoided.

Observation of Figure 2.14 shows that the sound pressure distribution is symmetric along the centre, the shape is similar to a parabola symmetric to the y-axis, and the vibration velocity distribution is consistent. This result is similar to the sound pressure distribution obtained in [7]. Numerically, the sound pressure at the centre of the circle of the ultrasonic transducer when the driving voltage is $4V_{pp}$, $6V_{pp}$, $8V_{pp}$ and $10V_{pp}$ is 0.5 MPa, 0.7 MPa, 0.9 MPa and 1.1 MPa respectively, and the sound pressure will increase by 0.2 MPa for every increase of $2V_{pp}$ in driving voltage.

The sound pressure results were measured near the vibrating surface of the PZT transducer, but did not show several peak distributions as in the vibration velocity measurements. The vibration velocity distribution results show that one wavelength is approximately 0.8 mm, which means that the measurement spacing needs to be less than 0.1 mm to measure the sound pressure distribution. Therefore, because the diameter of the probe tip is approximately 0.5 mm coarse, the distribution with a number of peaks cannot be measured and is averaged to show a quadratic distribution.

The upper limit of the vibration velocity of the PZT piezoelectric material is approximately 1 m/s, and the sound pressure in water is proportional to the vibration velocity, thus theoretically the sound pressure in water does not reach 1 MPa, but the diameter of the probe tip is approximately 0.5 mm, which may cause standing waves to increase the measurement results.

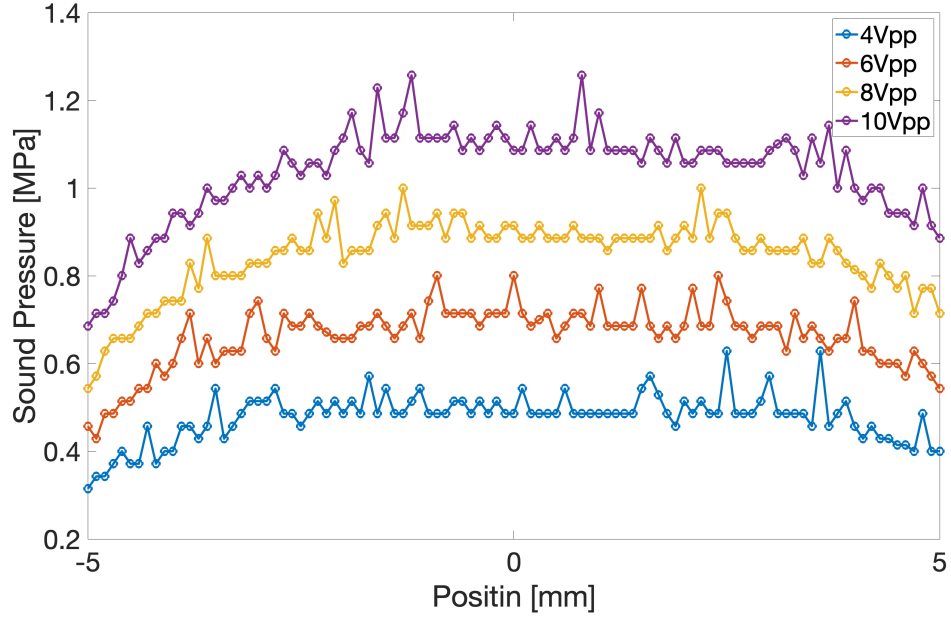


Figure 2.14: Measurement results of Sound Pressure.

2.6 Propulsion Force

2.6.1 Measurement System

In order to validate the calculation of the acoustic radiation pressure, it is necessary to measure the thrust characteristics generated by the ultrasonic actuator. Therefore, as shown in Figure 2.15 the dynamometer(Mark=10, M5-012) was fixed on a stand, then the ultrasonic actuator was fixed on the probe end of the dynamometer, and then the container with water was placed under the dynamometer and it was ensured that the surface of the ultrasonic wave transducer was completely and fully in contact with the water.

2.6.2 Results of Measurements

The propulsion force measurements are shown in Figure 2.16. The input power is almost linearly related to the propulsion force, and when the drive power is below 0.075 W, the propulsion force of the PZT underwater actuator cannot be measured. When the input power reaches 4.64w, the propulsion force of PZT underwater actuator can reach 1 mN.

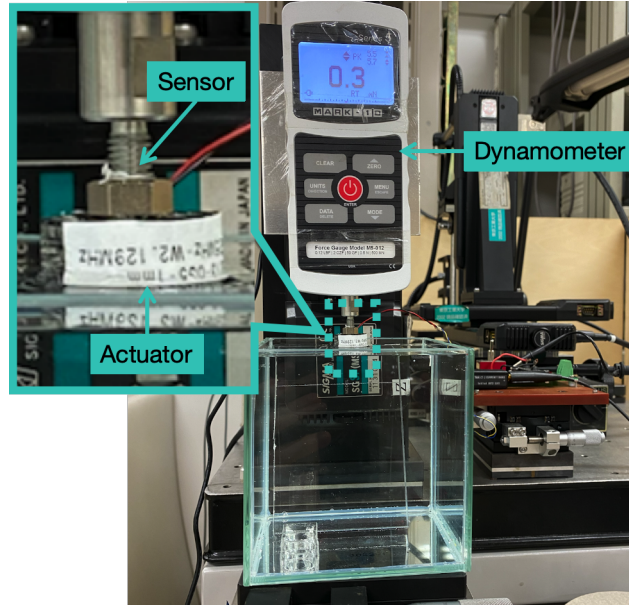


Figure 2.15: Zero-Speed propulsion force Measurement System.

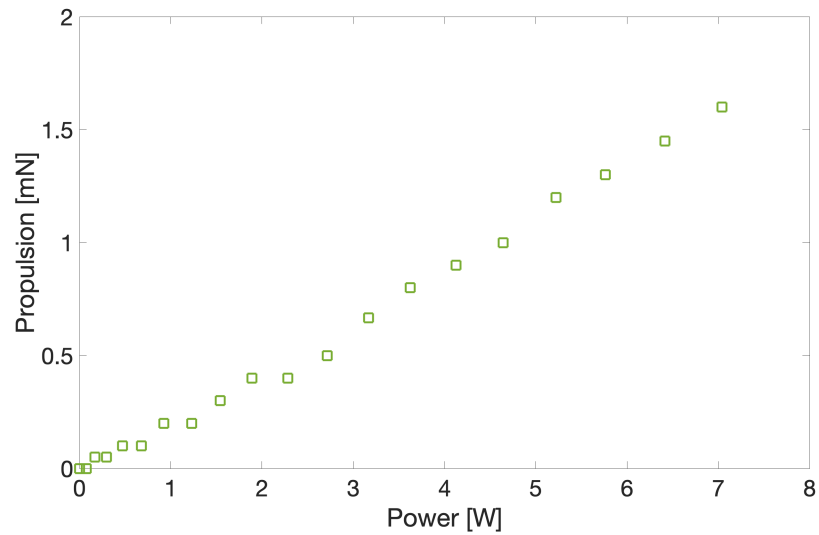


Figure 2.16: Measurement Result of Zero-Speed propulsion force.

2.7 No Load Speed

2.7.1 Ultrasonic Underwater Actuator

An acrylic prototype Ultrasonic underwater actuator was designed. An air cavity is used to create an unbalanced condition on one side of the PZT transducer. Unidirectional propulsion force is produced for one-direction moving. An explosion diagram of the Ultrasonic underwater actuator is shown in Figure 2.17(a). Three acrylic parts, fabricated by a three-dimensional (3D) printer [KEYENCE (Itasca, IL) AGILISTA-3100], are glued together. The PZT transducer is fixed in the groove with pure mechanical integration. Two symmetrical wire holes are set. A photograph of the acrylic prototype ultrasonic underwater actuator is shown in Figure 2.17(b). The length and diameter of the entire actuator are 18 and 14 mm, respectively. [8]

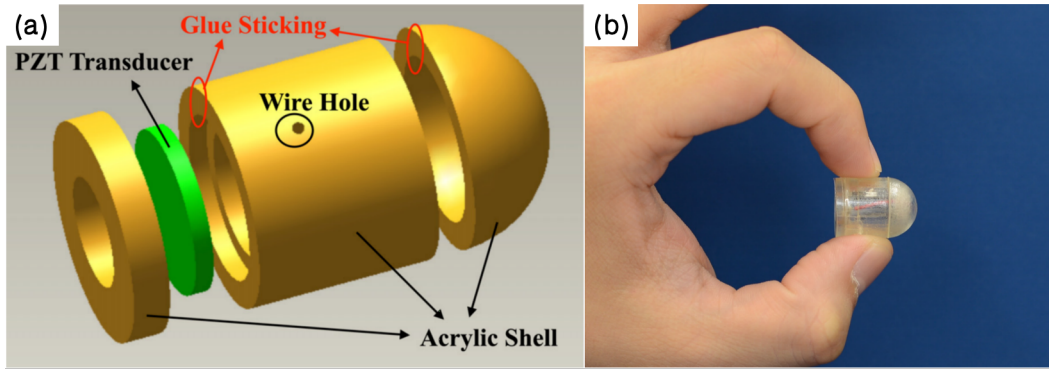


Figure 2.17: Ultrasonic underwater actuator.(a): Explosion diagram; (b): Photograph of acrylic prototype actuator.

2.7.2 No Load Speed Measurement

To analyze the prototype ultrasonic underwater actuator locomotion in water, No-Load-Speed(NLS) was investigated. As shown in Figure 2.18(a), a foam kickboard was attached to the underwater actuator to make it balance on the water surface. The prototype ultrasonic underwater actuator achieved horizontal propulsion. To suppress the wire traction effect, flexible wires [BLOCK (Verden, Germany), 926869, U0.1 mm] were used for radio frequency power in the measurements. The locomotion of the prototype ultrasonic underwater actuator was recorded by a camera, as shown in Figure 2.18(a)-(d), at an input voltage of $80.4V_{pp}$ and a driving frequency of 2.1 MHz. The movement of the ultrasonic underwater actuator was analyzed using an image recognition program in Matlab.

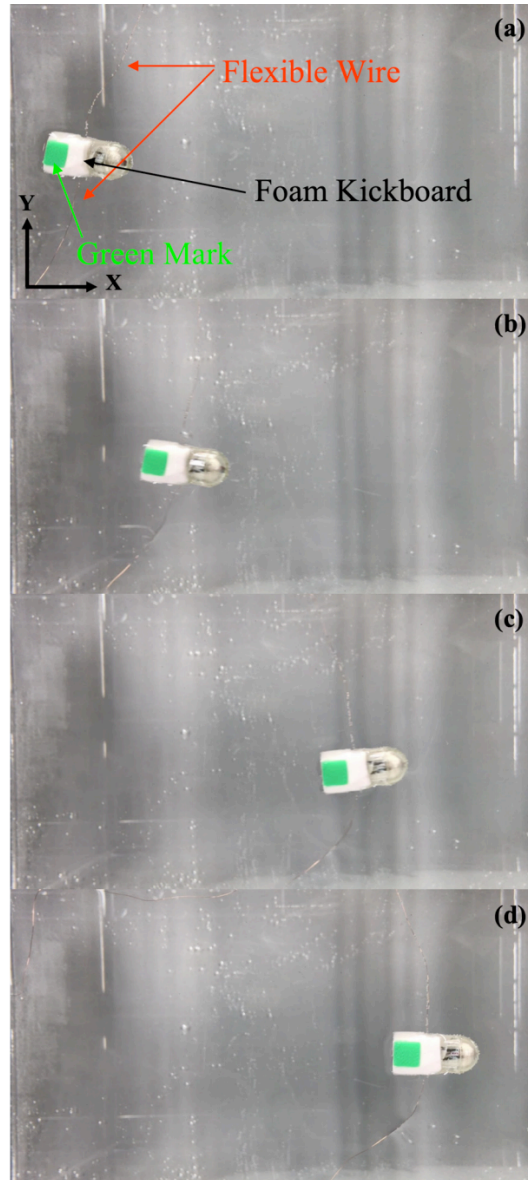


Figure 2.18: ultrasonic underwater actuator locomotion in water for input voltage of $80.4V_{pp}$ at 2.1 MHz recorded by a camera. A foam kickboard (green square) was used to keep balance. The length and width of frame are 270 and 152 mm.

As shown in 2.18(a), a green square acrylonitrile butadiene styrene (ABS) polymer plate was used as the identification mark. The length (270 mm) and width (152 mm) of a video frame were determined using a ruler before the measurements. The NLS changeover time in the x and y axis directions was then analyzed even though the trajectory of the ultrasonic underwater actuator was not a perfectly straight line. The NLS analysis results for an input voltage of $80.4V_{pp}$ are shown in Figure 2.19. The peak NLS value in water is 152.7 mm/s. The peak NLS value versus input voltage is shown in Figure 2.20. The NLS of the ultrasonic underwater actuator increased with increasing input voltage. For an input voltage of $12.4V_{pp}$, ultrasonic underwater actuator movement was first detected when the peak NLS value was 6.1 mm/s. For an input voltage of $89.6V_{pp}$, the peak NLS value was 171.9 mm/s. Because fluid resistance and wire traction affect the NLS, a higher NLS is expected for a given acoustic radiation force if the ultrasonic underwater actuator shape and the wireless power supply are optimized. The high power density of acoustic radiation pressure can be maintained when the scale of the acoustic underwater propulsion system is down with higher frequency because vibration velocity does not greatly change. Thus, a higher NLS is expected for a smaller-scale swimmer.

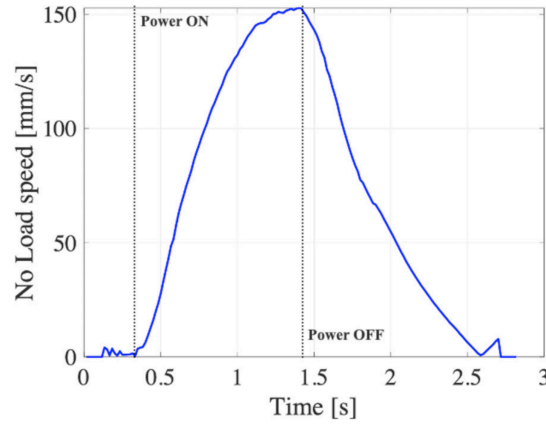


Figure 2.19: NLS values for input voltage of $80.4V_{pp}$. Peak value is 152.7 mm/s.

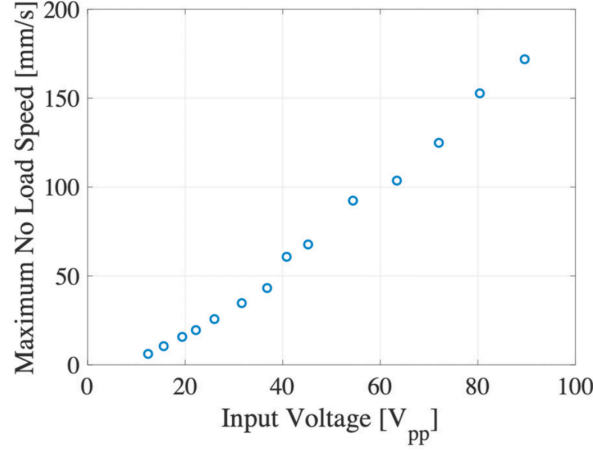


Figure 2.20: Maximum NLS values for swimmer in water for various input voltages.

2.8 Acoustic radiation force

Existing theories of propulsive force are examined in order to accurately determine the propulsion force. When sound waves penetrate the boundary between two media with different acoustic impedances, a constant, albeit minute, DC force is generated at the boundary. This is called the acoustic radiation force. [9–15]

The propulsion force of an ultrasonic underwater actuator is at the interface between the ultrasonic transducer and the water, which is more consistent with acoustic radiation force with the ultrasonic transducer on one side and the water on the other. Thus, the acoustic radiation force is thought to be responsible for the ability of the ultrasonic actuation to generate Propulsion force underwater.

The viscosity and thermal conductivity of the medium are neglected to simplify the calculations.

There is a way of describing the acoustic radiation pressure P_{ac} acting on an object in the sound field in terms of the time-averaged acoustic energy density $\langle E_i \rangle$ of the incident plane sound wave (when there is no object in the sound field and no reflection or scattering) [8]. In this case, the radiation pressure is given by

$$P_{ac} = \langle E_i \rangle \quad (2.4)$$

for objects of perfect absorption or

$$P_{ac} = (1 + R_p^2) \langle E_i \rangle \quad (2.5)$$

for objects with a sound pressure reflection coefficient R_p .

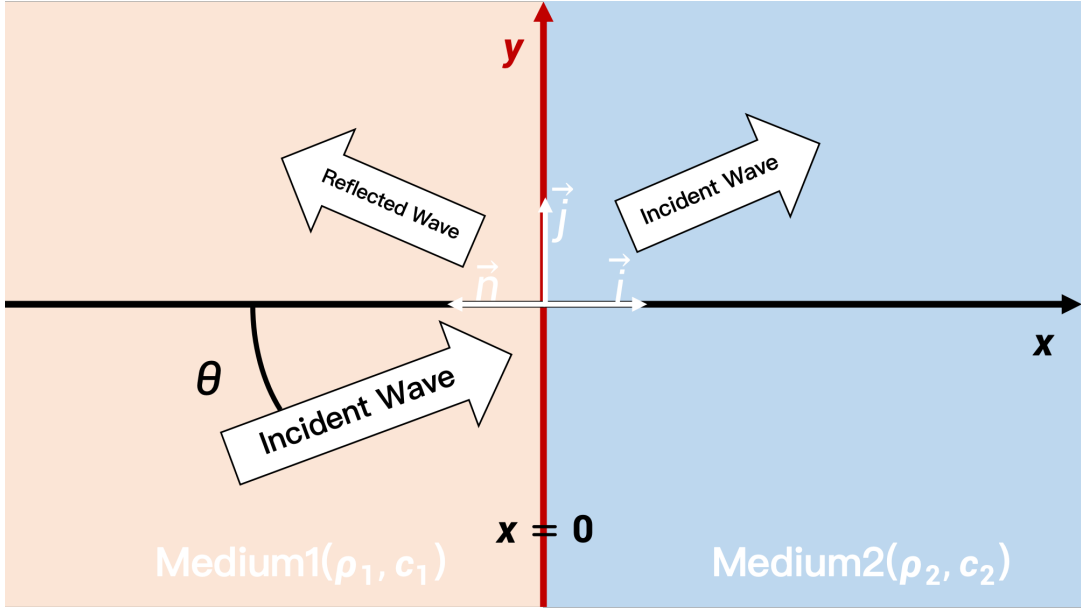


Figure 2.21: Sound waves are incident on a two-layer medium of different acoustic impedance.

As shown in Figure 2.21, there are different intrinsic acoustic impedance media on both sides of the boundary. If the plane sound wave has an angle of incidence $\theta = 0$, some of the sound wave is reflected by the different media and the rest is transmitted. therefore the sound pressure p is

$$p = \begin{cases} p_0 \sin(\omega t - k_1 x) + R_P \sin(\omega t + k_1 x), & x < 0 \\ p_0 T_P \sin(\omega t - k_2 x), & x > 0. \end{cases} \quad (2.6)$$

where $k_i = \frac{\omega}{c_i}$ ($i = 1, 2$) is the wavenumber in the respective medium.

The reflection and transmission coefficients of sound pressure, R_p and T_p , are $R_p = \frac{1-Z}{1+Z}$ and $T_p = \frac{2}{1+Z}$ respectively, expressed with the impedance ratio $Z = \frac{\rho_1 c_1}{\rho_2 c_2}$.

The particle $\vec{u}$ is only in the x-axis direction, which, if we let

$$u = \begin{cases} \frac{p_0}{\rho_1 c_1} \sin(\omega t - k_1 x) - R_P \sin(\omega t + k_1 x), & x < 0 \\ \frac{p_0 T_P}{\rho_2 c_2} \sin(\omega t - k_2 x), & x > 0. \end{cases} \quad (2.7)$$

If $\vec{i}$ and $\vec{n}$ are the unit vectors shown in the Figure 2.21, there is a relationship $\vec{u} = i\vec{u} = -n\vec{u}$. The acoustic radiation force is also given as

$$F_{ac} = \int \int_{S_0} \langle \mathcal{L} \rangle \vec{n} - \rho \langle \vec{u} (\vec{u} \cdot \vec{n}) \rangle dS \quad (2.8)$$

Where ρ_0 is medium density of static pressure with no sound waves present., Lagrangian $\mathcal{L} = \rho_0 \vec{u}_1 \cdot \frac{\vec{u}_1}{2} - \frac{p_1^2}{(2\rho_0 c_0^2)}$. $\vec{i}$ & $\vec{j}$ is the fundamental vector of the x- and y-axes, $\vec{n}$ is the normal vector.

$\mathcal{L} = K - U$ and $\rho \vec{u} (\vec{u} \cdot \vec{n}) = -\rho u^2 \vec{i} = 2K \vec{n}$ in the integral term of Equation 2.8, therefore the acoustic radiation pressure is

$$P_{ac} = -\langle (K - U) - 2K \rangle = \langle K + U \rangle. \quad (2.9)$$

Where kinetic energy density $K = \frac{\rho_0 u^2}{2}$, The potential energy density $U = \frac{p^2}{2\rho_0 c_0^2}$

From this, the time-averaged $\langle E \rangle$ of the sound energy density $E = K + U$ is

$$\langle E \rangle = \begin{cases} \langle E_i \rangle (1 + R_P^2), & x < 0 \\ \langle E_i \rangle \frac{c_1}{c_2} Z T_P^2, & x > 0. \end{cases} \quad (2.10)$$

Where, Incoming sound energy density $\langle E_i \rangle = \frac{p_0^2}{2\rho_1 c_1^2}$, sound pressure $P_0 = \rho_2 c_2 A \omega$, vibration amplitude A .

Equation 2.10 represents the pressure received on both sides of the boundary surface. The difference in energy density between the left and right sides of the boundary surface is the acoustic radiation pressure, thus eventually the acoustic radiation pressure per unit area from the left side of the boundary surface to the right side is

$$P_{ac} = \langle E \rangle|_{x<0} - \langle E \rangle|_{x>0} = \frac{2\langle E_i \rangle}{(1 + Z)^2} (1 + Z^2 - 2Z \frac{c_1}{c_2}) \quad (2.11)$$

From this, the acoustic radiation force is

$$F_{ac} = P_{ac} S \quad (2.12)$$

However, S is the area of the radiating surface. From this, the acoustic radiation propulsion $\vec{F}_{ARP}$ is

$$\overrightarrow{F_{ARP}} = -\overrightarrow{F_{ac}}. \quad (2.13)$$

Next, the acoustic radiation propulsion is calculated using the measured vibration velocity and sound pressure of the disc-shaped PZT tansducer. However, the effective radiating area is $S = 5^2 \times 10^{-6} \pi \text{m}^2$.

2.8.1 Calculation of Acoustic Propulsion Force

A new quadratic function is obtained by processing the results of acoustic pressure through Matlab's fitting function, and then the obtained quadratic function is substituted into the acoustic radiating pressure calculation formula 2.11, and the area division is carried out to get the calculation result of propulsion force.

Calculated results of thrust were obtained as 0.07 mN, 0.13 mN, 0.21 mN, and 0.31 mN when the driving power was 0.08 W (driving voltage $4V_{pp}$), 0.14 W (driving voltage: $6V_{pp}$), 0.30 W (driving voltage $8V_{pp}$), and 0.47 W (driving voltage $10V_{pp}$), respectively.

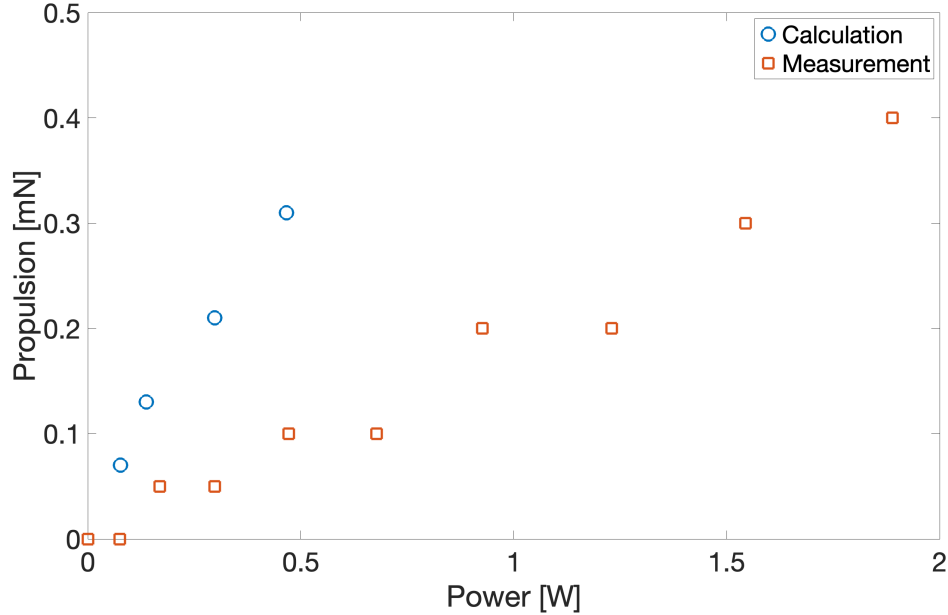


Figure 2.22: Calculation result of propulsion force.

Comparison of the calculation results with the measured results of the propulsion force is obtained as shown in Figure 2.22, where the calculated

results are greater than the measured results at the same driving power. At 0.08 W, the calculated thrust is 0.07 mN, while the measured result is 0mN. At 0.3 W, the calculated propulsion force is 0.16 mN larger than the measured result, and at 0.47 W, the calculated thrust is 0.21 mN larger than the measured result, which can be seen as the difference between the calculated result and the measured result becomes larger with the increase of the driving power. is getting bigger and bigger. Therefore, the calculated results do not correspond to the actual thrust characteristics.

The reasons may be in these points:

- 1, The calculation needs the value of the sound speed in the PZT transducer, which is calculated from the equivalent circuit model of longitudinal vibration, thus the value of the sound speed is different from the thickness vibration mode, which leads to the deviation of the calculation result.
- 2, Because the propulsion force is too small, the measured value will be deviated due to the external influence during the measurement.
- 3, Due to the lack of consideration of the viscosity, which causes the calculated values to be larger than measured values.

2.9 Conclusion

In this section, an ultrasonic underwater actuator with a PZT disc transducer is fabricated. Measurements of the admittance, vibration, propulsion force and acoustic pressure characteristics of the actuator were carried out. An ultrasonic actuator capable of moving underwater was fabricated based on the measurement actuator, and its speed was evaluated. Finally, using the acoustic pressure measurement results, an attempt was made to calculate the acoustic radiation force. The following conclusions were eventually reached.

1. Measurement results of admittance characteristics:

The resonance frequency of the PZT actuator in air and water was 2.13 and 2.11 MHz, respectively. The conductance of the PZT actuator in air and water was 85.8 and 49.9 mS at resonance frequency, respectively. the resonance frequency in water and air is almost constant, while the conductance at the resonance frequency is reduced by 35.87 mS.

2. Measurement results of vibration velocity in air:

The vibration velocities at the centre of the circle is 280 mm/s, 430 mm/s, 580 mm/s, 775 mm/s, thus for every $2V_{pp}$ increase in the peak-to-peak value of the voltage, the vibration speed at the centre increases by 150 mm/s, 150 mm/s, 195 mm/s, respectively.

The vibration amplitude distribution of the thickness vibration is distinguished from the pure longitudinal vibration by the presence of about

11 peaks, and it's centrally symmetrical, and the peak at the centre is the largest, and decreases further away from the centre. As the input voltage increases, the vibration amplitude distribution becomes clearer. This shows that the thickness vibration is formed by the combination of radial vibration and longitudinal vibration.

The phase difference between the drive voltage and the vibration velocity at different positions varies around -140° , but the maximum phase difference differs from the minimum phase difference by only 60° . Therefore, it can be determined from the measurement results of the phase difference between the drive voltage and the vibration velocity that the vibration distribution of the thickness vibration is indeed as shown in Figure 2.9.

3. Measurement results of vibration velocity in water:

Due to the underwater measurement of the signal is not stable enough to obtain the measurement data signal strength threshold is reduced, resulting in the distribution of the measurement results is not as clear as the air which, but still can be seen about 11 peaks, and the peak at the centre of the circle is the largest, and the farther away from the centre of the circle, the smaller the peak.

When the driving voltage is $4V_{pp}$, $6V_{pp}$, $8V_{pp}$ and $10V_{pp}$, the vibration velocity at the centre of the circle is $89.47mm/s$, $154.14mm/s$, $227.44mm/s$, $312.97mm/s$. Every $2V_{pp}$ increase of the driving voltage, the vibration speed at the centre of the circle is increased by 74 mm/s , 73.3 mm/s , 85.5 mm/s respectively.

4. Measurement results of sound pressure in water:

The sound pressure distribution is symmetric along the centre, the shape is similar to a parabola symmetric to the y-axis, and the vibration velocity distribution is consistent. This result is similar to the sound pressure distribution obtained in [7]. Numerically, the sound pressure at the centre of the circle of the ultrasonic transducer when the driving voltage is $4V_{pp}$, $6V_{pp}$, $8V_{pp}$ and $10V_{pp}$ is 0.5 MPa , 0.7 MPa , 0.9 MPa and 1.1 MPa respectively, and the sound pressure will increase by 0.2 MPa for every increase of $2V_{pp}$ in driving voltage.

5. Measurement results of propulsion force in water:

The input power is almost linearly related to the propulsion force, and the PZT underwater actuator starts to generate propulsion force only when the input power reaches 0.075 W or more. When the input power reaches 4.64 W , the propulsion force of PZT underwater actuator can reach 1 mN .

6. Measurement results of No-Load-Speed in water:

The NLS of the ultrasonic underwater actuator increased with increasing input voltage. For an input voltage of $12.4V_{pp}$, ultrasonic underwater actuator movement was first detected when the peak NLS value was 6.1 mm/s . For

an input voltage of $89.6V_{pp}$, the peak NLS value was 171.9 mm/s. Because fluid resistance and wire traction affect the NLS, a higher NLS is expected for a given acoustic radiation force if the ultrasonic underwater actuator shape and the wireless power supply are optimized. The high power density of acoustic radiation pressure can be maintained when the scale of the acoustic underwater propulsion system is down with higher frequency because vibration velocity does not greatly change. Thus, a higher NLS is expected for a smaller-scale swimmer.

7. Calculation results of propulsion force:

Calculated results of thrust were obtained as 0.07 mN, 0.13 mN, 0.21 mN, and 0.31 mN when the driving power was 0.08 W (driving voltage $4V_{pp}$), 0.14 W (driving voltage: $6V_{pp}$), 0.30 W (driving voltage $8V_{pp}$), and 0.47 W (driving voltage $10V_{pp}$), respectively. And the calculated results do not correspond to the actual propulsion characteristics.

The reasons may be in these points:

1, The calculation needs the value of the sound speed in the PZT transducer, which is calculated from the equivalent circuit model of longitudinal vibration, thus the value of the sound speed is different from the thickness vibration mode, which leads to the deviation of the calculation result.

2, Because the propulsion force is too small, the measured value will be deviated due to the external influence during the measurement.

3, Due to the lack of consideration of the viscosity, which causes the calculated values to be larger than measured values.

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Chapter 3

Miniature Underwater Actuator using (Bi, Na, Ba)(Ti, Mn)O₃ Disk-shaped Transducer

In this chapter, the environmental concept is introduced for the first time in an ultrasonic underwater actuator, and the ultrasonic underwater actuator is designed using (Bi, Na, Ba)(Ti, Mn)O₃, a non-lead piezoelectric material made in the research laboratory of the DOSHIDA Lab of ASHIKAGA university. Underwater propulsion system using PZT transducer, as a new type of underwater propulsion system, has been studied. (Bi, Na, Ba)(Ti, Mn)O₃, namely BNBTM lead-free piezoelectric ceramics also were noticed. Through comparative experiments, lead-free BNBTM piezoelectric ceramics are not only friendly to the environment and the human body, but it is also that the material properties of BNBTM piezoelectric ceramics are more stable than that of PZT piezoelectric ceramics.

Therefore, this chapter mainly discusses the feasibility of using a BNBTM transducer in the underwater propulsion systems by the measurement of admittance characteristic, propulsion characteristic, vibration characteristics and the No-load speed.

3.1 Miniature Underwater Actuator using BNBTM Transducer

$\text{Pb}(\text{Zr}, \text{Ti})\text{O}_3$, namely PZT piezoelectric ceramics, has been used in novel miniature underwater propulsion systems, and some results have been obtained. However, due to the PZT material characteristics, the transducer has some disadvantages, such as with the increase of vibration velocity, the temperature rises rapidly, the quality factor drops, the stiffness drops and so on, which are unfavorable to the underwater propulsion system that needs steady and continuous propulsion. [1–10] In addition, PZT transducer is harmful to the human body and the environment because it contains lead.

Therefore, $(\text{Bi}, \text{Na}, \text{Ba})(\text{Ti}, \text{Mn})\text{O}_3$, namely BNBTM lead-free piezoelectric ceramics were noticed. The BNBTM transducers have not yet been used for novel underwater propulsion systems. Compared to the PZT transducer, the BNBTM transducer is not only harmless to the human body and the environment due to no lead, but also more slowly temperature rise, stable quality factors and stable stiffness against the increases of vibration velocity. The stable BNBTM characteristics can help the underwater propulsion system to provide steady and continuous propulsion.

The purpose of this research is to set up a novel underwater propulsion system with BNBTM transducer to investigate the feasibility of the BNBTM transducer used in the novel underwater propulsion system. A thickness-vibration-mode BNBTM transducer was used for the novel underwater propulsion system. Firstly, in order to obtain the driving frequency of the maximum vibration velocity of the BNBTM transducer and the resistance under the driving frequency, the admittance characteristics of the transducer were measured. Secondly, in order to obtain the value of vibration velocity required for ARP calculation, the amplitude distribution along the specified route on the BNBTM surface was measured. Then, in order to compare the calculated propulsion value, the actual propulsion of the BNBTM actuator was measured by a force sensor. Finally, the feasibility of using BNBTM transducer in the novel underwater propulsion system was evaluated by measuring the BNBTM actuator's no load speed at different driving voltages and at the same driving frequency.

3.2 Design of underwater actuator

If the BNBTM transducer is placed directly in water, then according to the principle of acoustic radiation force, the two sides of the transducer will receive the same propulsion in opposite directions. These two forces cancel

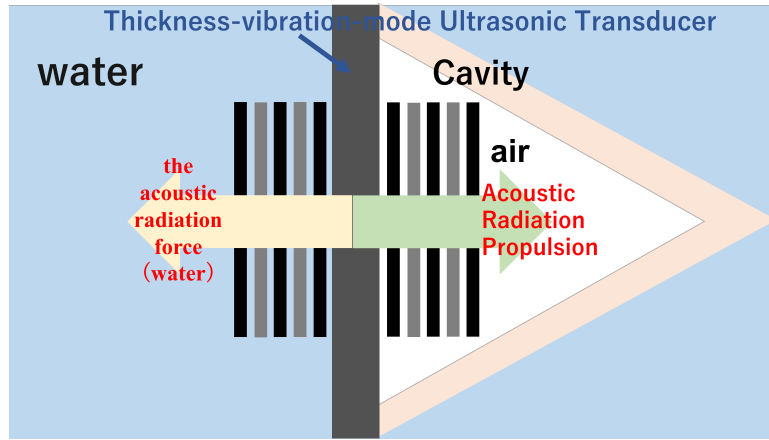


Figure 3.1: Structure of the underwater propulsion system using the BNBTM transducer, the actuator's cross section, and acoustic radiation propulsion principle diagram.

each other out, so that no propulsion can be generated. As shown in Figure 3.1, there is space on the right side of the transducer to store air so that the right surface of the BNBTM transducer is in full contact with air. The left side of the BNBTM transducer is in full contact with water. According to the design shown in Figure 3.1, the side of the transducer where contact with water will be subjected to acoustic radiation force. And the side in contact with air does not produce acoustic radiation force. Thus the force on the water from the side in contact with the water in turn generates a propulsion force perpendicular to the surface of the vibrator.



Figure 3.2: Thickness vibration mode disk-shaped BNBTM transducer; 8.4 mm diameter; 0.5 mm thickness

As shown in Figure 3.2, a thickness-vibration-mode BNBTM transducer (M13R from DOSHIDA Lab.) was used to set up a small underwater propulsion system. The size of BNBTM is a round plate with a diameter of 8.4 mm and a thickness of 0.5 mm. The diameter of the electrode section is approximately 5 mm.

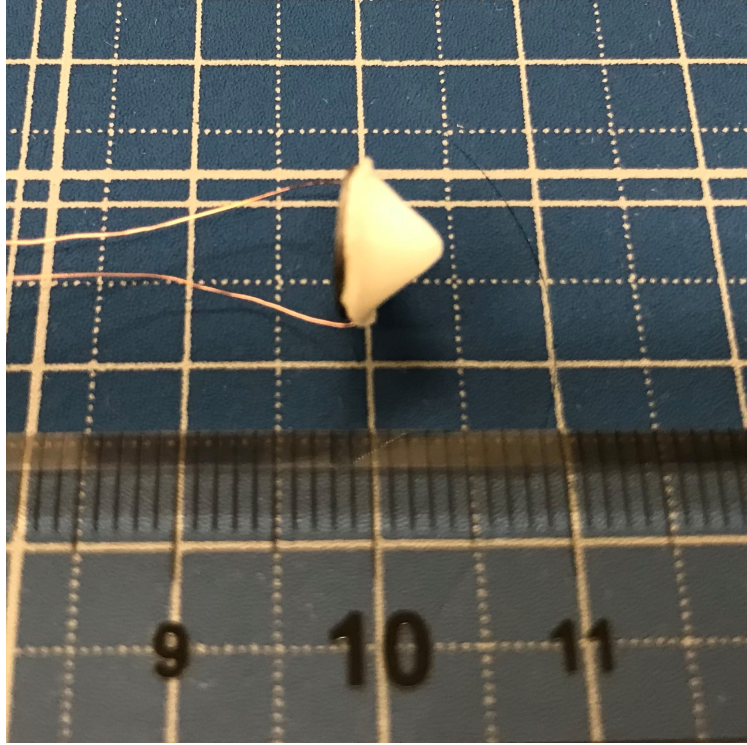


Figure 3.3: actuator with the underwater propulsion system using the BNBTM transducer. The total length of the actuator is about 7 mm.

To investigate the acoustic underwater propulsion system using the BNBTM transducer, a actuator with the underwater propulsion system was demonstrated as shown in Figure 3.3. A hollow plastic cone with a wall thickness of 1 mm was attached to the right side of the transducer to inhibit the acoustic radiation force on the right side of the transducer. The cone is made of acrylonitrile butadiene styrene (ABS) by the 3D Painter (MUTOH, Value3D MagiX MF-2200D). The horizontal acoustic radiation force is generated on the left of actuator. The length of the entire actuator was 7 mm.

Moreover, a foam sheet was attached to the top of this actuator to maintain balance in the water, then a green plastic sheet was attached to the top of the foam to make the swimmer easily recognized by the image recognition program of MATLAB. looking down from the top, as shown in Figure 3.9.

3.3 Admittance Characteristics of BNBTM Transducer

The admittance characteristics of the BNBTM transducer were measured using an impedance analyzer. As shown in Figure 3.4, the distributions were shown by blue and red lines, when the BNBTM transducer was in air and in water. Comparing the conductance characteristics in air and water, it can be seen that the resonance frequency in water is almost unchanged compared to that in air, which is about 4.22 MHz, while the conductance at the resonance frequency is 114.8 mS in air and 45.3 mS in water, which is a difference of 69.5 mS.

Neglecting the loss of the induced impedance as well as the air loading, the vibrational loss of the BNBTM actuator at the resonance frequency is about the loss of $R_m = 8.7\Omega$ resistance at the resonance frequency shown in the measurements of the admittance characteristics in air. The power on the resistance represented by the $R_{w-m} = 13.4\Omega$ difference between the resistance at the resonance frequency shown in the measurement results of the admittance characteristics in water and the resistance in air represents the output power of the BNBTM actuator in water. This means that the output efficiency of the BNBTM actuator is 60.7%.

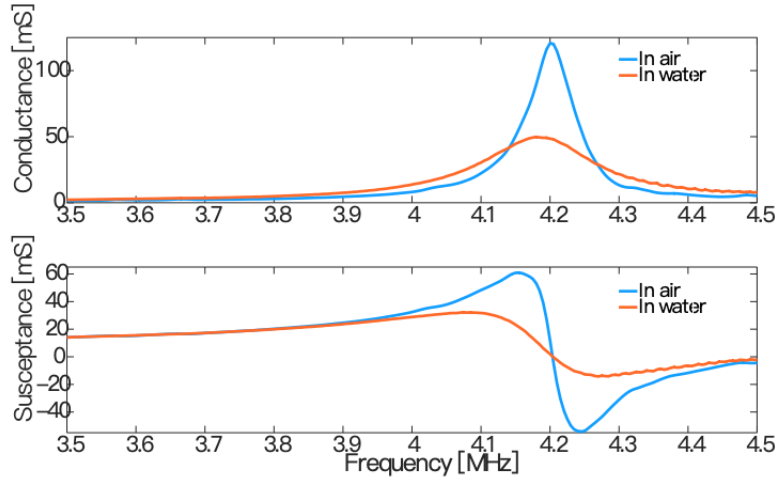


Figure 3.4: a Admittance characteristics of disc-shaped BNBTM transducer thickness vibration mode. The resonance frequency of the BNBTM transducer is about 4.22 MHz. The blue curve is the measurement result of the BNBTM transducer in air, and the orange curve is the measurement result of the BNBTM transducer in water.

3.4 Vibration Amplitude Distribution of BNBTM Transducer

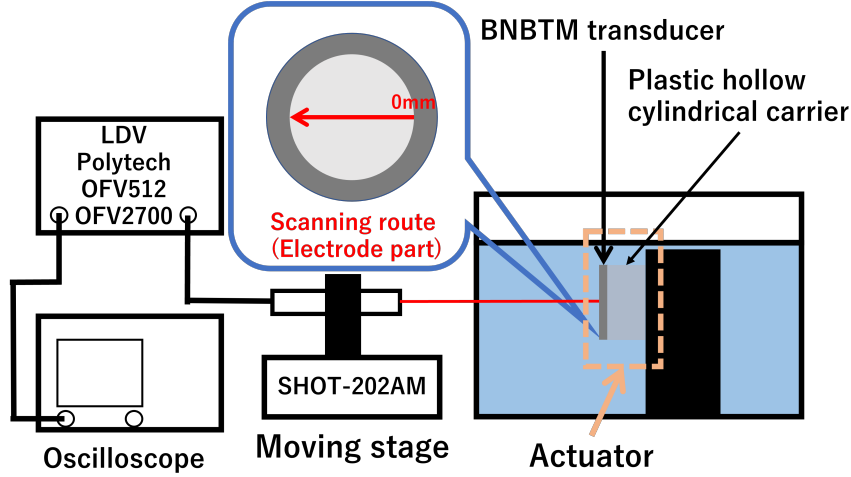


Figure 3.5: Measurement system of vibration amplitude.

The BNBTM transducer surface's vibration amplitude was measured using a laser Doppler velocimeter (PolyTech, OFV512, OFV2700). The measurement method is shown in Figure 3.5, the BNBTM transducer was attached on one side of an ABS plastic hollow cylindrical carrier. The size of this carrier is 9 mm in outer diameter, 6 mm in inner diameter, and 10 mm in height. The other side of the carrier was attached to the metal block, then the metal block was placed at the bottom of the glass container filled with water. Finally, the vibration amplitude on the specified route on the BNBTM transducer surface was measured by the LDV. As shown in Fig. 3.6, the distribution was displayed by the red line, when the BNBTM transducer was driven at $33.4V_{pp}$ driving voltage and at 4.22 MHz driving frequency in water. The position 0 mm is the point of the exposed electrode part marginal of the transducer. The electrode diameter was 6 mm, so that the 3 mm position in Figure 3.6 is the center of the BNBTM transducer. At an interval of 0.2 mm, the vibration amplitude values of 31 positions were measured along the measurement route. From these vibration amplitude values, it can be calculated that the average vibration amplitude of the BNBTM transducer surface is 40.3 nm, and by Equation. 3.1 it can be obtained that the maximum average vibration velocity is $1.02ms^{-1}$

$$v = A * 2 * \pi * f. \quad (3.1)$$

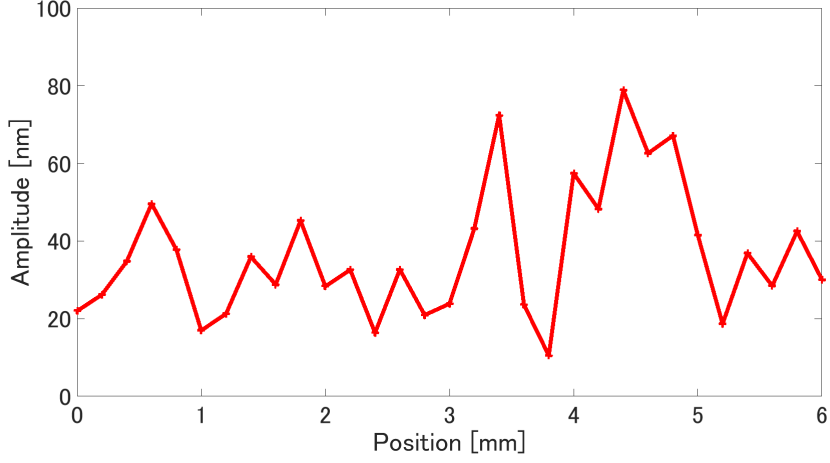


Figure 3.6: Vibration amplitude distribution on a scanning path of the BNBTM transducer. The driving frequency is 4.22 MHz and the driving voltage is 33.4V_{pp}

However, v is the vibration velocity, A is the vibration amplitude, f is the driving frequency.

For the estimation of the ARP, the acoustic radiation pressure discussed by Kamakura with the time average was needed. [11,12] The radiation pressure per unit area P_a can be calculated by

$$P_a = \frac{2E_i}{(1+Z)^2} (1+Z^2 - 2Z\frac{c_1}{c_2}), \quad (3.2)$$

$$Z = \frac{\rho_1 c_1}{\rho_2 c_2}, \quad (3.3)$$

$$E_i = \frac{(\rho_2 c_2 2\pi f A)^2}{2\rho_1 c_1^2}. \quad (3.4)$$

However, Z is the mechanical impedance ratio with solid and fluid, the density and the acoustic velocity of solid are ρ_1 and c_1 , the density and the acoustic velocity of fluid are ρ_2 and c_2 , E_i is acoustic energy density with time average. From Equation. 3.1, it can be concluded that the vibration velocity can be maintained by increasing the vibration frequency even if the vibration amplitude decreases due to the decrease of the vibrator's size. From Equation 3.4, it can be concluded that E_i has a linear relationship with v_2 . Thus, the acoustic energy density can be maintained while reducing the vibrator's size. Since acoustic radiation pressure per unit area P_a , F_{ARF} can be calculated by

$$F_{ARF} = P_a S. \quad (3.5)$$

However, S is the electrode area. The ARP is numerically equal to F_{ARF} but in the opposite direction, thus the ARP of the propulsion system using the BNBTM transducer can be calculated by

$$F_{ARP} = -F_{ARF} \quad (3.6)$$

However, F_{ARP} is the ARP of the underwater propulsion system using the BNBTM transducer.

In this paper, the calculation parameters were $\rho_1 = 5650\text{kgm}^{-3}$, $\rho_2 = 997\text{kgm}^{-3}$, $c_1 = 4630\text{ms}^{-1}$, $c_2 = 1484\text{ms}^{-1}$. The calculation result is shown by the red line in Figure 3.8.

3.5 Propulsion force characteristics

The propulsion of the BNBTM actuator with the underwater propulsion system was measured using a force sensor (Honeywell, FSG005WNPB). The sensor head was connected to the BNBTM actuator, as shown in Figure 3.7. This BNBTM actuator is the same as the actuator used to measure the vibration amplitude. The measurement result is shown by the blue round in Figure 3.8. Comparing the measurement and calculation propulsion values in Figure 3.8, it can be seen that there is still a large deviation between the measurement and calculation results. This calculation only considers the results of the time-averaged radiation pressure, and does not consider the viscosity and compressibility of water. Thus, the calculation method requires it to be further improved.

From the measurement results shown in the figure 3.8, it can be learnt that the BNBTM underwater actuator starts to generate thrust at an input voltage of about $6V_{pp}$, 0.2 mN at a driving voltage of $8.4V_{pp}$, 1.18 mN at a driving voltage of $20.4V_{pp}$, and 3.89 mN at a driving voltage of $36.4V_{pp}$. It can be seen that the thrust of the BNBTM underwater actuator, as the voltage rises, rises faster before reaching the limit.

3.6 No-load speed of the BNBTM underwater actuator

For the measurement of the BNBTM actuator's no load speed, the motion videos of the BNBTM actuator were collected. Then the position of

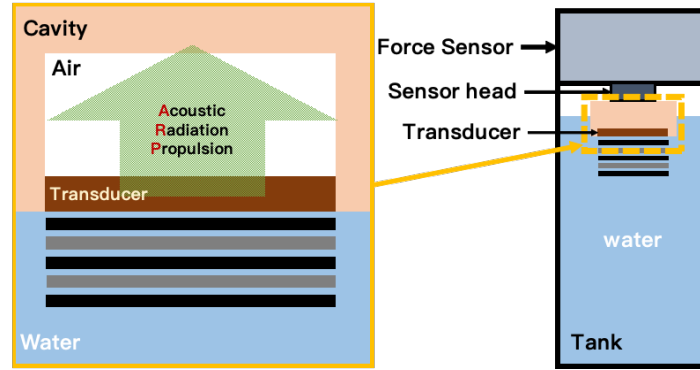


Figure 3.7: Schematic diagram of zero speed propulsion measurement setup. Head of actuator and sensor is fixed.

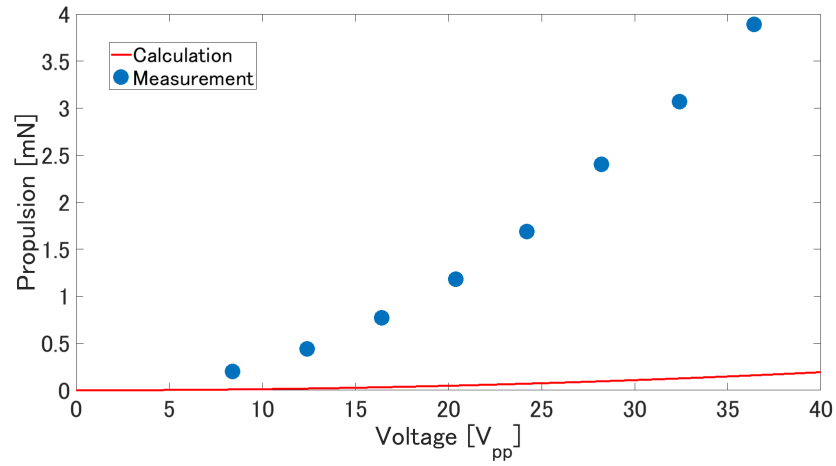


Figure 3.8: The calculation result and measurement result of the BNBTM transducer's propulsion.

the BNBTM actuator in each frame of the video was identified by the image recognition program of MATLAB. As shown in Figure 3.9, the red dot indicates that the BNBTM actuator has been identified. The BNBTM actuator's no load speed was calculated by using the displacement distance of the red dot between each frame and the time interval between each frame. Figure 3.9 shows the BNBTM actuator's movement from the top view. The measurement was carried out under different driving voltages at 4.22 MHz driving frequency.

The speed change curves of the BNBTM actuator at different driving voltages and 4.22 MHz driving frequency were shown in Figure 3.10. It can be seen that as the driving voltage rises, the curve of the accelerating section becomes steeper. The speed reached at the maximum around 0.7 s and then the feeding wires obstructed the actuator motion, so that the speed was slightly decreased. Between 1.7 and 2.4 s, the driving voltage shut down, then the speed decreased to null in an instant. The maximum no load speed value of each speed change curve in Fig. 10 was taken out; The relationship between driving voltage and the BNBTM actuator's no load speed was obtained as shown in Figure 3.11. Three points of null speed in Figure 3.11 indicates that when the driving voltages were $8.4V_{pp}$, $12.4V_{pp}$, and $16.4V_{pp}$, the BNBTM actuator could not provide enough propulsion to drive the BNBTM actuator in water.

The conductance of the BNBTM transducer at the resonance frequency was 45.30mS, as shown in Figure 3.4. The driving electric power can be calculated by the driving voltage and conductance. The relationship between the BNBTM actuator's no load speed and input electric power was obtained as shown in Figure 3.1 by the blue circle. The relationship between the propulsion of the BNBTM actuator and input electric power was also obtained as shown in Figure 3.12 by the orange square. It can be concluded from Figure 3.12 that the BNBTM actuator required only about 1 mN propulsion for motion; The BNBTM actuator required only about 0.4 W of electric power input to provide 1 mN propulsion. The result shows that the BNBTM actuator can be driven by batteries. It can be seen that the use of BNBTM transducer will be helpful for wireless control.

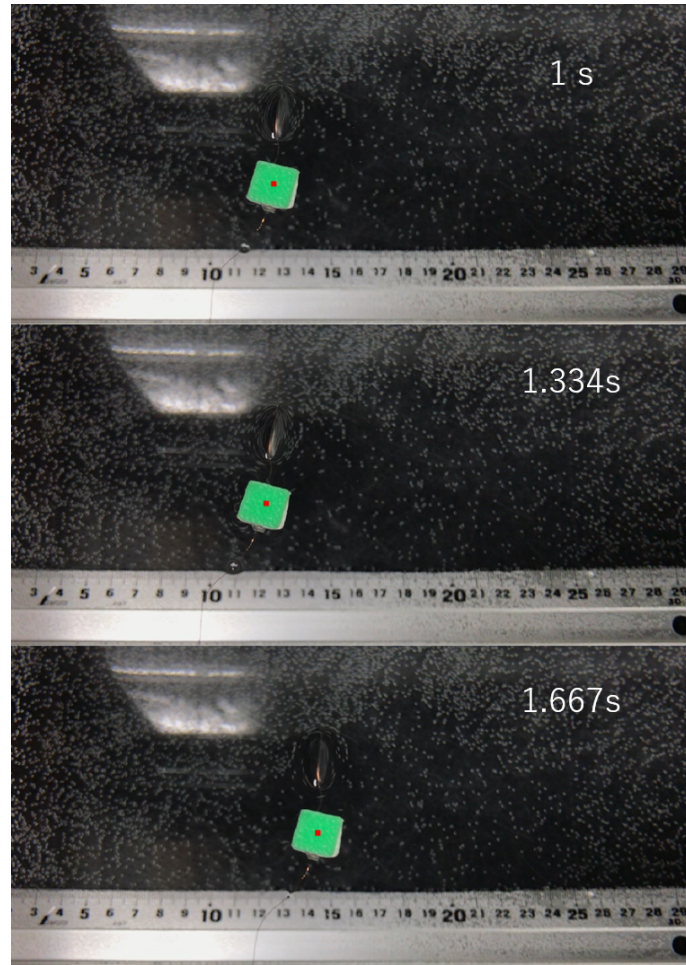


Figure 3.9: Actuator's movement in water from the top view. A green plastic sheet was attached to the top of the BNBTM actuator to maintain balance in the water. The red dots indicate that the actuator has been identified by the image recognition program of MATLAB. The driving voltage is $36.4V_{pp}$, driving frequency is 4.22 MHz.

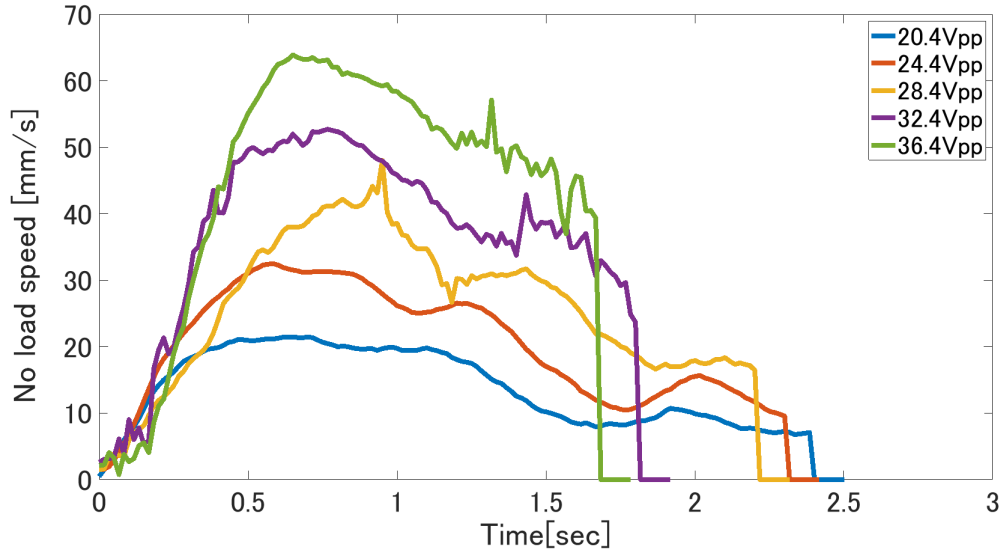


Figure 3.10: Speed change curve of the swimmer at driving voltages of $20.4V_{pp}$, $24.4V_{pp}$, $28.4V_{pp}$, $32.4V_{pp}$, and $36.4V_{pp}$.

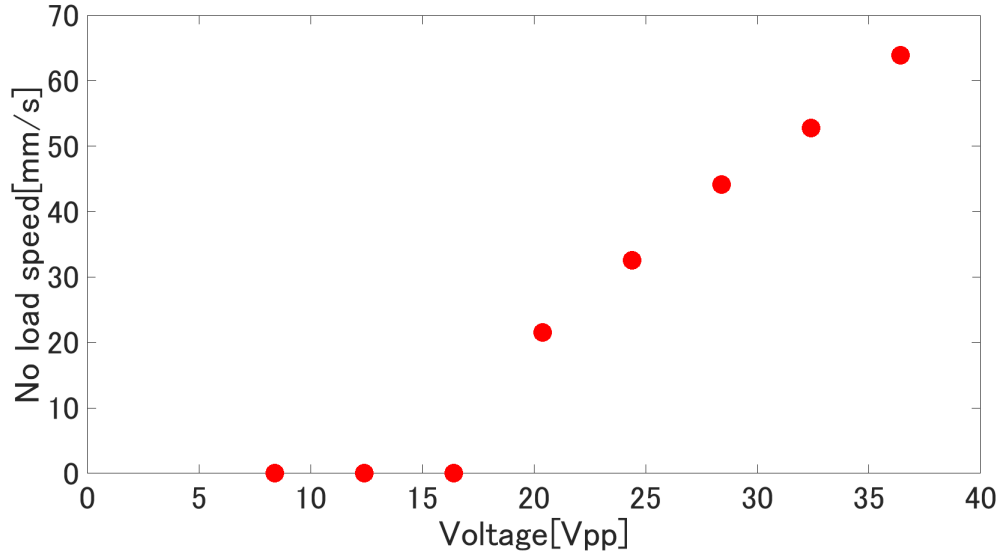


Figure 3.11: Maximum no load speed of the swimmer at 4.22 MHz.

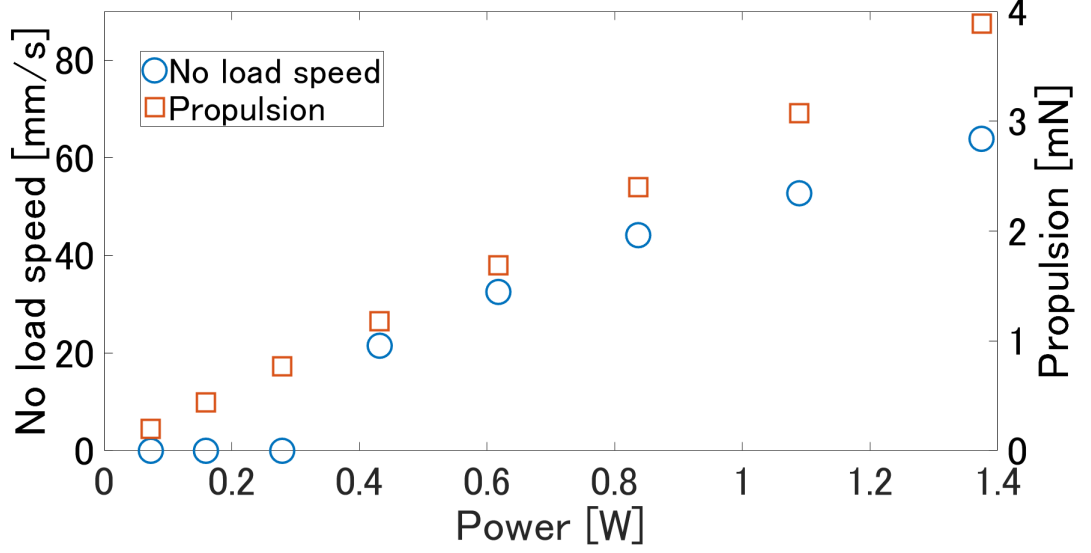


Figure 3.12: Maximum no load speed of the BNBTM actuator, and the propulsion of the BNBTM actuator at $4.22MHz$ under different driving powers.

3.7 Conclusion

The BNBTM transducer was used for the first time in the novel underwater propulsion system in this paper. The BNBTM actuator was demonstrated by the propulsion measurement. The BNBTM swimmer was demonstrated by no load speed measurement. From measurement results, the BNBTM actuator at 0.4 W input electric power can provide about 1 mN propulsion to drive the BNBTM actuator. Thus, the feasibility of the BNBTM transducer used in the underwater propulsion system has been verified. By comparing the calculated and measured results of the propulsion, the ARP calculation method still has a lot of defects. In the future, the BNBTM transducer will then be further miniaturized to build the underwater propulsion system and the ARP calculation method will be further improved.

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Chapter 4

Miniature Underwater Actuator using a PZT Ring Transducer

The Miniature ultrasonic underwater propulsion system with disk shaped vibrator was introduced in the last two chapters, and although it has good underwater propulsion capability, it still has several problems. Structurally, because the vibrator is a disk-shaped design, small holes must be cut on the side of the actuator for wire connection in case of wired connection to the external power supply, which results in the wire bending to form a protrusion at the small hole of the actuator that is not easy to maintain the balance, and considering that it is an application with vascular robotics as the ultimate goal, the protrusion on the outer side is prone to cut the vascular wall and cause damages. Regarding the vibration characteristics, it is known from the vibration distribution measurements of the disk-shaped vibrator in the previous two chapters that there are multiple peak distributions and that the vibration velocity at the troughs is extremely small, meaning that little propulsion force is generated at these troughs, which affects the drive efficiency. Therefore, ring transducers were applied to improve the above problems.

This study investigates a scheme utilizing a ring transducer for an acoustic underwater propulsion system. Acoustic underwater propulsion systems are well suited for the inspection and repair of underwater robots due to their small size, high power density, and simple structure. Previous research has focused on self-propelled swimmers utilizing disc transducers. However, the radial vibration component of disc transducers makes it difficult to provide propulsion for an acoustic underwater propulsion system driven by an acous-

tic driving force. Pure longitudinal vibration requires a greater thickness to achieve the same vibration area, resulting in higher impedance and reduced driving efficiency. In this chapter, simulation, and measurements of vibration distribution demonstrate that a ring transducer exhibits a vibration distribution closely resembling pure longitudinal vibration. A prototype underwater actuator using a ring transducer was fabricated for experimental evaluation through measurements of admittance characteristics, zero-speed propulsion, and no-load speed in water.

4.1 Ring Ultrasonic Transducer

In this study, a PZT ring transducer with a vibration distribution closely resembling pure longitudinal vibration was studied. To design a plate transducer with an approximate longitudinal vibration distribution, the vibration mode of a PZT ring transducer was simulated through finite element method analysis software (COMSOL Multiphysics 5.6).

The two-dimensional axisymmetric model, depicted in Figure 4.1, is 0.9 mm wide and 1.4 mm high with PZT (FUJI Ceramics Corporation, PZTc-204) material and electrodes situated at the top and bottom for natural frequency analysis. The complete simulation model is depicted in Figure 4.1. It features a PZT ring transducer characterized by an outer diameter of 5 mm, an inner diameter of 3.2 mm, and a thickness of 1.4 mm.

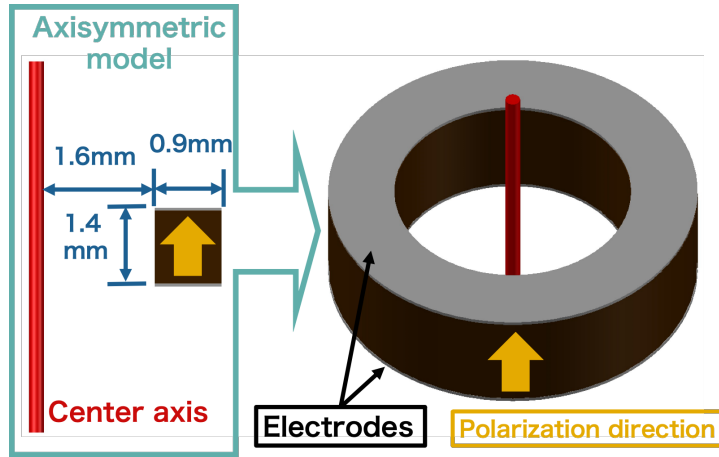


Figure 4.1: Analysis conditions for the ring transducer. The left diagram illustrates a two-dimensional symmetrical model surrounding the central axis in the ring transducer, while the simulation model of the ring transducer is depicted on the right. The electrodes are positioned on both the top and bottom surfaces of the ring transducer.

The simulation results at a frequency of 1.11 MHz are presented in Figure 4.2. Figure 4.2-(a) shows the ring transducer contracted in the thickness direction, while Figure 4.2-(b) shows the state of the ring transducer stretched in the thickness direction. The color gradient from blue to red represents increasing displacement. The simulation results demonstrate that the ring transducer made of PZT material with an outer diameter of 5 mm, an inner diameter of 3.2 mm, and a thickness of 1.4 mm exhibits a vibration distribution of approximately pure longitudinal vibration when driven at a frequency of 1.11 MHz.

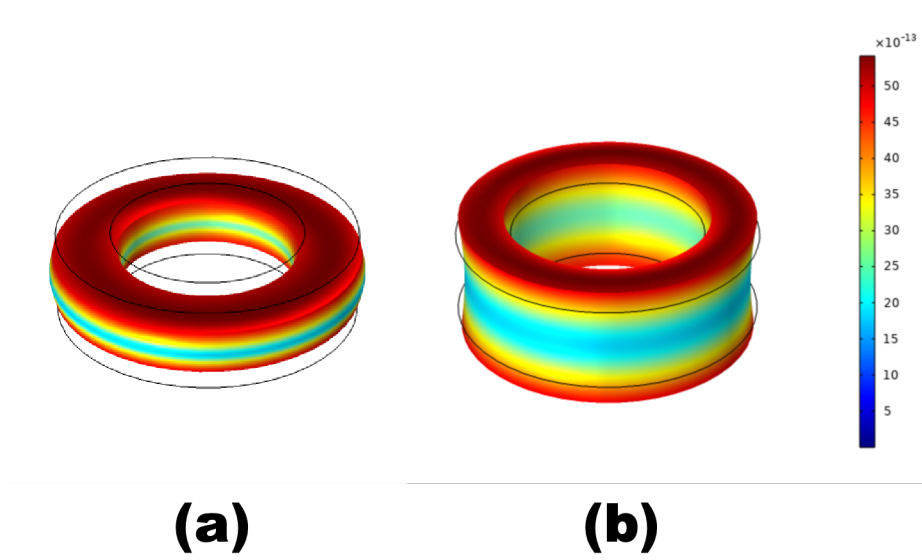


Figure 4.2: Vibration mode simulation of the PZT ring transducer at 1.11 MHz (outer diameter 5 mm; inner diameter 3.2 mm; thickness 1.4 mm).

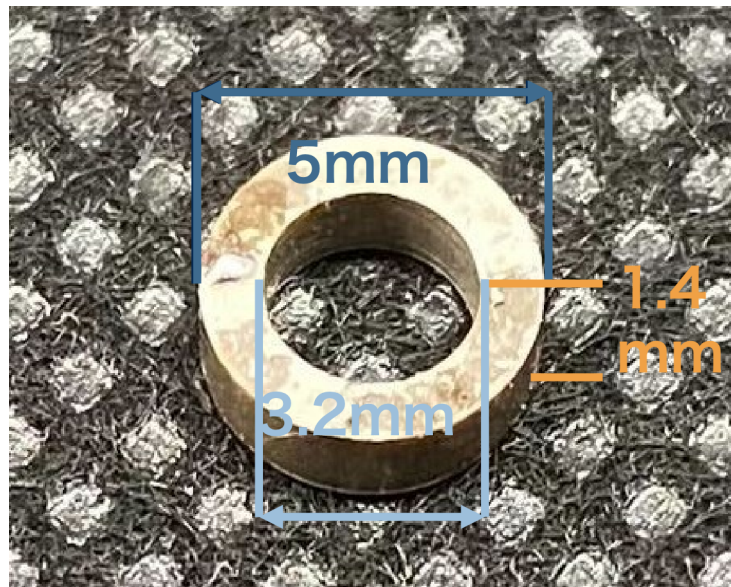


Figure 4.3: PZT ring ultrasonic transducer (outer diameter 5 mm; inner diameter 3.2 mm; thickness 1.4 mm).

A PZT ring transducer with an outer diameter of 5 mm, an inner diameter of 3.2 mm, and a thickness of 1.4 mm with electrodes placed on the bottom and top surfaces was fabricated to validate the simulation results based on the obtained outcomes, as shown in Figure 4.3.

4.1.1 Admittance Characteristics

The admittance characteristics of the PZT ring transducer in the air were measured using an impedance analyzer (Agilent 4294A), as depicted in Figure 4.4. At the resonance frequency of 1.09 MHz, the PZT ring transducer exhibited a conductance value of 32.5 mS in air.

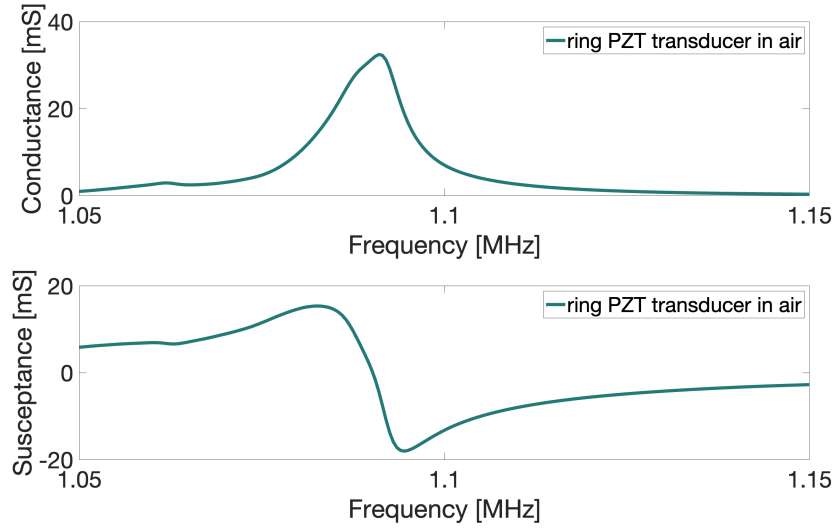


Figure 4.4: Admittance characteristics of the PZT ring transducer in air. The conductance at the resonance frequency of 1.09 MHz is 32.5 mS.

4.1.2 Vibration Velocity Distribution

The vibration velocities on the surface of the PZT ring transducer were measured at the resonance frequency of 1.09 MHz and different voltages using a laser Doppler vibrometer (Polytec, OFV512, OFV2700). The setup for measuring vibration velocity is depicted in Figure 4.5. The vibration velocities of the electrode surface in the direction of the diameter were measured by placing the PZT ring transducer on top of two pieces of rubber. With the same fixation, the vibration velocities of the peripheral surface of the PZT ring transducer in the thickness direction were also measured.

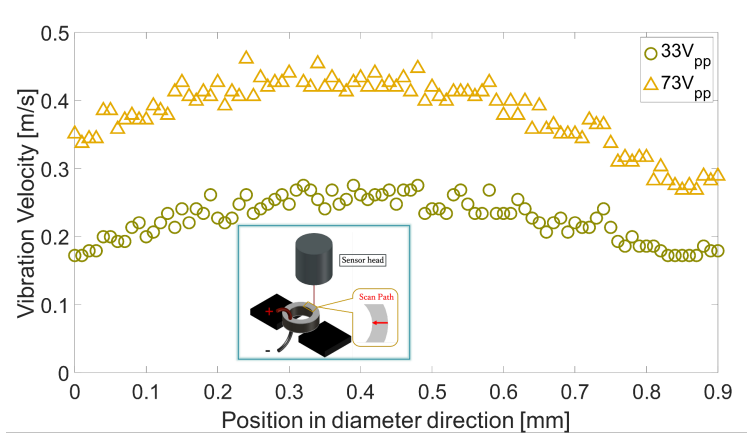


Figure 4.5: Vibration velocities of the PZT transducer's electrode surface. The PZT ring transducer was placed on top of two rubber sheets and measured with a laser Doppler vibrometer at different voltages along the electrode surface diameter direction (the scan path indicated by the red arrow). The vibration velocity distribution presents a peak shape with a difference of 0.18ms^{-1} between the maximum and the minimum vibration velocities at $73V_{pp}$ driving voltage.

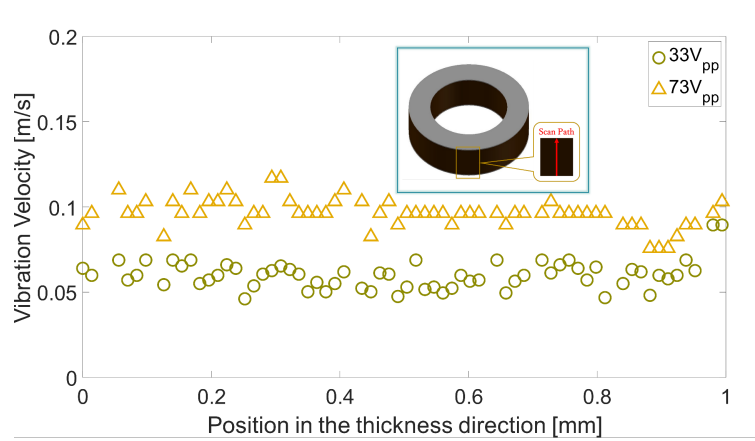


Figure 4.6: Radial vibration velocity at the peripheral surface of the PZT ring transducer. The PZT ring transducer was placed on top of two rubber sheets and measured with a laser Doppler vibrometer at different voltages in the direction of thickness (the scan path indicated by the red arrow). The radial vibration velocity at the peripheral surface of the PZT ring transducer was an average of 0.1ms^{-1} at $73V_{pp}$ driving voltage

The results for the vibration velocity of the electrode surface of the PZT ring transducer are shown in Figure 4.5. The uneven spacing of the measuring points in the figure can be attributed to surface defects on the PZT ring transducer, such as scratches and dirt. These imperfections hinder accurate measurements of vibration velocity. When driven by the voltage of 33 V_{pp} , a peak appears in the vibration velocity distribution with a maximum value of 0.27ms^{-1} and a difference of 0.11ms^{-1} between the maximum and minimum values. On increasing to a driving voltage of 73V_{pp} , a peak appears in the vibration velocity distribution with a maximum value of 0.46ms^{-1} and a difference of 0.18ms^{-1} between maximum and minimum values.

The vibration velocity measurements in the radial direction on the peripheral surface of the PZT ring transducer are shown in Figure 4.6. The vibration velocity distributions on the peripheral surface of the PZT ring transducer shown in Figure 4.6 are approximately linear. When the driving voltage is 33V_{pp} the average vibration velocity is 0.06ms^{-1} , while when the driving voltage is increased to 73V_{pp} the average vibration velocity increases to 0.1ms^{-1} . At a driving voltage of 73V_{pp} , the ratio of the maximum vibration velocity of 0.46ms^{-1} in the longitudinal direction to the average vibration velocity of 0.1ms^{-1} in the radial direction is 4.6:1.

The results of vibration velocity measurements confirm that the vibration velocity distribution of the PZT ring transducer closely resembles pure longitudinal vibration. A PZT ring transducer with a predominantly longitudinal vibration distribution was designed and fabricated, and the validity of the simulation results was confirmed by measuring the admittance characteristics and vibration velocities.

4.2 Prototype Underwater Actuator

The structural design of the swimmer is shown in Figure 4.7. A tubular component was positioned at the center of the PZT ring transducer, while a cylinder consisting of two semicylindrical components was placed around the PZT ring transducer to securely hold it in place, forming a sealed swimmer configuration where one side of the PZT ring transducer remains exposed to air.

The wiring of the electrode on the air-contacting side of the PZT ring transducer passed through the hole in the central tubular component and out from the center of the tubular component to the rear side of the swimmer, while the wiring of the electrode on the water-contacting side was exposed directly to water. An adhesive (Cemedine EP001N) was used to secure the PZT ring transducer and to assemble the swimmer structure to ensure that

the swimmer is waterproof.

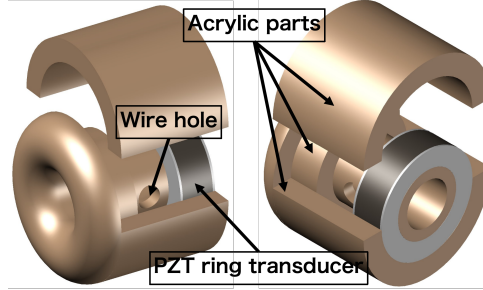


Figure 4.7: Swimmer design (front view left, rear view right). A tubular component was positioned at the center of the PZT ring transducer, while a cylinder consisting of two semi-cylindrical components was placed around the PZT ring transducer to hold it in place. The wiring of the electrode on the air-contacting side of the PZT ring transducer passed through the hole in the central tubular component and out from the center of the tubular component to the rear side of the swimmer, while the wiring of the electrode on the water-contacting side was exposed directly to water.

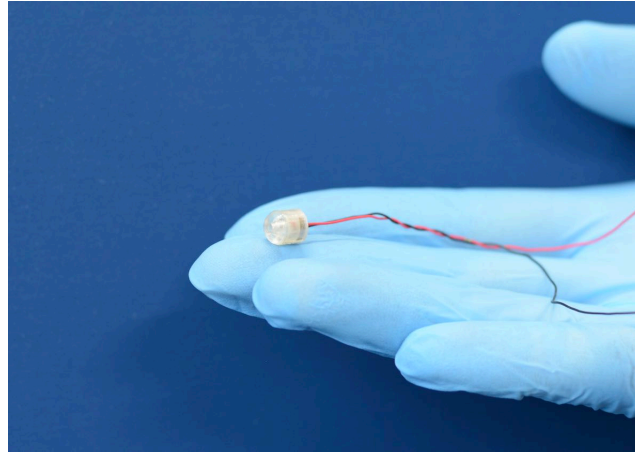


Figure 4.8: The prototype swimmer.

All fixing elements were fabricated using acrylic resin with a 3D printer. The swimmer had a length of 7 mm and a diameter of 7 mm, as shown in Figure 4.8. The structural configuration of the swimmer induced an asymmetry in the propulsive forces acting on either side of the PZT ring transducer, resulting in a propulsive force directed towards the surface of the transducer.

4.2.1 Admittance characteristics

The admittance characteristics of the swimmer are presented in Figure 4.9, where the blue and orange lines represent the swimmer in air and water, respectively. When the PZT ring transducer was fixed in the swimmer the resonance frequency was observed to be 1.08 MHz, accompanied by a decrease in conductance at the resonance frequency to 13.1 mS. Submerging the swimmer in water resulted in a shift of the resonance frequency to 1.07 MHz and a further reduction in conductance at the resonance frequency to 5.2 mS.

Based on the measured results for admittance characteristics, it is evident that the resonance frequency exhibits a difference of 0.01 MHz due to the fixed mode and the influence of water. However, the conductance under resonance frequency displays high sensitivity towards both fixed mode and water, leading to an apparent decrease. As conductance is indicative of the vibration velocity on the surface of the transducer, proportional to current, reduced conductance leads to decreased vibration velocity under identical driving voltage conditions.

Neglecting the loss of the induced impedance as well as the air loading, the vibrational loss of the PZT ring actuator at the resonance frequency is about the loss of $R_m = 76.3\Omega$ resistance at the resonance frequency shown in the measurements of the admittance characteristics in air. The power on the resistance represented by the $R_{w-m} = 116.0\Omega$ difference between the resistance at the resonance frequency shown in the measurement results of the admittance characteristics in water and the resistance in air represents the output power of the PZT ring actuator in water. This means that the output efficiency of PZT ring actuator is 60.3%.

4.2.2 Vibration Velocity Distribution

The vibration velocity along the diameter direction on the exposed side electrode surface of the PZT ring transducer, secured to the swimmer, was measured at a resonance frequency of 1.08 MHz and different voltages using a laser Doppler vibrometer (Polytec, OFV512, OFV2700). As shown in Figure 4.10, the maximum vibration velocity on the electrode surface after fixing the PZT ring transducer to the swimmer was approximately 0.1ms^{-1} slower than when the transducer was measured alone. The fixation method caused a shift in the vibration velocity distribution from a peak distribution to a sloped distribution.

After fixation, the reduction in vibration velocity on the electrode surface of the PZT ring transducer is attributed to the applied pressure on the in-

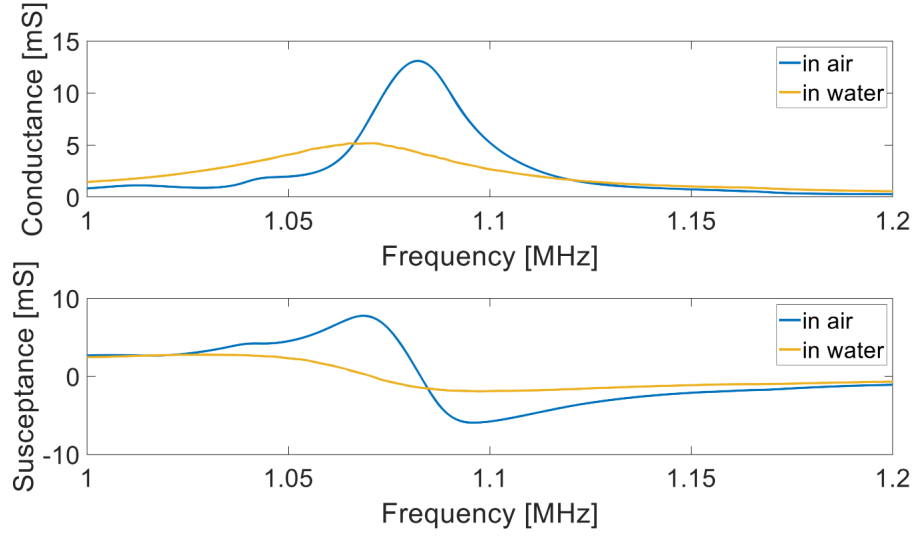


Figure 4.9: Admittance characteristics of the swimmer in air and water. The result in air is shown by a blue line and the result in water is shown by an orange line. In air conductance at the resonance frequency of 1.08 MHz is 13.1 mS. In water conductance at the resonance frequency of 1.07 MHz is 5.2 mS.

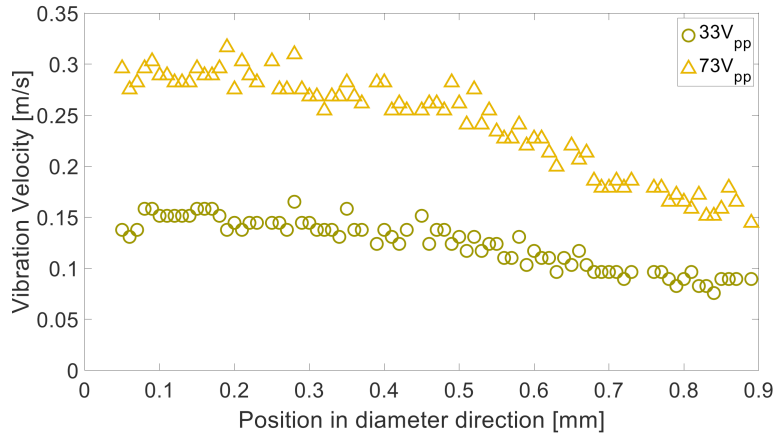


Figure 4.10: Vibration velocities of the surface of the fixed PZT transducer. The PZT ring transducer was fastened to the swimmer and measured with a laser Doppler vibrometer at different voltages along the electrode surface diameter direction (the scan path was the same as in the vibration velocity measurement of the PZT ring transducer). The vibration velocity distribution is close to a slope with a difference of 0.17ms^{-1} between maximum and minimum vibration velocities at $73V_{pp}$ driving voltage.

ner and outer diameter surfaces, which suppresses elastic deformation. The measured conductance results also indicate a decrease in conductance at the resonance frequency of the fixed PZT ring transducer compared with before fixation. Specifically, under the same voltage drive, there was a decrease in current. Given that current was directly proportional to vibration velocity, this contributes to the observed slowing down of the vibration velocity for the fixed PZT ring transducer compared with its unfixed state.

When driven by a voltage of 33 V_{pp}, the maximum vibration velocity reached 0.16ms⁻¹, with a difference of 0.09ms⁻¹ between the maximum and minimum values. Increasing the driving voltage to 73V_{pp} resulted in a maximum vibration velocity of 0.32ms⁻¹, with a difference of 0.17ms⁻¹ between the maximum and minimum values. Measurements of the vibration velocity distribution demonstrate that the PZT ring transducer maintains a vibration distribution that approximates purely longitudinal vibration even after being fixed to the swimmer.

4.2.3 Zero-Speed Propulsion

The ZSP of the swimmer driven at various voltages at a resonance frequency of 1.07 MHz in water was measured using a force gauge (Mark-10, M5-012), as shown in Figure 4.11. The swimmer was attached to the sensor head. The results illustrating the swimmer's ZSP are presented in Figure 4.11. A roughly linear relationship between the ZSP and driving voltage is observed. When the drive voltage is below 17V_{pp}, the ZSP becomes indiscernible for measurement, resulting in an NSP of 0 mN. At a driving voltage of 80V_{pp}, the swimmer achieves a ZSP of 1 mN at the resonance frequency of 1.07 MHz. The ZSP of the swimmer was measured independent of water resistance and wire traction, thereby confirming the efficacy of the acoustic underwater propulsion system using a PZT ring transducer to generate the acoustic propulsive force.

4.2.4 No-load Speed

To maintain the swimmer's equilibrium in water, a foam float and a carbon fiber kickboard with a diameter of 3 cm were placed on top of the swimmer, as shown in Figure 4.12. The identification point was marked on the kickboard with a square of red tape. The measurement technique for NLS is shown in Figure 4.12. A camera was employed to capture a motion video of a swimmer driven by varying voltages at a resonance frequency of 1.07 MHz, followed by analysis using an image recognition program in MATLAB to evaluate the NLS.

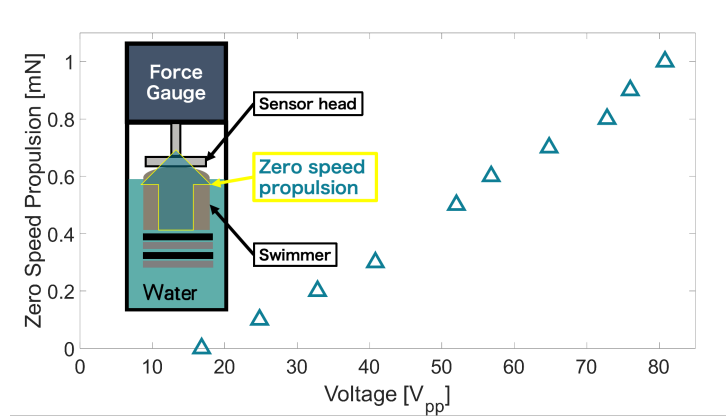


Figure 4.11: Zero-speed propulsion of the swimmer under different input voltages at 1.07 MHz. At a driving voltage of $80V_{pp}$, the swimmer achieved 1 mN.

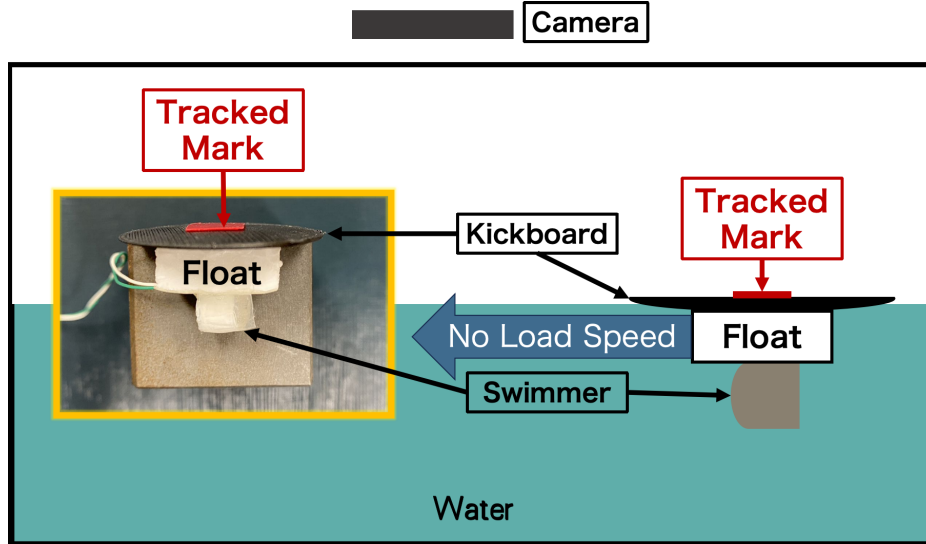


Figure 4.12: Experimental diagram of no-load speed measurement. A foam float and a kickboard made of carbon fiber are placed on top of the swimmer to maintain balance. A tracked mark is used to record the movement of the swimmer with a camera.

The results of NLS measurement for the swimmer at the resonance frequency of 1.07 MHz are depicted by distinct color curves under varying input voltages (Figure 4.13). The activation of the input voltage occurs at approximately 1.3 s, leading to a gradual acceleration of the swimmer with minimal wire traction until it reaches the maximum NLS at around 4 s. With an increase in input voltage, a corresponding augmentation in acceleration is observed. The NLS gradually decreases after 4 s because the traction of the wire increases with increasing distance. Ultimately, after the swimmer had been propelled for approximately 15 s the process of image recognition was terminated, resulting in the NLS value being reset to 0ms^{-1} .

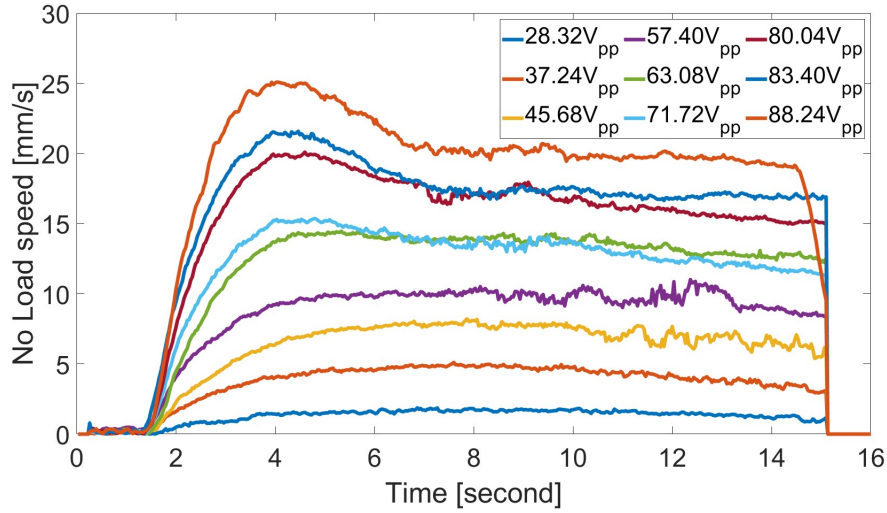


Figure 4.13: NLS of the swimmer under different input voltages at a resonance frequency of 1.07 MHz (1.3 s, input driving voltage; maximum NLS reached at 4 s; image recognition program terminated at 15 s).

The measurements of the maximum NLS of the swimmer at a resonance frequency of 1.07 MHz, obtained under different input voltages, are depicted in Figure 4.14. The maximum NLS increases in proportion to the input voltage, exhibiting a nearly linear relationship. When the driving voltage was below $27V_{pp}$, the propulsive force generated by the swimmer was insufficient to overcome both the wire traction and the resistance of water, resulting in an inability to measure NLS at input voltages lower than $27V_{pp}$. The maximum NLS was 19.79mm/s when the input voltage was $80V_{pp}$ at a resonance frequency of 1.07 MHz.

The NLS can be measured to show that the propulsive force generated by the acoustic underwater propulsion system using a PZT ring transducer

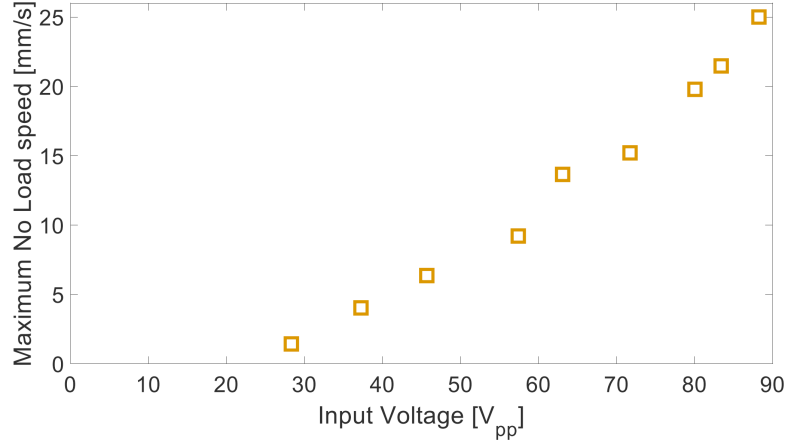


Figure 4.14: Maximum NLS of the swimmer under different input voltages at 1.07 MHz. The maximum NLS is 19.79mms^{-1} when the input voltage is 80 Vpp. When the driving voltage is below $27V_{pp}$, the propulsive force generated by the swimmer is insufficient to overcome both the wire traction and the resistance of water, resulting in an inability to measure NLS at input voltages lower than $27V_{pp}$

is sufficient for the swimmer to overcome water resistance and wire traction for underwater motion.

4.3 Conclusion

In this study, an acoustic underwater propulsion system utilizing a PZT ring ultrasonic transducer was investigated. The first step involves conducting simulations on the vibration mode and measuring the vibration velocities of the PZT ring transducer. Subsequently, a swimmer equipped with a PZT ring transducer was designed and fabricated. Finally, the propulsive characteristics of the swimmer were evaluated through measurements of admittance characteristics, vibration velocities, zero-speed propulsion (ZSP), and no-load propulsion (NLS).

Through simulation of the vibration mode, it was observed that the PZT ring transducer exhibits a simple vibration distribution, closely resembling pure longitudinal vibration. The vibration velocity measurements confirmed that the PZT ring transducer exhibited a vibration distribution resembling pure longitudinal vibration. Specifically, at the resonance frequency of 1.09 MHz and driving voltage of $73V_{pp}$, the longitudinal vibration velocity distribution of the electrode surface of the PZT ring transducer along the diameter

direction presents a peak shape with a peak value of 0.46ms^{-1} , and the radial vibration velocity distribution of the peripheral surface of the PZT ring transducer along the thickness direction presents an approximately straight line with an average value of 0.1ms^{-1} . As a result, at the resonance frequency of 1.09 MHz and driving voltage of $73V_{\text{pp}}$, the ratio of the maximum vibration velocity in the longitudinal direction of the PZT ring transducer to the average vibration velocity in the radial direction is 4.6:1.

A swimmer utilizing a PZT ring transducer was designed and fabricated with resonance frequencies of 1.08 MHz and 1.07 MHz in air and water, respectively, accompanied by conductance at the resonance frequencies of 13.1 mS and 5.2 mS. At the resonance frequency of 1.08 MHz and a driving voltage of $73V_{\text{pp}}$, the vibration velocity distribution along the diameter direction on the exposed electrode surface of the PZT ring transducer, which was fixed to the swimmer, showed a slope with a maximum vibration velocity of 0.32ms^{-1} . Although the vibration velocity distribution of the PZT ring transducer changes due to the change in the fixation method it still approximates to a pure longitudinal vibration distribution.

At the resonance frequency of 1.07 MHz and a driving voltage of $80V_{\text{pp}}$, the acoustic underwater propulsion system using the ring PZT transducer provided a ZSP of 1 mN, enabling the swimmer to achieve a maximum NLS of 19.79mms^{-1} by overcoming water resistance and wire traction. Future research will delve into a more efficient transducer and suppression of the influence of the wire.

Chapter 5

Equivalent Circuit of Disk Transducer

In order to facilitate the design of vibrators that improve the driving efficiency of ultrasonic underwater actuators without the need to spend a lot of effort on experimentation and selection of suitable vibrators, it is necessary to analyse the factors affecting the output efficiency of the vibrator by examining the theoretical model of the vibrator. Electromechanical equivalent circuits are often used as models for studying the electromechanical properties of vibrators. [1–6] Therefore, in this chapter, the electromechanical equivalent circuit model of a vibrator in the disk-shaped thickness vibration mode is studied in conjunction with the measurement results of the vibrator in the disk-shaped thickness vibration mode in Chapter 2.

An examination of the literature shows that, in order to simplify calculations, electromechanical equivalent circuits are generally only discussed for cases where all parameters other than the direction of interest are omitted. Examples include longitudinal vibration and radial symmetrical vibration. [7–20] Based on the measurement results of the vibration velocity distribution in Chapter 2, it can be assumed that the thickness vibration of the disc-shaped vibrator is caused by a combination of longitudinal and radial vibrations. Therefore, in this chapter, the electromechanical equivalent circuit of thickness vibration combining longitudinal and radial directions is discussed for the first time, referring to the process of deriving the electromechanical equivalent circuit of longitudinal vibration.

5.1 Longitudinal vibration

In this section, an attempt is made to explore thickness vibrations using the equivalent circuit of longitudinal vibrations. The longitudinal vibration is excited by an AC electric field parallel to the polarisation direction of the disc-shaped vibrator, with the polarisation direction of the disc-shaped vibrator as the thickness direction. The longitudinal vibration is excited under constant electric flux density, and the electric field distribution is related to the position z . In addition, the effect of the anti-electric field inside the transducer due to the vibration occurs.

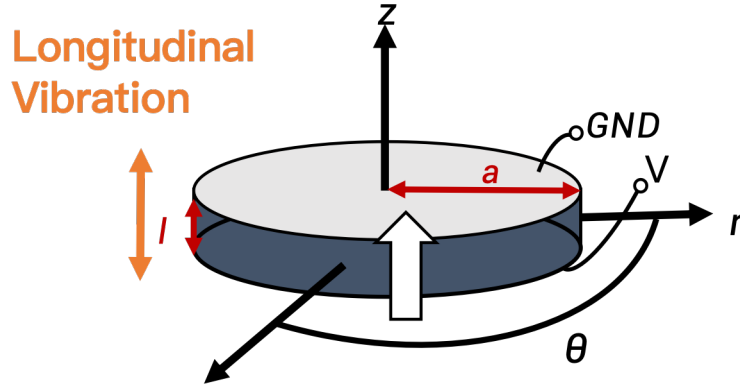


Figure 5.1: Longitudinal Vibration

The coordinates are taken in the direction of the polarisation direction in the z -axis (3-axis direction), in accordance with the general rules for considering piezoelectric vibrations. The size of the vibrator is assumed to be of radius a and thickness l . The mechanical boundary conditions are assumed to be free at both ends. In the case of longitudinal vibration, an electric field is applied in the z -axis direction (3-axis direction) and the distortion in the direction parallel to this is considered, resulting in 33 effects.

To ensure that the sign of the electric field E_z is also positive when the sign of the input voltage V is positive. The electrical boundary conditions are as follows: the upper electrode at $z = l/2$ is electrically grounded and the voltage $V = V_0 e^{j\omega t}$ with angular frequency ω is input to the lower electrode at $z = -l/2$.

The calculation is performed by converting from the Cartesian coordinate system to the polar coordinate system in the case of a disk-shaped piezoelectric material. [17] In the case of longitudinal vibration, if the piezoelectric equation is in the piezoelectric e form and the stress $[T]$ and flux density $[D]$ are set on the left-hand side, the calculation development becomes sim-

ple. However, in the piezoelectric e -form, S_{rr} and $S_{\theta\theta}$ other than S_{zz} are included in the equation for stress T_{zz} , which makes the calculation complicated. Therefore, the piezoelectric d -form is first taken up and the following equation is obtained.

$$\begin{bmatrix} S_{rr} \\ S_{\theta\theta} \\ S_{zz} \\ S_{\theta z} \\ S_{rz} \\ S_{r\theta} \end{bmatrix} = \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 \\ s_{12}^E & s_{11}^E & s_{13}^E & 0 & 0 & 0 \\ s_{13}^E & s_{13}^E & s_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66}^E \end{bmatrix} \begin{bmatrix} T_{rr} \\ T_{\theta\theta} \\ T_{zz} \\ T_{\theta z} \\ T_{rz} \\ T_{r\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \\ 0 & d_{15} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_r \\ E_\theta \\ E_z \end{bmatrix} \quad (5.1)$$

$$\begin{bmatrix} D_r \\ D_\theta \\ D_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_{rr} \\ T_{\theta\theta} \\ T_{zz} \\ T_{\theta z} \\ T_{rz} \\ T_{r\theta} \end{bmatrix} + \begin{bmatrix} \epsilon_{11}^T & 0 & 0 \\ 0 & \epsilon_{11}^T & 0 \\ 0 & 0 & \epsilon_{33}^T \end{bmatrix} \begin{bmatrix} E_r \\ E_\theta \\ E_z \end{bmatrix} \quad (5.2)$$

However, $s_{66}^E = 2(s_{11}^E - s_{12}^E)$. Matrix $[s^E]$ represents compliance. Matrix $[d]$ represents the piezoelectric constant. Matrix $[\epsilon^T]$ represents the dielectric constant. For the electric field, flux density and stress, because of longitudinal vibration, all components other than those in the vibration direction (z -axis direction) are set to 0, leaving only E_z , D_z and T_{zz} . It is important to note that even though S_{rr} , $S_{\theta\theta}$ and S_{zz} have values for strain, the third equation in Equation 5.1 does not include anything other than S_{zz} .

In other words, in the third equation of Equation 5.1 and the third equation of Equation 5.2, substituting $T_{rr} = T_{\theta\theta} = 0$ in the two equations and transforming the left-hand side so that the stress T_{zz} and the flux density D_z , the basic equation is

$$T_{zz} = \overline{c_{33}^E} S_{zz} - \overline{e_{33}} E_z \quad (5.3)$$

$$D_z = \overline{e_{33}} S_{zz} + \overline{\epsilon_{33}^{S_{zz}}} E_z. \quad (5.4)$$

However, $\overline{c_{33}^E} = \frac{1}{s_{33}^E}$, $\overline{e_{33}} = \frac{d_{33}}{s_{33}^E}$, $\overline{\epsilon_{33}^{S_{zz}}} = \epsilon_{33}^T - \frac{d_{33}^2}{s_{33}^E}$ were assumed.

The equation of motion for a micro-mass $\rho a^2 \pi dz$ when taking a micro-segment dz in the z -axis direction is

$$\rho a^2 \pi dz \frac{\partial^2 u_z}{\partial t^2} = \rho a^2 \pi \frac{\partial T_{zz}}{\partial z} dz \quad (5.5)$$

$$\rho \frac{\partial^2 u_z}{\partial t^2} = \frac{\partial T_{zz}}{\partial z}, \quad (5.6)$$

with the displacement in the z -axis direction at each position z as $u_z(z, t)$. Because the stress T_{zz} is a function of the electric field as well as the strain, as shown by the piezoelectric equation in equation 5.3, the equation of motion is

$$\rho \frac{\partial^2 u_z}{\partial t^2} = \overline{c_{33}^E} \frac{\partial S_{zz}}{\partial z} - e_{33} \frac{\partial E_z}{\partial z}. \quad (5.7)$$

In addition, the piezoelectric body does not have an internal charge, which gives $\frac{\partial D_z}{\partial z} = 0$. Therefore, differentiating both sides of Equation 5.4 by z gives

$$\frac{\partial D_z}{\partial z} = \overline{e_{33}} \frac{\partial S_{zz}}{\partial z} + \overline{\epsilon_{33}^{S_{zz}}} \frac{\partial E_z}{\partial z} = 0, \quad (5.8)$$

which gives

$$-\frac{\overline{e_{33}}}{\overline{\epsilon_{33}^{S_{zz}}}} \frac{\partial S_{zz}}{\partial z} = \frac{\partial E_z}{\partial z} \quad (5.9)$$

as the condition for the electric field. In longitudinal oscillations, the electric field is a function of the strain, which is the origin of the anti-field.

Substituting $\frac{\partial E_z}{\partial z}$ in Equation 5.9 into the equation of motion in Equation 5.7 yields the wave equation for the displacement $u_z(z, t)$ as follows.

$$\frac{\partial^2 u_z}{\partial t^2} = v_z^2 \frac{\partial^2 u_z}{\partial z^2} \quad (5.10)$$

However, $\overline{c_{33}^D} = \frac{\overline{c_{33}^E}}{1 - k_{33}^2}$, $v_z = \sqrt{\frac{\overline{c_{33}^D}}{\rho}}$ because $\frac{1}{\overline{c_{33}^E}}(1 - k_{33}^2) = \frac{1}{\overline{c_{33}^D}}$ rather than $\frac{\overline{e_{33}}^2}{\overline{c_{33}^E} \overline{\epsilon_{33}^{S_{zz}}}} = \frac{k_{33}^2}{1 - k_{33}^2}$ and $s_{33}^E(1 - k_{33}^2) = s_{33}^D$ [9].

Because the differential equation presented in Equation 5.10 is in the form of a wave equation, the separation of variables method yields the general solution as follows.

$$u_z = [A \cos(\frac{\omega}{v_z} z) + B \sin(\frac{\omega}{v_z} z)] e^{j\omega t} \quad (5.11)$$

However, A and B are constants and $v_z = \sqrt{\frac{\overline{c_{33}^D}}{\rho}}$.

Because the mechanical boundary conditions are free at both ends, the stress $T_{zz} = 0$ must always be independent of time t when $z = \pm \frac{l}{2}$ is substituted in the general solution of Equation 5.11. Because the stress is a function of strain and flux density, from the two piezoelectric equations in 5.3, 5.4, it is

$$T_{zz} = \overline{c_{33}^D} S_{zz} - \frac{\overline{e_{33}}}{\varepsilon_{33}^{S_{zz}}} D_z \quad (5.12)$$

From the general solution for $u_z(z, t)$ in Equations 5.12 and 5.11, the two relations

$$\begin{aligned} T_{zz}|_{z=-\frac{l}{2}} &= \overline{c_{33}^D} \left(\frac{\omega}{v_z} \right) [A \sin(\frac{\omega l}{2v_z}) + B \cos(\frac{\omega l}{2v_z})] e^{j\omega t} - \frac{\overline{e_{33}}}{\varepsilon_{33}^{S_{zz}}} D_z = 0 \\ T_{zz}|_{z=\frac{l}{2}} &= \overline{c_{33}^D} \left(\frac{\omega}{v_z} \right) [-A \sin(\frac{\omega l}{2v_z}) + B \cos(\frac{\omega l}{2v_z})] e^{j\omega t} - \frac{\overline{e_{33}}}{\varepsilon_{33}^{S_{zz}}} D_z = 0 \end{aligned} \quad (5.13)$$

are obtained, because D_z is constant regardless of position in order to satisfy the boundary condition $T_{zz} = 0$ at $z = \pm \frac{l}{2}$. From equation 5.13,

$$\begin{aligned} A &= 0 \\ B \cos(\frac{\omega l}{2v_z}) e^{j\omega t} &= \frac{v_z}{\omega} \frac{\overline{e_{33}}}{\overline{c_{33}^D} \varepsilon_{33}^{S_{zz}}} D_z. \end{aligned} \quad (5.14)$$

This leaves only B , which is the coefficient of the odd function, for the general solution of $u_z(z, t)$.

Consider the case where the input voltage is given. In the case of longitudinal vibration, the resonant angular frequency ω_r cannot be determined from the second equation in Equation 5.14 because the flux density D_z depends on the displacement amplitude B of the vibration mode being excited. Therefore, the electric field distribution E_z and the electric flux density D_z are need to be determined. According to Equation 5.8, there is a relationship

$$\frac{\partial E_z}{\partial z} = -\frac{\overline{e_{33}}}{\varepsilon_{33}^{S_{zz}}} \frac{\partial^2 u_z}{\partial z^2} \quad (5.15)$$

between the electric field and displacement. Integrating both sides by z yields

$$E_z = -\frac{\overline{e_{33}}}{\varepsilon_{33}^{S_{zz}}} \frac{\partial u_z}{\partial z} + N, \quad (5.16)$$

with the integration constant N . The integration constant N is determined by the potential difference. In the case shown in 5.1, the voltage $+V_0 e^{j\omega t}$ is applied to the lower electrode ($z = -\frac{l}{2}$), thus the potential difference from $z = -\frac{l}{2}$ to $z = \frac{l}{2}$ is $-V_0 e^{j\omega t}$. Using $u_z(z, t) = B \sin(\frac{\omega}{v_z} z) e^{j\omega t}$ and Equation 5.16, with attention to the direction of integration,

$$\begin{aligned} 0 - V_0 e^{j\omega t} &= - \int_{-\frac{l}{2}}^{\frac{l}{2}} E_z dx \\ &= 2B \frac{\overline{e_{33}}}{\varepsilon_{33}^{S_{zz}}} \sin\left(\frac{\omega l}{2v_z}\right) e^{j\omega t} - Nl \end{aligned} \quad (5.17)$$

is obtained. By this potential condition, the integration constant N is found to be

$$N = \left[\frac{V_0}{l} + 2B \frac{\overline{e_{33}}}{\varepsilon_{33}^{S_{zz}} l} \sin\left(\frac{\omega l}{2v_z}\right) \right] e^{j\omega t}. \quad (5.18)$$

By this constant, the electric field E_z , shown by Equation 5.16, is expressed as

$$E_z = \left[\frac{V_0}{l} - B \frac{\overline{e_{33}}}{\varepsilon_{33}^{S_{zz}}} \frac{\omega}{v_z} \cos\left(\frac{\omega}{v_z} z\right) + 2B \frac{\overline{e_{33}}}{\varepsilon_{33}^{S_{zz}} l} \sin\left(\frac{\omega l}{2v_z}\right) \right] e^{j\omega t} \quad (5.19)$$

using $u_z(z, t) = B \sin(\frac{\omega}{v_z} z) e^{j\omega t}$. The second term on the right-hand side of Equation 5.19 is a function of position, which depends on the piezoelectric vibration distribution, while the third term is independent of position. This third term has the meaning of an anti-electric field correction term.

Furthermore, substituting the general solution $u_z(z, t) = B \sin(\frac{\omega}{v_z} z) e^{j\omega t}$ for strain and Equation 5.19 for the electric field distribution E_z into the

piezoelectric equation in Equation 5.4 to obtain the electric flux density D_z , which is

$$D_z = [\overline{\varepsilon_{33}^{Szz}} \frac{V_0}{l} + 2B \frac{\overline{e_{33}}}{l} \sin(\frac{\omega l}{2v_z})] e^{j\omega t}. \quad (5.20)$$

This confirms that the flux density D_z is in a form independent of z and that the condition $\frac{\partial D_z}{\partial z} = 0$ is satisfied. In Equation 5.20, $\overline{\varepsilon_{33}^{Szz}} \frac{V_0}{l}$ corresponds to the charge density in the absence of non-piezoelectric or piezoelectric distortion. On the other hand, $2B \frac{\overline{e_{33}}}{l} \sin(\frac{\omega l}{2v_z}) e^{j\omega t}$ is the flux density component corresponding to the charge supplied by the power source to generate the correction term $2B \frac{\overline{e_{33}}}{\varepsilon_{33}^{Szz} l} \sin(\frac{\omega l}{2v_z}) e^{j\omega t}$ for the electric field.

The second equation in Equation 5.14 is then used to obtain the displacement amplitude

$$B = \frac{v_z \overline{e_{33}}}{\omega \overline{c_{33}^D}} \frac{1}{\cos(\frac{\omega l}{2v_z}) [1 - \frac{\overline{e_{33}}^2}{c_{33}^D \varepsilon_{33}^{Szz}} \frac{\tan(\frac{\omega l}{2v_z})}{\frac{\omega l}{2v_z}}]} \frac{V_0}{l}. \quad (5.21)$$

Using this displacement amplitude B , the general solution for the vibrational displacement distribution is

$$u_z(z, t) = \frac{v_z \overline{e_{33}}}{\omega \overline{c_{33}^D}} \frac{1}{\cos(\frac{\omega l}{2v_z}) [1 - \frac{\overline{e_{33}}^2}{c_{33}^D \varepsilon_{33}^{Szz}} \frac{\tan(\frac{\omega l}{2v_z})}{\frac{\omega l}{2v_z}}]} \frac{V_0}{l} \sin(\frac{\omega}{v_z} z) e^{j\omega t}. \quad (5.22)$$

The equation for the displacement function therefore shows that the displacement u_z does not depend on the position in the r direction and the vibration distribution on the electrodes on both sides ($z = \pm \frac{l}{2}$) of the thickness direction of the disc-shaped vibrator is uniform. However, this is clearly not in agreement with the measured results of the vibration distribution presented in Chapter 2.

Although it is known that the displacement function obtained for longitudinal vibration does not satisfy the measurement results, the electrical-mechanical equivalent circuit for longitudinal vibration is then continued to be completed in order to investigate the admittance characteristic.

5.1.1 Equivalent Circuit

The general solution for the beam density from the displacement amplitude B is

$$D_z = \left[1 + \frac{\frac{\overline{e_{33}^2}}{c_{33}^D \overline{\varepsilon_{33}^{S_{zz}}}} \frac{\tan(\frac{\omega l}{2v_z})}{\frac{\omega l}{2v_z}}}{1 - \frac{\overline{e_{33}^2}}{c_{33}^D \overline{\varepsilon_{33}^{S_{zz}}}} \frac{\tan(\frac{\omega l}{2v_z})}{\frac{\omega l}{2v_z}}} \right] \overline{\varepsilon_{33}^{S_{zz}}} \frac{V_0}{l} e^{j\omega t}. \quad (5.23)$$

The current i supplied by the power supply can be area divided by the time derivative of the flux density equation, which is

$$\begin{aligned} i &= \int_0^a 2\pi r \frac{\partial D_z}{\partial z} dr. \\ &= j\omega \overline{\varepsilon_{33}^{S_{zz}}} \frac{a^2 \pi}{l} V_0 e^{j\omega t} + j \frac{2a^2 \pi v_z \overline{e_{33}^2}}{l^2 c_{33}^D} \frac{1}{\cot(\frac{\omega l}{2v_z}) - \frac{\overline{e_{33}^2}}{c_{33}^D \overline{\varepsilon_{33}^{S_{zz}}}} (\frac{\omega l}{2v_z})} V_0 e^{j\omega t}. \end{aligned} \quad (5.24)$$

Therefore, the admittance $Y(= \frac{i}{V})$ is

$$\begin{aligned} Y &= \frac{i}{V_0 e^{j\omega t}} \\ &= j\omega C_d + \frac{1}{\frac{1}{-j\omega C_d} + \frac{1}{Y_m}}. \end{aligned} \quad (5.25)$$

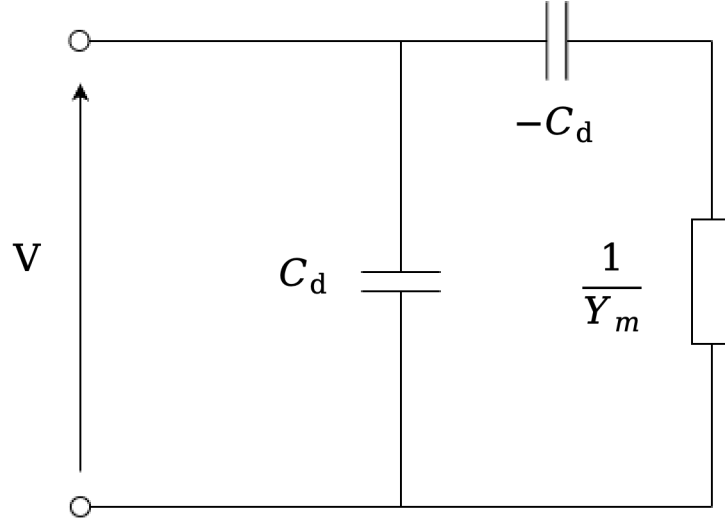


Figure 5.2: Equivalent Circuit of the Longitudinal Vibration

However, $C_d = \overline{\varepsilon_{33}^{S_{zz}}} \frac{a^2 \pi}{l}$ and $Y_m = j \frac{2a^2 \pi v_z \overline{e_{33}^2}}{l^2 c_{33}^D} \tan(\frac{\omega l}{2v_z})$. At full admittance, as shown in Fig. 5.2, this series circuit of $-C_d$ and Y_m is an equivalent circuit that is further connected in parallel with the braking capacitance C_d .

In addition, whereas in a normal electrical circuit the capacitance of a capacitor is always positive, in the equivalent circuit for longitudinal vibration, a $-C_d$ with a negative sign is connected in series with Y_m , which represents mechanical vibration. This corresponds to compensating for the influence of the anti-electric field generated by the piezoelectric vibration on the potential difference between the electrodes, as in the calculation of the electric field distribution Equation 5.19 inside the vibrator.

Next section, the admittance characteristics with no load are calculated from this equivalent circuit.

5.1.2 Admittance Characteristics without External Load

In this section, the admittance characteristics for a diameter of 10 mm and a thickness of 1 mm will be calculated based on the equivalent circuit without external load derived in the previous section using the PZTc204(FUJI CERAMICS) material parameters mentioned in Table 5.1.

Table 5.1: Material Parameters of PZTc204 (FUJI CERAMICS)

Electro-mechanical coupling coefficient		Relative permittivity			Equivalent piezoelectric constant [$\times 10^{-12} \text{m/V}$]	
k_p	k_{33}	k_t	$\varepsilon_{11}^T/\varepsilon_0$	$\varepsilon_{33}^T/\varepsilon_0$	d_{31}	d_{33}
0.63	0.74	0.52	2240	2200	-190	435
Voltage output coefficient [$\times 10^{-3} \text{V} \cdot \text{m/N}$]		Elastic constant [$\times 10^{10} \text{N/m}^2$]		Frequency constant [$\text{m} \cdot \text{Hz}$]		
g_{31}	g_{33}	$1/s_{11}^E$	$1/s_{33}^E$	N_p	N_{33}	N_t
-9.7	22.0	7.2	5.7	2110	1450	2070
Density [$\times 10^3 \text{kg/m}^3$]		Mechanical quality factor		Dielectric loss		Poisson's ratio
ρ		Q_m		$\tan\delta$		σ
7.65		520		0.0025		0.29

Returning to the Equation 5.25 for the admittance, Y_m can be transformed into

$$Y_m = j\omega C_d k_{33}^2 \frac{\tan(\frac{\omega l}{2v_z})}{\frac{\omega l}{2v_z}} \quad (5.26)$$

because of $\frac{\overline{\varepsilon_{33}}}{c_{33}^D \varepsilon_{33}^{S_{zz}}} = k_{33}^2$, $C_d = \overline{\varepsilon_{33}^{S_{zz}}} \frac{a^2 \pi}{l}$. Therefore Equation 5.25 can be transformed into

$$Y = j\omega C_d \frac{1}{1 - k_{33}^2 \frac{\tan(\frac{\omega l}{2v_z})}{\frac{\omega l}{2v_z}}}. \quad (5.27)$$

In the resonance state, when the current amplitude is ∞ for a constant input amplitude voltage, which means that $Y \rightarrow \infty$, the denominator in Equation 5.27 is

$$1 - k_{33}^2 \frac{\tan(\frac{\omega l}{2v_z})}{\frac{\omega l}{2v_z}} = 0, \quad (5.28)$$

and

$$\frac{\tan(\frac{\omega_r l}{2v_z})}{\frac{\omega_r l}{2v_z}} = \frac{1}{k_{33}^2} \quad (5.29)$$

holds for the angular frequency ω_r . Thus, the electromechanical coupling coefficient k_{33} is included in the equation for ω_r , and it can be seen that the resonant angular frequency varies with piezoelectricity.

From the admittance loop, it can be seen that the mechanical series resonance angular frequency ω_s , the resonance angular frequency ω_r and the angular frequency ω_m at which the absolute value of admittance reaches its maximum are not the same due to the presence of C_d , but the mechanical quality factor Q_m is sufficiently large that the conductance at the mechanical series resonance angular frequency ω_s (diameter of the admittance loop) is sufficiently large that the change in admittance due to C_d is negligible. Therefore, $\omega_s \approx \omega_r$. [10] And it can be learnt from Equation 5.25 that the mechanical series resonance angular frequency ω_s is consistent with the conditions for solving the resonance angular frequency ω_r .

The anti-angular frequency ω_a , on the other hand, is when the impedance is ∞ and no current flows. which means that $Y \rightarrow 0$, so from Equation 5.27,

$$\tan(\frac{\omega_a l}{2v_z}) = \infty \quad (5.30)$$

holds. In other words, because of

$$\frac{\omega_a l}{2v_z} = \frac{(2n - 1)\pi}{2}, \quad (5.31)$$

the anti-resonant angular frequency is simpler than the resonant frequency and can be obtained from the thickness l of the vibrator and the sound speed v_z . However, n is a natural number.

When $X = \frac{\omega l}{2v_z}$ and $\tan(X) = \frac{1}{k_{33}^2}X$, it can be obtained from Equation 5.29, and a graph of the functions on both sides of the equation yields Figure 5.3. The orange curve represents the function $y = \tan(X)$ and the yellow curve represents the function $y = \frac{1}{k_{33}^2}X$. The resonance angular frequency can be obtained from the intersection points of the two curves (*blue**) at $X_{rn}(= \frac{\omega_{rn}l}{2v_z})$. Taking the first intersection point of the graph, $\omega_{r1} = 2 * \pi * f_{r1}$ gives $f_{r1} = 1.44$ MHz, which is approximated by the frequency factor $f_r = N_{33} * l = 1.45$ MHz. Equation 5.31 also shows that the X of the anti-resonant frequency is $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ and that the anti-resonant frequency is always higher than the resonant frequency.

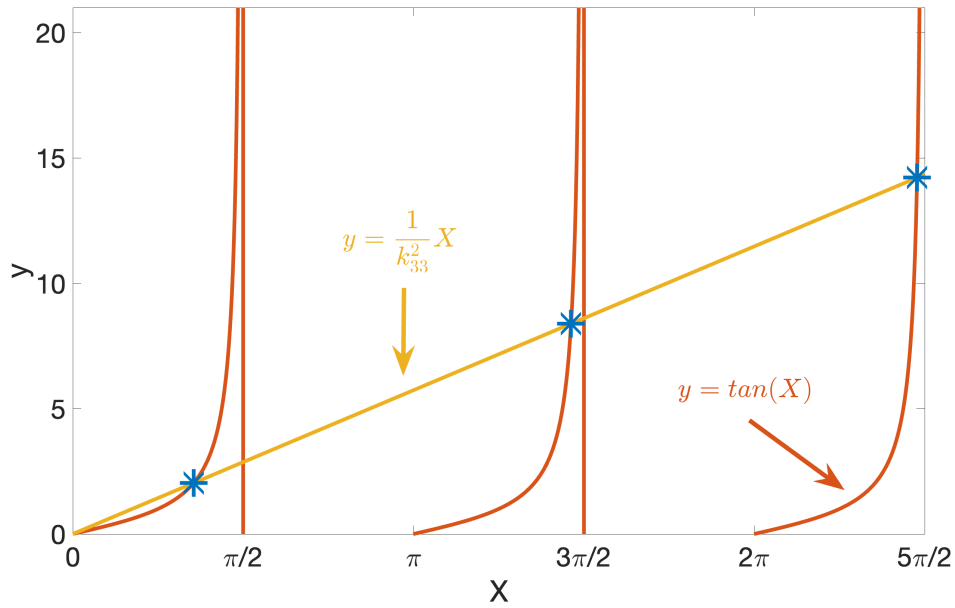


Figure 5.3: Resonance and Anti-Resonance Angular Frequencies of Longitudinal Vibration

The above mentioned exact solution for the resonant frequency, however, the existence of the $\tan(X)$ function prevents the calculation results of the admittance from converging, thus the calculation by taking an approximation of the $\tan(X)$ function is considered as follows. This allows the calculation results to converge, although the error in the calculation results will be larger.

Returning to the dynamic admittance Y_m in Equation 5.26, by Laurent

expansion of $\tan(\frac{\omega l}{2v_z})$, it can be expressed in a circuit with LC series components corresponding to each vibration mode component connected in parallel. Laurent expansion of equation 5.26 allows from

$$\tan(\frac{\omega l}{2v_z}) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2 \frac{\pi^2}{8} \frac{2v_z}{\omega l} - \frac{\omega l}{4v_z}} \quad (5.32)$$

to be

$$Y_m = A^2 \sum_{n=1}^{\infty} \frac{1}{j\omega L_{2n-1} + \frac{1}{j\omega C_{2n-1}}}. \quad (5.33)$$

However, $A = \frac{2a^2\pi\overline{e_{33}}}{l}$ is the force coefficient, $L_{2n-1} = \frac{\rho a^2\pi l}{2}$ is the equivalent mass and $C_{2n-1} = \frac{2l}{(2n-1)^2\pi^2 a^2\pi\overline{c_{33}^D}}$ is the equivalent compliance. The equivalent mass L_{2n-1} is independent of the mode order and is half of the actual mass $\rho a^2\pi l$, whereas the equivalent compliance C_{2n-1} depends on the mode order and decreases at a rate of $(2n-1)^2$ as the mode order increases. The equivalent spring constant is the reciprocal of C_{2n-1} , which means that the stiffness increases as the mode order increases.

Considering only the first-order mode, the equivalent circuit is an L_1C_1 series circuit through transformer A connected in series with a capacitor $-C_d$ with negative capacitance. The dynamic admittance Y_m can be converted to

$$Y_m = A^2 \frac{1}{j\omega L_1 + \frac{1}{j\omega C_1}}, \quad (5.34)$$

However, $L_1 = \frac{\rho a^2\pi l}{2}$ and $C_1 = \frac{2l}{\pi^3 a^2\overline{c_{33}^D}}$.

To obtain the full admittance characteristic of PZTc204 disk-shaped vibrator, the resistance R_m of the vibration loss and the resistance R_d of the dielectric loss must be determined in the electromechanical equivalent circuit.

Because the mechanical quality factor Q_m and the Mechanical series angular resonance frequency $\omega_s = \sqrt{LC}$ are known, R_m in series with $L_m = \frac{L_1}{A^2}$ can be obtained from

$$R_m = \frac{\omega_s L_m}{Q_m}. \quad (5.35)$$

Because the induced loss $\tan\delta = 0.0025$ can be found from Table 5.1, the braking capacitance C_d and the resistance Rd in parallel can be obtained from

$$R_d = \frac{1}{\tan \delta \omega C_d} \quad (5.36)$$

Finally, the electromechanical equivalent circuit is obtained as shown in Figure 5.4.

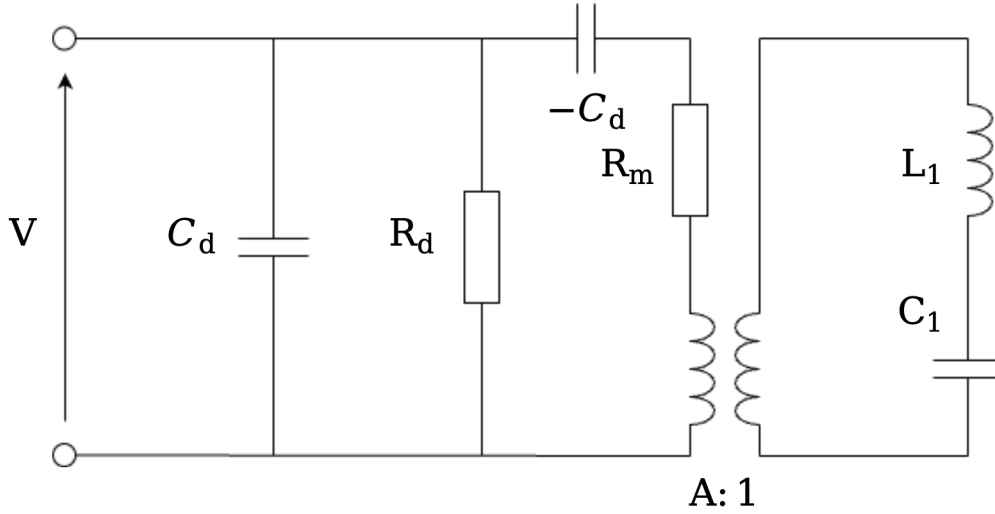


Figure 5.4: Equivalent Circuit of the Longitudinal Vibration with losses

From this electromechanical equivalent circuit the admittance characteristics can be calculated as shown in Fig. 5.5. The resonance frequency is 1.50 MHz and the conductance at the resonance frequency is 2.06 S. As the calculation only expands the $\tan(X)$ function to the first order, the calculated resonance frequency of 1.5 MHz differs by 0.06 MHz from the 1.44 MHz obtained from the exact solution, and theoretically, increasing the expansion order of the tan function will bring it closer to the exact solution. The admittance characteristics of the actuator using the PZTc204 disc-shaped vibrator with a diameter of 10 mm and a thickness of 1 mm, measured in air, are shown in Fig. 5.6, with a resonance frequency of 2.13 MHz and a conductance at the resonance frequency of 85.8 mS.

Comparing the results of the admittance calculations with the measurement results, not only are the resonance frequencies different, but the admittance values are also very different. It follows that this electrical equivalence circuit for longitudinal vibration without load is not applicable to thickness vibration.

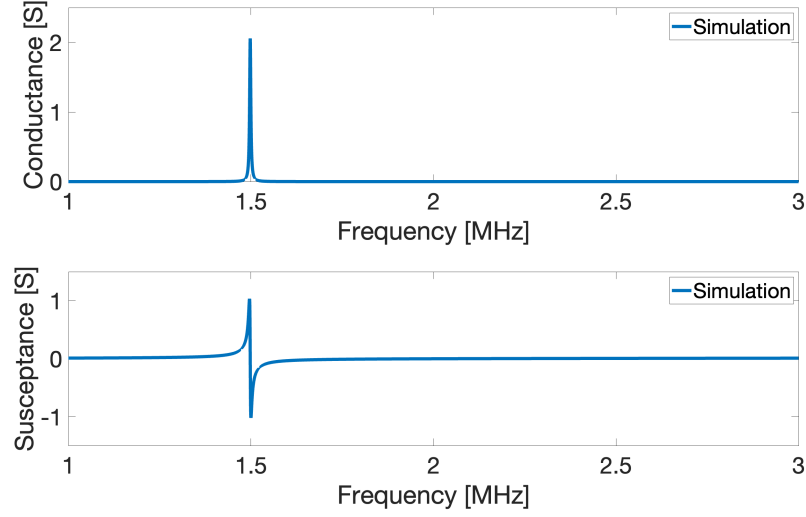


Figure 5.5: Calculation result for the Admittance of PZTc204 Disc-shaped 10mm Diameter Transducer in the longitudinal vibration equivalent circuit without load (Resonance frequency: 1.50 MHz; Conductance: 2.06 S)

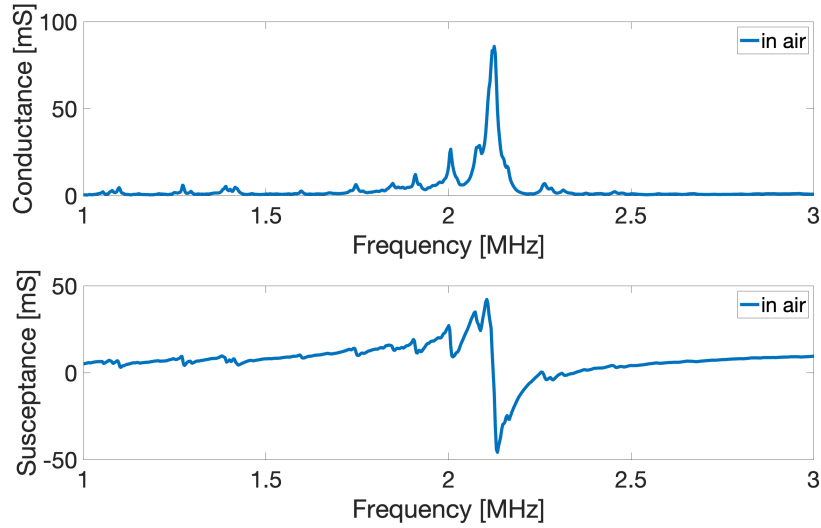


Figure 5.6: Measurement result for the Admittance of the Actuator using PZTc204 Disc-shaped 10mm Diameter 1mm Thickness Transducer in Air (Resonance Frequency: 2.13 MHz; Conductance: 85.8 mS)

As the calculations are for a disk-shaped vibrator with free end faces, it will be investigated whether the results can be improved by adding loads to both end faces. In order to verify this, it is necessary to create an electromechanical equivalent circuit (Mason equivalent circuit), which can deal with arbitrary boundary conditions, and the Mason equivalent circuit for longitudinal vibration was investigated in the next section.

5.1.3 Mason Equivalent Circuit

Finding electromechanical equivalent circuits for centralised constant systems is constrained by the fact that the boundary conditions are fixed. In this section, Mason equivalent circuit is represented for the longitudinal vibration that can accommodate arbitrary boundary conditions.

Take the inward positive force in the z -axis direction of the vibrator and define each parameter as F_1 and F_2 , respectively, as shown in Fig. 5.7.

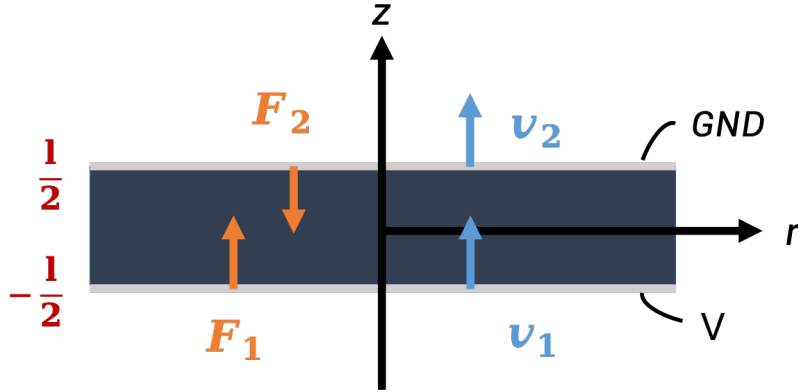


Figure 5.7: Definition of each parameter of the transfer matrix of longitudinal vibrations.

From the wave equation for longitudinal vibration, the general solution for displacement is

$$u_{z,t} = [A \cos(\frac{\omega}{v_z} z) + B \sin(\frac{\omega}{v_z} z)] e^{j\omega t}. \quad (5.37)$$

However, $v_z = \sqrt{\frac{c_{33}^D}{\rho}}$. Because no boundary conditions such as freedom at both sides are given here, $A = 0$ is not used and even functions are included. The electric field in this case agrees with the case of boundary conditions such as two sides free, since the even functions disappear when the potential

is calculated in the same way as in Equation 5.17. Thus, from Equation 5.16, the electric field is

$$E_z = \left[\frac{V_0}{l} - \frac{\overline{e_{33}}}{\varepsilon_{33}^{S_{zz}}} \frac{S_{zz}}{e^{j\omega t}} + 2B \frac{\overline{e_{33}}}{\varepsilon_{33}^{S_{zz}} l} \sin\left(\frac{\omega l}{2v_z}\right) \right] e^{j\omega t}. \quad (5.38)$$

In this derivation process, the mechanical boundary conditions of the vibrator are not included, thus it holds for all boundary conditions and angular frequencies ω . By substituting the general solution of the displacement and the expression for the electric field into the piezoelectric equation T_{zz} , the stress T_{zz} is

$$T_{zz} = \left\{ -\overline{e_{33}} \frac{V_0}{l} + \overline{c_{33}^D} \frac{\omega}{v_z} \left[-A \sin\left(\frac{\omega}{v_z} z\right) + B \cos\left(\frac{\omega}{v_z} z\right) \right] - 2B \frac{\overline{e_{33}}^2}{\varepsilon_{33}^{S_{zz}} l} \sin\left(\frac{\omega l}{2v_z}\right) \right\} e^{j\omega t}. \quad (5.39)$$

Noting that the stress is positively tensile, whereas the force is positively compressive, F_1 at $z = -\frac{l}{2}$ is

$$\begin{aligned} F_1 &= -a^2 \pi T_{zz} \Big|_{z=-\frac{l}{2}} \\ &= -a^2 \pi \left\{ \overline{c_{33}^D} \frac{\omega}{v_z} \left[A \sin\left(\frac{\omega l}{2v_z}\right) + B \cos\left(\frac{\omega l}{2v_z}\right) \right] - 2B \frac{\overline{e_{33}}^2}{\varepsilon_{33}^{S_{zz}} l} \sin\left(\frac{\omega l}{2v_z}\right) - \overline{e_{33}} \frac{V_0}{l} \right\} e^{j\omega t}. \end{aligned} \quad (5.40)$$

and F_2 at $z = \frac{l}{2}$ is

$$\begin{aligned} F_2 &= -a^2 \pi T_{zz} \Big|_{z=\frac{l}{2}} \\ &= -a^2 \pi \left\{ \overline{c_{33}^D} \frac{\omega}{v_z} \left[-A \sin\left(\frac{\omega l}{2v_z}\right) + B \cos\left(\frac{\omega l}{2v_z}\right) \right] - 2B \frac{\overline{e_{33}}^2}{\varepsilon_{33}^{S_{zz}} l} \sin\left(\frac{\omega l}{2v_z}\right) - \overline{e_{33}} \frac{V_0}{l} \right\} e^{j\omega t}. \end{aligned} \quad (5.41)$$

On the other hand, the vibration velocities v_1 and v_2 on both the top and bottom sides are

$$\begin{aligned} v_1 &= \frac{\partial u_z}{\partial t} \Big|_{z=-\frac{l}{2}} = j\omega \left[A \cos\left(\frac{\omega l}{2v_z}\right) - B \sin\left(\frac{\omega l}{2v_z}\right) \right] e^{j\omega t} \\ v_2 &= \frac{\partial u_z}{\partial t} \Big|_{z=\frac{l}{2}} = j\omega \left[A \cos\left(\frac{\omega l}{2v_z}\right) + B \sin\left(\frac{\omega l}{2v_z}\right) \right] e^{j\omega t}, \end{aligned} \quad (5.42)$$

because the general solution of the displacements in Equation 5.37 can be differentiated with time t .

Substituting Equation 5.1 into Equations 5.40 and 5.41, and $V = V_0 e^{j\omega t}$, gives

$$F_1 = jS_u Z_0 \left[v_1 \tan\left(\frac{\omega l}{2v_z}\right) - \frac{v_1 - v_2}{\sin\left(\frac{\omega l}{v_z}\right)} \right] + \frac{1}{j\omega C_d} \left(\frac{a^2 \pi \bar{e}_{33}}{l} \right)^2 (v_2 - v_1) + \frac{a^2 \pi \bar{e}_{33}}{l} V \quad (5.43)$$

$$F_2 = jS_u Z_0 \left[-v_2 \tan\left(\frac{\omega l}{2v_z}\right) - \frac{v_1 - v_2}{\sin\left(\frac{\omega l}{v_z}\right)} \right] + \frac{1}{j\omega C_d} \left(\frac{a^2 \pi \bar{e}_{33}}{l} \right)^2 (v_2 - v_1) + \frac{a^2 \pi \bar{e}_{33}}{l} V \quad (5.44)$$

and eliminates the constants A and B that were included in the equations representing external forces F_1 and F_2 . However, the acoustic impedance $Z_0 = \rho v_z$, the cross-sectional area $S_u = a^2 \pi$ and $C_d = \frac{\bar{\epsilon}_{33}^{S_{zz}} a^2 \pi}{l}$. If the force coefficient $A = \frac{a^2 \pi \bar{e}_{33}}{l}$, the relationship between force and velocity is

$$F_1 = jS_u Z_0 \left[v_1 \tan\left(\frac{\omega l}{2v_z}\right) - \frac{v_1 - v_2}{\sin\left(\frac{\omega l}{v_z}\right)} \right] + \frac{1}{j\omega C_d} A^2 (v_2 - v_1) + AV \quad (5.45)$$

$$F_2 = jS_u Z_0 \left[-v_2 \tan\left(\frac{\omega l}{2v_z}\right) - \frac{v_1 - v_2}{\sin\left(\frac{\omega l}{v_z}\right)} \right] + \frac{1}{j\omega C_d} A^2 (v_2 - v_1) + AV \quad (5.46)$$

By the force coefficient A corresponding to the ideal transformer, the input voltage is converted into a force, which is then used for mechanical vibration.

In this calculation process, the stress was determined from the force component and the relationship with the vibration velocity was determined, thus the part of the braking capacitance C_d from the power supply side is not represented. The braking capacitance enters in parallel with the input voltage, as can be seen from the consideration in the concentrated constant system, and if the losses are considered in the same way as in the concentrated constant system, the equivalent circuit for is shown in Figure 5.8. However, $Z_1 = -j \frac{S_u Z_0}{\sin\left(\frac{\omega l}{v_z}\right)}$ and $Z_2 = j S_u Z_0 \tan\left(\frac{\omega l}{2v_z}\right)$. The dielectric loss R_d and vibration loss R_m are calculated in a similar way to the equivalent circuit of a concentrated constant system.

The vibration loss R_m can be calculated from the Mason equivalent circuit short-circuited at both ends (free state at both end faces). Moreover,

the Mason's equivalent circuit after short-circuiting at both ends can be converted to the equivalent circuit of the centralised fixed number system, thus R_m and R_d are the same as in the equivalent circuit of the centralised fixed number system. [10]

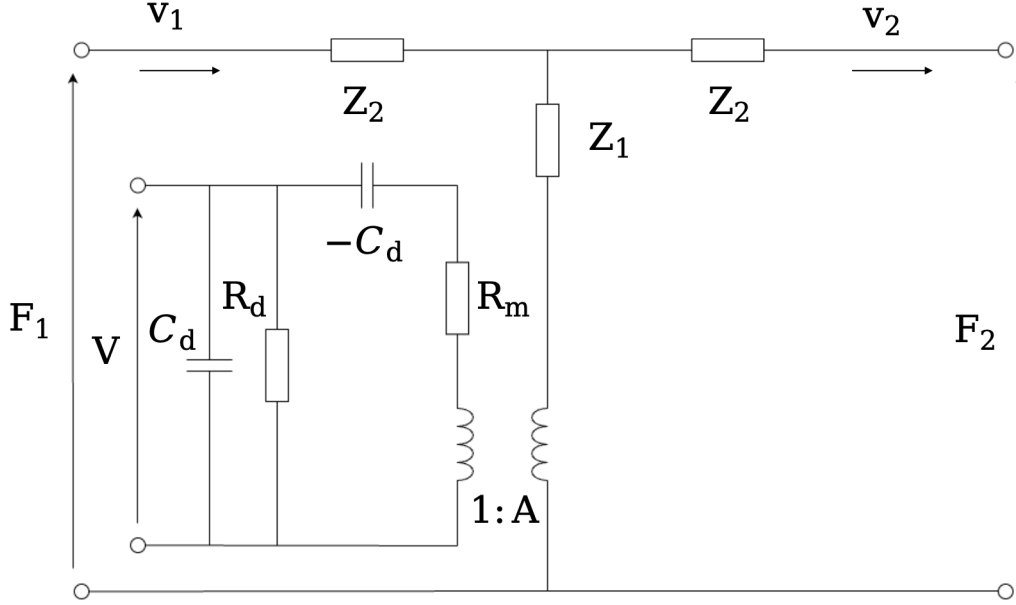


Figure 5.8: Mason Equivalent Circuit of the Longitudinal Vibration with losses.

Because the obtained two-port Mason equivalent circuit can correspond to arbitrary boundary conditions, the next section will consider adding loadings to the Mason equivalent circuit to calculate the admittance.

5.1.4 Admittance Characteristics with Exterbal Load

In the case where the top and bottom sides of the vibrator are in contact with air, the air load will be added to the Mason equivalent circuit with air acoustic impedance. In this way, a Mason equivalent circuit with air load can be obtained, as shown in Fig. 5.9. However, $Z_{air} = S_u \rho_{air} v_{air}$. The $\rho_{air} = 1.184 kg/m^3 (25^\circ C)$ is the density of air. The $v_{air} = 346.3 m/s (25^\circ C)$ is sound speed in air.

The calculated admittance characteristics of the Mason equivalent circuit with air loads at both ends are shown in Figure 5.10. The mechanical series resonance frequency f_s (the resonance frequency at which the conductance reaches its maximum) is reduced by 0.08 MHz to 1.42 MHz compared to

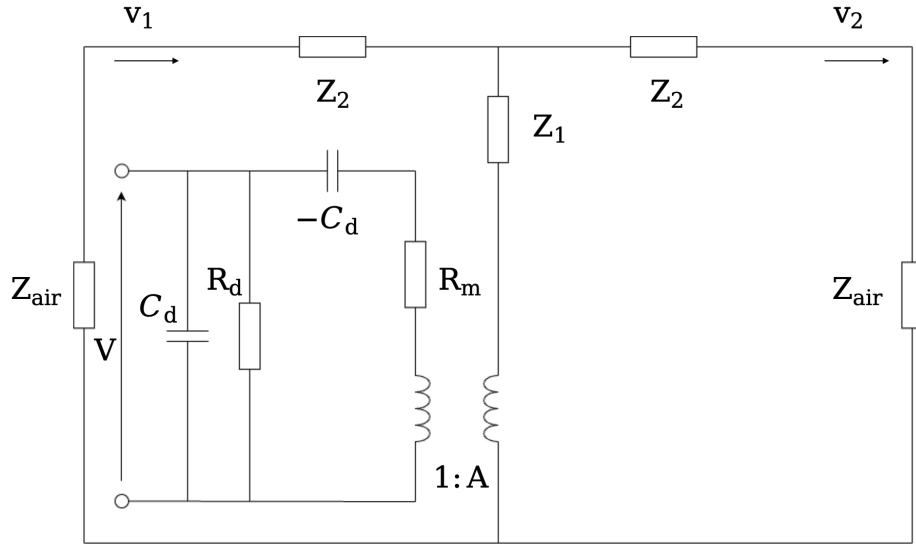


Figure 5.9: Mason Equivalent Circuit of the Longitudinal Vibration with Air loads.

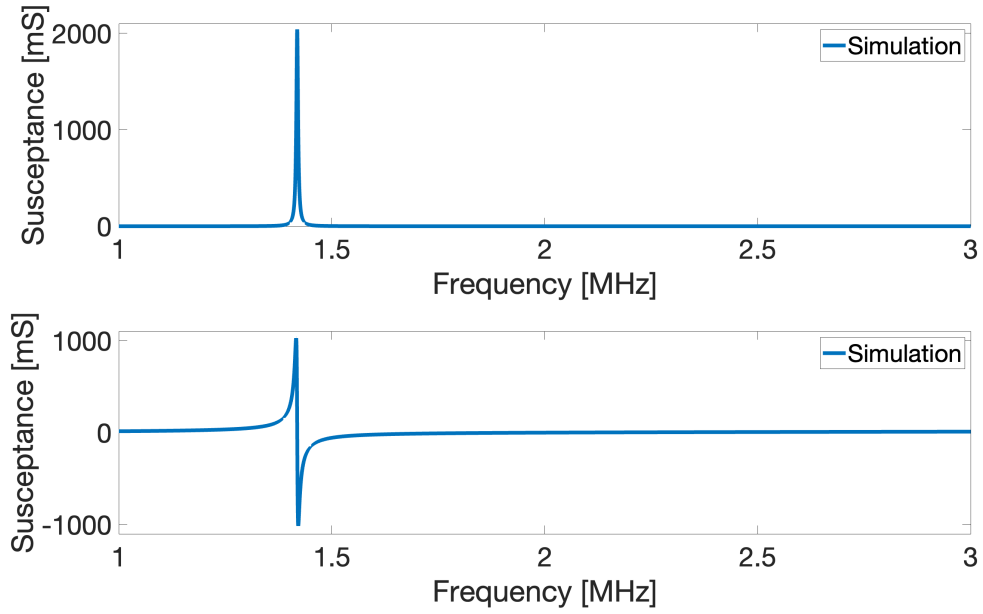


Figure 5.10: Calculation result for the admittance of PZTc204 disc-shaped 10mm diameter transducer in the longitudinal vibration mason equivalent circuit with air loads (Resonance frequency: 1.42MHz; Conductance: 2041.14mS)

the case without air loads, and the conductance at the resonance frequency f_s is reduced to 2041.14 mS.

The above results show that the addition of an air load changes the admittance characteristics of the Mason equivalent circuit. The next step is to investigate how the addition of a water load changes the admittance characteristics of the Mason equivalent circuit.

In the case where the top is contact with water and the bottom side is in contact with air, the water and air loads will be added to the Mason equivalent circuit with water acoustic impedance and air acoustic impedance. In this way, a Mason equivalent circuit with air and water loads can be obtained, as shown in Figure 5.11. However, $Z_{air} = S_u \rho_{air} v_{air}$ and $Z_{water} = S_u \rho_{water} v_{water}$. The $\rho_{water} = 997.05 \text{ kg/m}^3 (25^\circ\text{C})$ is the density of water. The $v_{water} = 1497 \text{ m/s} (25^\circ\text{C})$ is sound speed in water.

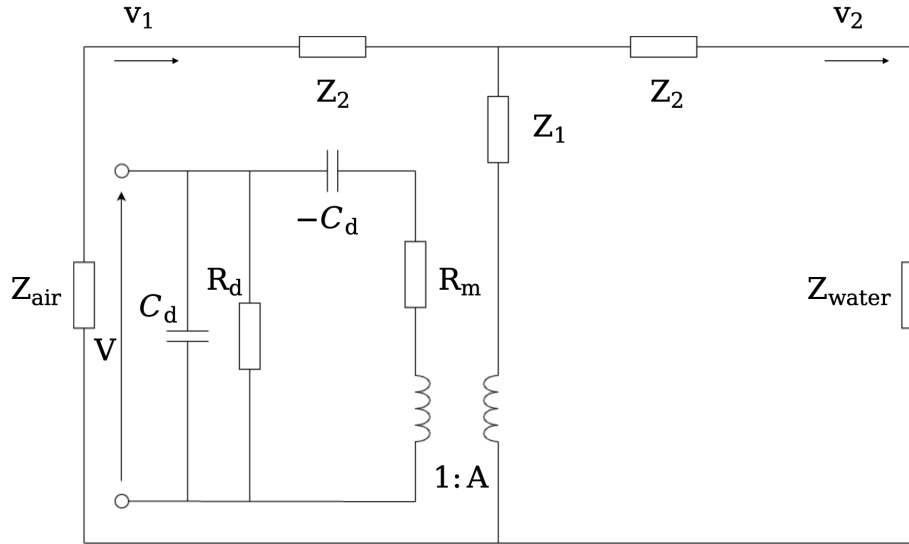


Figure 5.11: Mason Equivalent Circuit of the Longitudinal Vibration with Air and Water loads.

From this Mason equivalent circuit with air and water loads the admittance characteristics can be calculated as shown in Figure 5.12. The resonance frequency is 1.42 MHz and the conductance at the resonance frequency is 121.0 mS. The admittance characteristics of the actuator using the PZTc204 disc-shaped vibrator with a diameter of 10 mm and a thickness of 1 mm, measured in air, are shown in Figure 5.13, with a resonance frequency of 2.11 MHz and a conductance at the resonance frequency of 49.9 mS.

Comparing Figures 5.10 and 5.12, It can be seen that after adding a water load to one port and an air load to the other port of the Mason equivalent

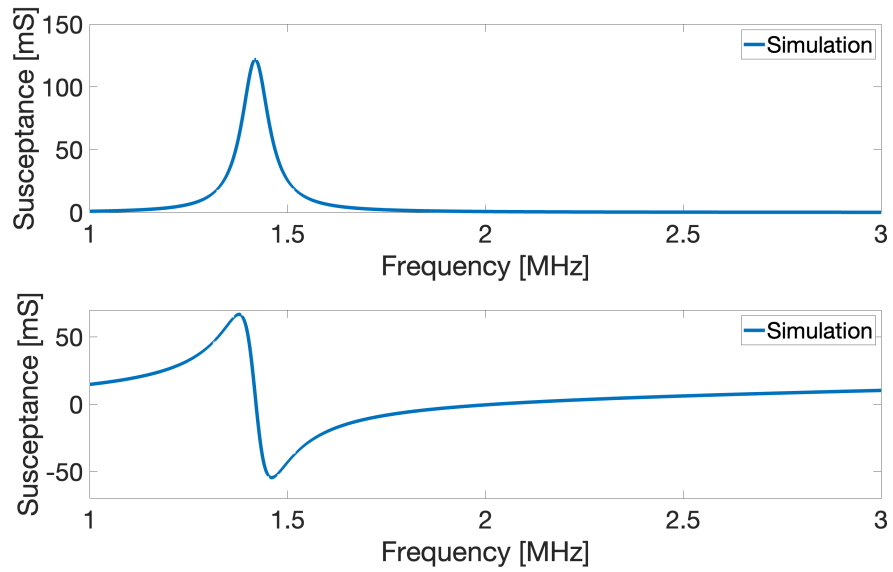


Figure 5.12: Calculation result for the admittance of PZTc204 disc-shaped 10mm diameter 1mm thickness transducer in the longitudinal vibration mason equivalent circuit with air and water loads (Resonance frequency: 1.42MHz; Conductance: 120.97mS)

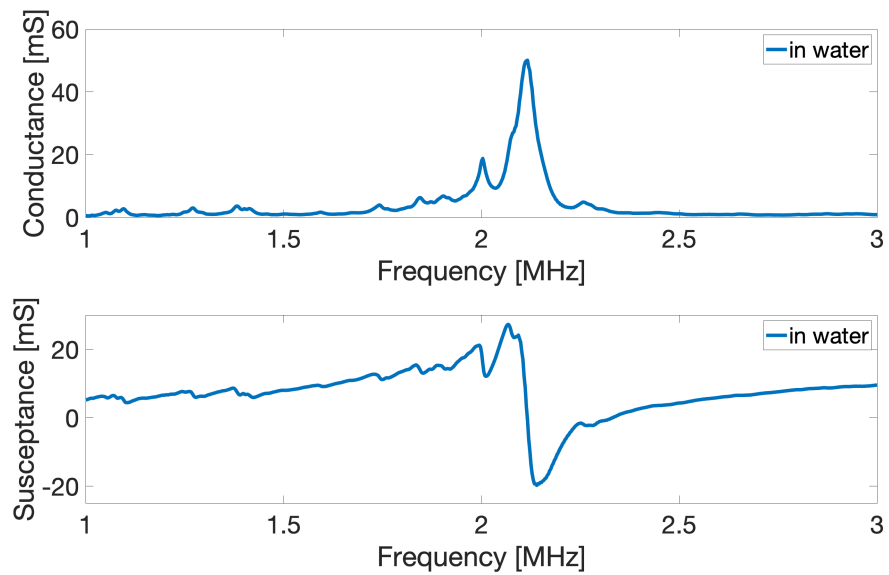


Figure 5.13: Measurement result for the admittance of the actuator using PZTc204 disc-shaped 10mm diameter 1mm thickness transducer in water (Resonance Frequency: 2.11MHz; Conductance: 49.92mS)

circuit, the conductance at the resonance frequency decreases from 2041.14 mS to 121.0 mS compared to the case when both ports are air loaded, even though the resonance frequency remains unchanged. Comparing Figures 5.6 and 5.13, it can be seen that the admittance characteristic of the actuator in water is 0.2 MHz lower at the mechanical series resonance frequency f_s and the conductance at the resonance frequency f_s is 35.7 mS lower than that of the actuator in air.

In conclusion, although there is still a significant difference between the admittance characteristics of water and air calculated by Mason equivalent circuit and the measured admittance characteristics, the pattern of decrease in conductance at the resonance frequency f_s and the resonance frequency f_s was found to be the same.

It can be seen above that by applying different loads to the two ports of the mason equivalent circuit, the upper and bottom boundary states of the disc-shaped vibrator can be represented. As shown in Figure 5.8, the currents flowing through the two ports have vibration velocities v_1 and v_2 , which means that the vibration velocity distributions at both ends of the disc-shaped vibrator at the resonant frequency can be calculated using the mason equivalent circuit under resonant conditions. Therefore, the calculation of the vibration velocity distribution using the mason equivalent circuit is attempted in the next section.

5.1.5 Vibration Velocity Distribution

According to Figure 5.8, the F_1 and F_2 ends of the Mason equivalent circuit are subjected to loads representing the boundary conditions between the upper and bottom end faces of the disc-shaped vibrator, and the currents at the two ports can represent the vibration velocity of the upper and bottom end faces of the disc-shaped vibrator. The vibration velocity distribution (2) is then calculated according to Figures 5.9 and 5.11 for the case where the bottom end face is in contact with air only and the case where the upper end face is in contact with air and water, respectively.

The calculated vibration velocity v_2 of the F_2 port in the two cases shown in Figures 5.9 and 5.11 is shown in Figure 5.14. In this case, the origin (0, 0) represents the centre of the circle on the top end face of the disc-shaped vibrator. The blue colour shows the distribution of the vibration velocity v_2 with respect to the diameter of the top end face in the case of Figure 5.9, and the orange colour shows the distribution of the vibration velocity v_2 with respect to the diameter of the top end face in the case of Figure 5.11. Equation 5.42 shows that the vibration velocity is independent of position ($'z'$ or $'r'$), thus the vibration velocity distribution appears linear.

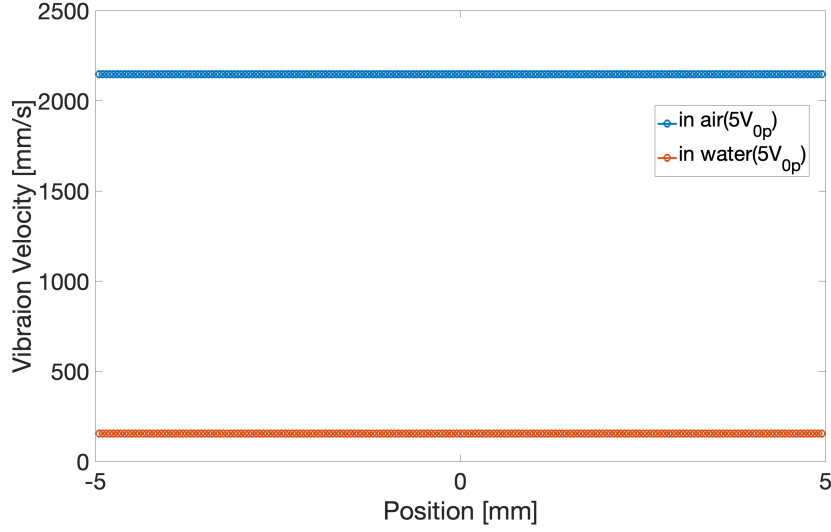


Figure 5.14: Calculation result for the vibration velocity distribution of PZTc204 disc-shaped 10mm diameter 1mm thickness transducer in air(blue) and water(orange). (Input Voltage: resonance frequency [MHz]: 1.42(air) and 1.42(water); amplitude [V_{op}]: 5)

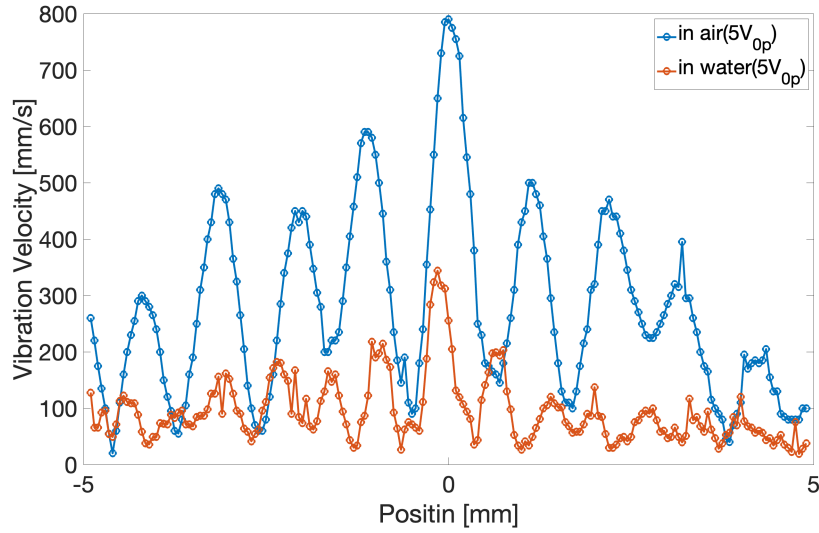


Figure 5.15: Measurement result for the admittance of the actuator using PZTc204 disc-shaped 10mm diameter 1mm thickness transducer in air(blue) and water(orange) (Input Voltage: resonance frequency [MHz]: 2.13(air) and 2.11(water); amplitude [V_{op}]: 5)

The vibration velocity v_2 when the upper end surface is in contact with air is 2147.5 mm/s at a frequency of 1.42 MHz and a drive voltage of $5V_{0p}$ amplitude, while the vibration velocity v_2 when the upper end surface is in contact with water is 156.1 mm/s at a frequency of 1.42 MHz and a drive voltage of $5V_{0p}$ amplitude. The vibration velocity of the upper end surface when the upper end surface is in contact with water is higher than the vibration velocity of the upper end surface when in contact with air. The vibration velocity is 1991.4 mm/s faster than the vibration velocity when in contact with water.

For comparison, the same conditions as described above for the calculation of the vibration velocity were maintained with the bottom surface of the disc-shaped vibrator always in contact with air and the upper surface of the disc-shaped vibrator in contact with air and water respectively. The measured vibration velocity distribution over the diameter of the top surface at a frequency of 2.13 MHz (in air) and 2.11 MHz (in water) and a drive voltage of $5V_{0p}$ amplitude is shown in Figure 5.15. Blue shows the vibration distribution on the diameter when the top surface is in contact with air. Orange shows the vibration distribution on the diameter when the top surface is in contact with water. The coordinate origin (0,0) indicates the centre of the circle of the top surface.

The measurement results shown in Figure 5.15 indicate that the vibration velocity distribution on the surface of a circular plate vibrator of PZTc204 with a diameter of 10 mm and thickness of 1 mm has about 11 peaks for both air and water in terms of shape, with the peak at the centre of the circle increases. The peaks at the centre of the circle are the largest and decrease with increasing distance from the centre of the circle. Numerically, the value of the largest peak in air is 790.0 mm/s and the value of the largest peak in water is 344.0 mm/s, with a difference of 446.0 mm/s.

This subsection compares calculated and measured vibration velocity distributions and shows that Mason longitudinal vibration equivalent circuit model does not accurately represent the vibration velocity distribution of a 10 mm diameter, 1 mm thickness circular disc-shaped PZTc204 vibrator, either in the form of a distribution or numerically. Therefore, the electromechanical equivalent circuit model needs to be improved.

5.1.6 Conclusion

In this section, thickness vibration is considered through an equivalent circuit model of longitudinal vibration. In the case of PZTc204, a disk-shaped vibrator with a diameter of 10 mm and a thickness of 1 mm was used to compare the calculation results of the equivalent circuit model of longi-

tudinal vibration with the measured results, and it was concluded that the equivalent circuit model of longitudinal vibration cannot accurately represent the thickness vibration characteristics.

1 Calculation results for admittance characteristics:

In air: resonance frequency f_s : 1.42 MHz; Conductance at resonance frequency: 2041.1 mS.

In water: resonant frequency f_s : 1.42 MHz; Conductance at resonance frequency: 121.0 mS.

2.1 Calculation results for vibration velocity distribution:

In air ($5V_{0p}$): Shape: straight line; Maximum vibration velocity: 2147.5 mm/s.

In water ($5V_{0p}$): Shape: straight line; Maximum vibration velocity: 156.1 mm/s.

2.2 Measurement results for vibration velocity distribution:

In air ($5V_{0p}$): Shape: approximately 11 peaks, with the largest peak at the centre of the circle and smaller peaks away from the centre of the circle; Maximum vibration velocity: 790 mm/s.

In water ($5V_{0p}$): Shape: approximately 11 peaks, with the largest peak at the centre of the circle, smaller peaks away from the centre of the circle; Maximum vibration velocity: 344 mm/s.

From the above, it was found that the equivalent circuit model considering only longitudinal vibration cannot accurately represent thickness vibration. As shown in Figure 5.1, in addition to longitudinal (z) vibration, there is also radial (r) and rotational (θ) vibration. From the vibration distribution measurements presented in Chapter 2, it is known that the vibration distribution is centrosymmetrical, thus the vibration in the direction of rotation (θ) can be neglected, leaving only the vibration in the radial direction (r).

The next section is therefore to consider whether radial vibration is present and longitudinal vibration may show multiple peaks in the distribution. For ease of calculation, longitudinal vibrations are ignored and only the radial vibration characteristics of the disc-shaped vibrator in the piezoelectric transverse effect are considered.

5.2 Radial Vibration

Consider the disk-shaped vibrator shown in Figure 5.16, which is subjected to an electric field in the polarised thickness direction and symmetrical elongational vibration in the radial direction. The coordinates are the same as for longitudinal vibration, with the direction of polarisation in the z -axis direction and the vibration direction in the r -axis direction. The disk-shaped vibrator has a diameter of 10 mm and a thickness of 1 mm, and the boundary condition is that the side surface is free. In order to make the electric field positive in the same upward direction as the polarisation, the upper electrode is electrically grounded and a voltage $V = V_0 e^{j\omega t}$ at an angular frequency ω is given to the bottom electrode as input.

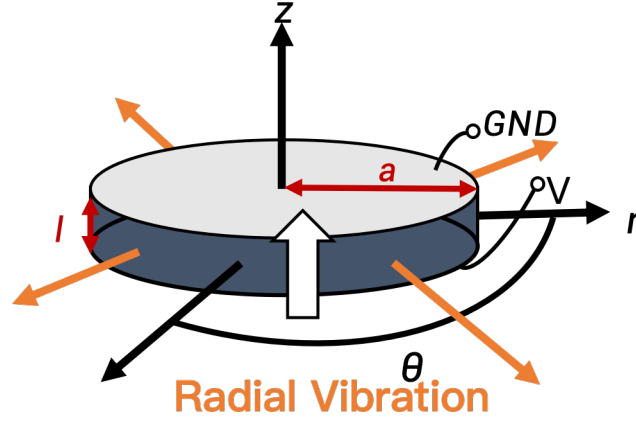


Figure 5.16: Radial Vibration

In symmetric elongational vibration, the distortion is considered S_{rr} in the vibration direction and $S_{\theta\theta} = \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}$ which contains the displacement function $u_r(r, t)$ in the r direction, and zero otherwise. In other words, with the condition that stresses other than T_{rr} and $T_{\theta\theta}$, flux densities other than D_z and electric fields other than E_z are zero, and without using the approximation $S_{zz} = 0$, from Equations 5.1 and 5.2, the relationship

$$\begin{aligned} S_{rr} &= s_{11}^E T_{rr} + s_{12}^E T_{\theta\theta} + d_{31} E_z \\ S_{\theta\theta} &= s_{12}^E T_{rr} + s_{11}^E T_{\theta\theta} + d_{31} E_z \\ D_z &= d_{31} T_{rr} + d_{31} T_{\theta\theta} + \varepsilon_{33}^T E_z \end{aligned} \quad (5.47)$$

is obtained. Transforming Equations 5.47 in piezoelectric d -form so that the left-hand side is the stresses T_{rr} and $T_{\theta\theta}$ and the flux density D_z gives

$$\begin{aligned}
T_{rr} &= \overline{c_{11}^E} S_{rr} + \overline{c_{12}^E} S_{\theta\theta} - \overline{e_{31}} E_z \\
T_{\theta\theta} &= \overline{c_{12}^E} S_{rr} + \overline{c_{11}^E} S_{\theta\theta} - \overline{e_{31}} E_z \\
D_z &= \overline{e_{31}} S_{rr} + \overline{e_{31}} S_{\theta\theta} + \overline{\varepsilon_{33}^{S_{rr}}} E_z.
\end{aligned} \tag{5.48}$$

However, $\overline{c_{11}^E} = \frac{s_{11}^E}{(s_{11}^E)^2 - (s_{12}^E)^2}$, $\overline{c_{12}^E} = \frac{-s_{12}^E}{(s_{11}^E)^2 - (s_{12}^E)^2}$, $\overline{e_{31}} = \frac{d_{31}}{s_{11}^E + s_{12}^E}$ and $\overline{\varepsilon_{33}^{S_{rr}}} = \varepsilon_{33}^T - \frac{2d_{31}^2}{s_{11}^E + s_{12}^E}$. Also, by $\frac{\partial T_{r\theta}}{\partial \theta} = \frac{\partial T_{rz}}{\partial z} = 0$, the wave equation is

$$\rho \frac{\partial^2 u_r}{\partial t^2} = \frac{\partial T_{rr}}{\partial r} + \frac{T_{rr} - T_{\theta\theta}}{r}. \tag{5.49}$$

As a piezoelectric body is a dielectric,

$$\frac{\partial D_z}{\partial z} = 0 \tag{5.50}$$

follows from Gauss's divergence theorem, assuming that there is no isolated charge inside it. Substituting this into the third equation in 5.48,

$$\frac{\partial D_z}{\partial z} = \overline{e_{31}} \left(\frac{\partial S_{rr}}{\partial z} + \frac{\partial S_{\theta\theta}}{\partial z} \right) + \overline{\varepsilon_{33}^{S_{rr}}} \frac{\partial E_z}{\partial z} = \overline{\varepsilon_{33}^{S_{rr}}} \frac{\partial E_z}{\partial z} = 0 \tag{5.51}$$

was obtained. The fact that the strain S_{rr} and $S_{\theta\theta}$ are independent of z ($\frac{\partial S_{rr}}{\partial z} + \frac{\partial S_{\theta\theta}}{\partial z} = 0$) was used to deform this equation. Equation 5.51 results in $\frac{\partial E_z}{\partial z} = 0$, which shows that the electric field E_z is independent of z and constant. The relationship between the electric field and the input voltage, taking into account that the bottom electrode $V_0 e^{j\omega t}$ is added, is

$$0 - V_0 e^{j\omega t} = - \int_{-\frac{l}{2}}^{\frac{l}{2}} E_z dz. \tag{5.52}$$

Therefore,

$$E_z = \frac{V_0 e^{j\omega t}}{l} \tag{5.53}$$

and the electric field are obtained.

And

$$\begin{aligned}
S_{rr} &= \frac{\partial u_r}{\partial r} \\
S_{\theta\theta} &= \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} = \frac{u_r}{r},
\end{aligned} \tag{5.54}$$

thus the wave equation is

$$\rho \frac{\partial^2 u_r}{\partial t^2} = \overline{c_{11}^E} \left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} \right). \tag{5.55}$$

Here, the electric field E_z is constant with respect to the r -axis direction ($\frac{\partial E_z}{\partial r} = 0$).

Let

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\omega^2}{v_r^2} \frac{\partial^2 u_r}{\partial t^2} = 0 \tag{5.56}$$

be the displacement $u_{r,t} = u(r)e^{j\omega t}$ in Equation 5.55. However, $v_r = \sqrt{\frac{c_{11}^E}{\rho}}$. Therefore, this solution is given by

$$u_r(r, t) = A J_1\left(\frac{\omega}{v_r} r\right), \tag{5.57}$$

using a function of the cylindrical coordinates. However, A is constant and $J_1(x)$ is a Bessel function of the first kind.

Because boundary conditions are

$$\begin{aligned}
r = 0 &\Rightarrow u_r = 0 : \text{automatically satisfied;} \\
r = a &\Rightarrow T_{rr} = 0,
\end{aligned}$$

When $r = a$,

$$\overline{c_{11}^E} \frac{\partial u_r}{\partial r} + \overline{c_{12}^E} \frac{u_r}{r} = \overline{e_{31}} E_z. \tag{5.58}$$

Here, because of

$$J'_n(x) = -\frac{n}{x} J_n(x) + J_{n-1}(x) \tag{5.59}$$

the undetermined constant A is given by

$$A = \frac{\overline{e}_{31} E_z a}{\overline{c}_{11}^E(\frac{\omega a}{v_r}) J_0(\frac{\omega a}{v_r}) + (\overline{c}_{12}^E - \overline{c}_{11}^E) J_1(\frac{\omega a}{v_r})}. \quad (5.60)$$

Substituting the value of A into equation 5.57,

$$u_r(r, t) = \frac{\overline{e}_{31} E_z a}{\overline{c}_{11}^E(\frac{\omega a}{v_r}) J_0(\frac{\omega a}{v_r}) + (\overline{c}_{12}^E - \overline{c}_{11}^E) J_1(\frac{\omega a}{v_r})} J_1(\frac{\omega}{v_r} r) \quad (5.61)$$

becomes a function of the displacement distribution. The resonance condition is

$$\overline{c}_{11}^E(\frac{\omega a}{v_r}) J_0(\frac{\omega a}{v_r}) = (\overline{c}_{11}^E - \overline{c}_{12}^E) J_1(\frac{\omega a}{v_r}) \quad (5.62)$$

from the condition where $u_r(r, t)$ is maximal.

The resonance conditions and displacement functions u_r obtained, the material parameters in Table 5.1 can be used to investigate the radial vibration distribution.

5.2.1 Vibration Distribution

In this section, the vibration distribution of the radial vibration is examined based on the displacement function $u_r(r, t)$ derived in the previous section. Equation 5.61 contains a first-order Bessel function, which is often used to describe vibration distributions, and is characterised by the presence of multiple peaks and the tendency for the peaks to gradually decrease in size, which is consistent with the characteristics of the vibration distribution shown in Figure 5.15. Therefore, the influence of the radial displacement function on the longitudinal vibration distribution is investigated to test whether the radial vibration may deform the longitudinal piston vibration into a multi-peak vibration distribution.

If $X = \frac{\omega a}{v_r}$, from the resonance condition equation 5.62, the functions on both sides of the equation can be displayed as in Figure 5.17, and the value of the co-ordinates of the intersection of the two curves (blue '*') can be expressed as a multiple solution of equation $X_n (n = 1, 2, 3, \dots)$ giving the n -th resonance angular frequency.

Analysis of the displacement function shows that the constant A determines the amplitude and the first-order Bessel function determines the vibration distribution. As only the vibration distribution is considered here,

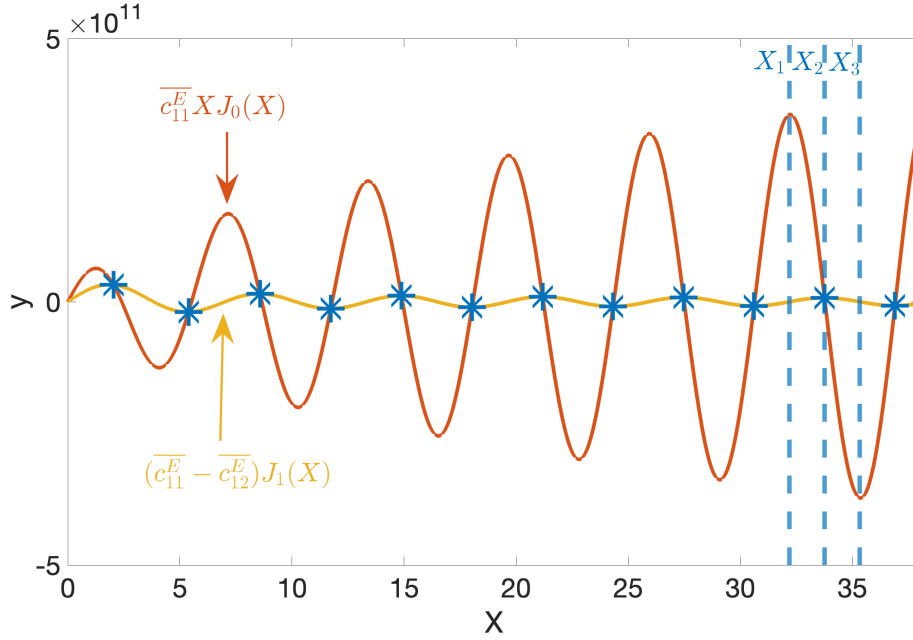


Figure 5.17: Resonance Angular Frequency of Radial Vibration

it is sufficient to examine only the first-order Bessel function part. Substituting X_n obtained from Figure 5.17 into the first-order Bessel function of the displacement function gives $J_1(\frac{X}{a}r)$, and taking the radius r from 0 to a , the vibration distribution exhibited by the radial vibration at the n -th resonant angular frequency can be determined. If a vibration distribution with about 11 peaks as shown in Figure 5.15, a vibration distribution with about 6 peaks on the radius (including the centre of the circle), can be obtained at the angular frequency between the intersection of the 10th and 12th blue '*' in Figure 5.17, it can be seen that a vibration distribution with 6 peaks is obtained.

However, considering only the vibration distribution in the radial direction, it is not intuitive to determine whether the expected vibration distribution in the longitudinal direction (z -direction) can be obtained. Therefore, the influence of the radial vibration on the longitudinal vibration distribution is investigated.

Transforming the piezoelectric equation in equation 5.1 into the form $[T] = [c][S] - [e][E]$ yields

$$T_{zz} = c_{13}^E(S_{rr} + S_{\theta\theta}) + c_{33}^E S_{zz} - e_{33}E_z, \quad (5.63)$$

where c_{13}^E is the stiffness coefficient, which is negative in PZT piezoelectric

materials because of the strong coupling between stress and strain. To simplify the calculation, the term E_z is deleted as T_{zz} is not affected by E_z , and assuming $T_{zz} = 0$,

$$\frac{\partial u_z}{\partial z} = -\frac{c_{13}^E}{c_{33}^E} \left(\frac{\partial u_r}{\partial r} - \frac{u_r}{r} \right) \quad (5.64)$$

can be obtained by simplifying the equation using Equation 5.59 and integrating both sides of the equation with z to obtain

$$u_z = -\frac{c_{13}^E}{c_{33}^E} A \left(\frac{\omega}{v_r} \right) \int J_0 \left(\frac{\omega}{v_r} r \right) dz \quad (5.65)$$

Here, u_z represents the longitudinal displacement function.

Analysis of the u_z displacement function shows that the zeroth-order Bessel function determines the vibration distribution and the remaining terms determine the vibration amplitude. Noting the positive and negative vibration amplitudes are also related to the vibration distribution and cannot be ignored. Observing Equation 5.60, it is known that the numerator e_{31} is negative, but the denominator is $+$, ∞ and $-$ throughout Figure 5.17, thus the constant A is $-$, $-\infty$ and $+$ as shown in Figure 5.17 for X_1 , X_2 and X_3 .

In summary, combining the positives and negatives of the coefficients as well as $J_0(\frac{X_{1,2,3}}{a}r)$ under X_1 , X_2 and X_3 results in different distributions of u_z in different cases, as shown in Figures 5.18, 5.19 and 5.20.

As can be seen from Figures 5.18, 5.19 and 5.20, the vibration in the radial direction can indeed affect the vibration distribution in the longitudinal direction. In particular, as shown in Figure 5.18 and 5.19, although there are multiple peaks in the longitudinal vibration distribution at the negative value of the constant A or at the resonance angular frequency of the vibration in the radial direction, it can be clearly found that the measured results of the vibration distribution as shown in Figure 5.15 are positively or negatively reversed in shape. The vibration distribution shown in Figure 5.20 is similar in shape to the measurement results of the vibration distribution shown in Figure 5.15, with six peaks from the centre of the circle to the circumference, and the peaks further away from the centre of the circle are smaller.

In summary, it can be seen that in the case where the constant A is positive and the frequency is between the two neighbouring resonance frequencies of radial vibrations, the radial vibration can make the purely longitudinal piston vibration distribution change to the vibration distribution of thickness vibration.

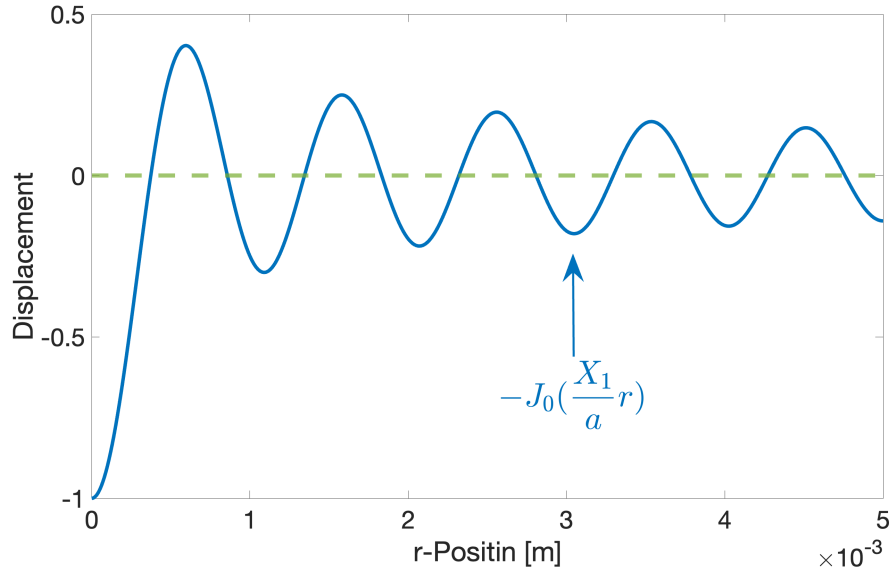


Figure 5.18: Longitudinal vibration distribution caused by radial vibration. ((0,0) is the centre of the circle on the surface of the vibrator in the longitudinal (z -axis) direction; X_1 : between the 10th and 11th resonance angular frequency).

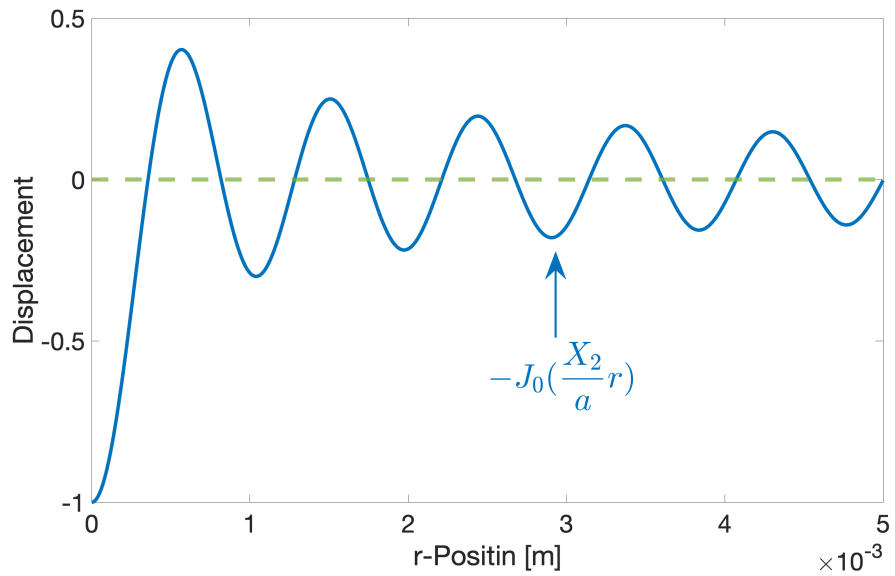


Figure 5.19: Longitudinal vibration distribution caused by radial vibration. ((0,0) is the centre of the circle on the surface of the vibrator in the longitudinal (z -axis) direction; X_2 : 11th resonance angular frequency).

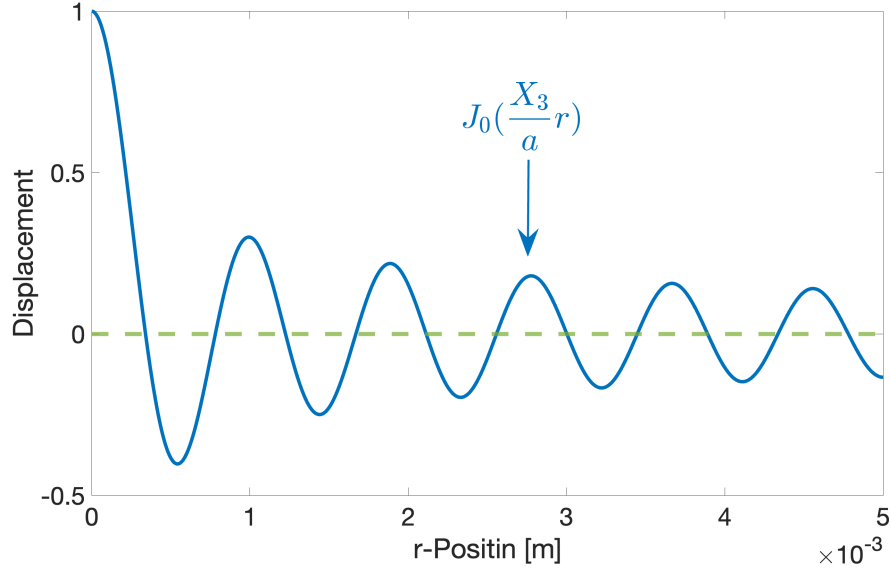


Figure 5.20: Longitudinal vibration distribution caused by radial vibration. $((0, 0)$ is the centre of the circle on the surface of the vibrator in the longitudinal (z -axis) direction; X_3 : between the 11th and 12th resonance angular frequency).

5.2.2 Conclusion

In this section, the displacement function $u_r(r, t)$ of the radially symmetric extension vibration of a disc-shaped vibrator is derived using the piezoelectric equation considering only the parameters in the radial direction. After analysing this displacement function u_r and converting it to longitudinal displacement, it is concluded that the symmetric extension vibration in the radial direction does have the possibility of transforming the piston vibration in the longitudinal direction into a thickness vibration. In other words, the thickness vibration may be a combination of the symmetric extension vibration in the radial direction and the piston vibration in the longitudinal direction.

Therefore, in the next section, thickness vibration will be investigated using the piezoelectric equation that preserves the parameters in the longitudinal (z) and radial (r) directions.

5.3 Thickness Vibration

In this section, an attempt will be made to derive the electromechanical equivalent circuit for thickness vibrations using the piezoelectric equation.

The piezoelectric longitudinal effect, where the polarisation direction of the disc-shaped vibrator is in the thickness direction and the thickness vibration is excited by an AC electric field parallel to the polarisation direction, is taken up. The same thickness vibration as the longitudinal vibration is excited under constant electric flux density, but the electric field distribution is a function of position ' z ' and ' r '. The effect of the anti-electric field inside the piezoelectric body associated with the same thickness vibration as the longitudinal vibration also occurs.

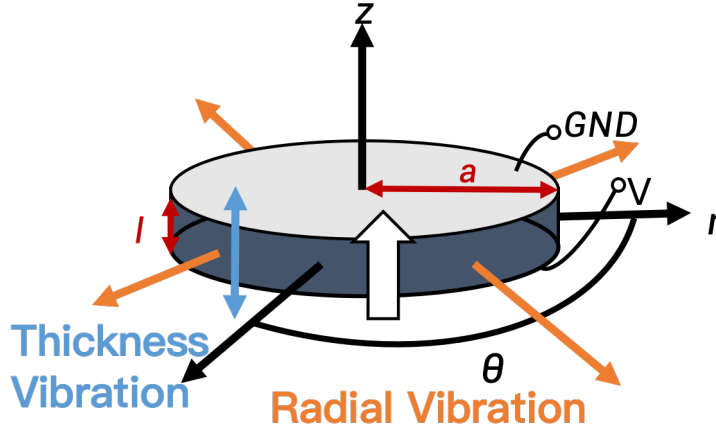


Figure 5.21: Thickness Vibration

The coordinates are taken in accordance with the general rules for considering piezoelectric vibrations, with the direction of polarisation in the z -axis direction (3-axis direction). The size of the vibrator is assumed to be radius ' a ' and thickness ' l '. The mechanical boundary conditions are free at both surfaces. In the case of thickness vibration, the effect is the piezoelectric longitudinal effect, which is the 33 effect because the electric field is applied in the 3-axis direction and the distortion in the z -axis direction (3-axis direction) parallel to this is considered.

To ensure that the sign of the electric field E_z is also positive when the sign of the input voltage V is positive. The electrical boundary conditions are as follows: the upper electrode at $z = l/2$ is electrically grounded and the voltage $V = V_0 e^{j\omega t}$ with angular frequency ω is input to the lower electrode at $z = -l/2$.

Because of the thickness vibration, the electric field of E_r and E_θ , the flux density of D_r and D_θ , and the stress of $T_{\theta z}$ and $T_{r\theta}$ are set to zero, leaving S_{rr} , $S_{\theta\theta}$, S_{zz} , S_{rz} , E_z and D_z in the end because the strains S_{rr} , $S_{\theta\theta}$, S_{rz} have r and z components. In addition, T_{rr} and $T_{\theta\theta}$ are not excited by the electric field E_z , but are expected to be excited by the stress T_{zz} , thus remove d_{31} from the piezoelectric equations.

$$\begin{aligned} T_{rr} &= c_{11}^E S_{rr} + c_{12}^E S_{\theta\theta} + c_{13}^E S_{zz} \\ T_{\theta\theta} &= c_{12}^E S_{rr} + c_{11}^E S_{\theta\theta} + c_{13}^E S_{zz} \\ T_{zz} &= c_{13}^E S_{rr} + c_{13}^E S_{\theta\theta} + c_{33}^E S_{zz} - e_{33} E_z \\ T_{rz} &= c_{44}^E S_{rz} \\ D_z &= e_{33} S_{zz} + \varepsilon_{33}^S E_z \end{aligned} \quad (5.66)$$

can be obtained by converting $[S] = [s][T] + [d][E]$ to $[T] = [c][S] - [e][E]$. However, The matrix $[c]$ represents the material stiffness parameters, $c_{11}^E = \frac{s_{11}^E s_{33}^E - (s_{13}^E)^2}{(s_{11}^E - s_{12}^E)[s_{33}^E(s_{11}^E + s_{12}^E) - 2(s_{13}^E)^2]}$, $c_{12}^E = \frac{-s_{12}^E s_{33}^E + (s_{13}^E)^2}{(s_{11}^E - s_{12}^E)[s_{33}^E(s_{11}^E + s_{12}^E) - 2(s_{13}^E)^2]}$, $c_{13}^E = \frac{-s_{13}^E}{s_{33}^E(s_{11}^E + s_{12}^E) - 2(s_{13}^E)^2}$, $c_{33}^E = \frac{s_{11}^E + s_{12}^E}{s_{33}^E(s_{11}^E + s_{12}^E) - 2(s_{13}^E)^2}$, $c_{44}^E = \frac{1}{s_{44}^E}$, $e_{33} = 2d_{31}c_{13}^E + d_{33}c_{33}^E$, $\varepsilon_{33}^S = \varepsilon_{33}^T - 2d_{31}e_{31} - d_{33}e_{33}$. [7] Because of the thickness vibration, displacements $u_r(r, z, t)$ and $u_z(r, z, t)$ are considered. In addition, $T_{r\theta} = 0$ and $T_{\theta z} = 0$, thus the equation of motion is given by [7, 8]

$$\rho \frac{\partial^2 u_r}{\partial t^2} = \frac{\partial T_{rr}}{\partial r} + \frac{\partial T_{rz}}{\partial z} + \left(\frac{T_{rr} - T_{\theta\theta}}{r} \right) \quad (5.67)$$

$$\rho \frac{\partial^2 u_z}{\partial t^2} = \frac{\partial T_{rz}}{\partial r} + \frac{\partial T_{zz}}{\partial z} + \frac{T_{rz}}{r}. \quad (5.68)$$

By substituting 1st, 2nd, 3rd, 4th equations of Equation 5.66 for Equation 5.67 and 5.68, the equation of motion becomes

$$\rho \frac{\partial^2 u_r}{\partial t^2} = c_{11}^E \frac{\partial S_{rr}}{\partial r} + c_{12}^E \frac{\partial S_{\theta\theta}}{\partial r} + c_{13}^E \frac{\partial S_{zz}}{\partial r} + c_{44}^E \frac{\partial S_{rz}}{\partial z} + (c_{11}^E - c_{12}^E) \frac{S_{rr} - S_{\theta\theta}}{r} \quad (5.69)$$

$$\rho \frac{\partial^2 u_z}{\partial t^2} = c_{44}^E \frac{\partial S_{rz}}{\partial r} + c_{13}^E \frac{\partial S_{rr}}{\partial z} + c_{13}^E \frac{\partial S_{\theta\theta}}{\partial z} + c_{33}^E \frac{\partial S_{zz}}{\partial z} - e_{33} \frac{\partial E_z}{\partial z} + c_{44}^E \frac{S_{rz}}{r}. \quad (5.70)$$

In addition, the piezoelectric material has no internal charge, thus

$$\text{div} \begin{pmatrix} D_r \\ D_\theta \\ D_z \end{pmatrix} = 0, \quad (5.71)$$

and the only component of the electric flux density is D_z , thus $\frac{\partial D_z}{\partial z} = 0$ is given. Then, the differential of both sides of the 5th equation of 5.66 by z leads to

$$\frac{\partial D_z}{\partial z} = e_{33} \frac{\partial^2 u_z}{\partial z^2} + \varepsilon_{33}^S \frac{\partial E_z}{\partial z} = 0 \quad (5.72)$$

which gives

$$-\frac{\partial E_z}{\partial z} = \frac{e_{33}}{\varepsilon_{33}^S} \frac{\partial^2 u_z}{\partial z^2} \quad (5.73)$$

as the condition for the electric field. In the piezoelectric longitudinal effect, the electric field is a function of distortion, which is the origin of the anti-electric field.

By substituting 5.73 into the equation of motion for 5.70, and because [9]

$$\begin{aligned} S_{rr} &= \frac{\partial u_r}{\partial r}, \\ S_{\theta\theta} &= \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} = \frac{u_r}{r} \\ S_{rz} &= \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \\ S_{zz} &= \frac{\partial u_z}{\partial z}, \\ c_{33}^D &= c_{33}^E + \frac{e_{33}^2}{\varepsilon_{33}^S}. \end{aligned} \quad (5.74)$$

The wave equations for the displacement functions $u_r(r, z, t)$ and $u_z(r, z, t)$ are obtained as follows.

$$\rho \frac{\partial^2 u_r}{\partial t^2} = c_{11}^E \left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} \right) + c_{44}^E \frac{\partial^2 u_r}{\partial z^2} + (c_{44}^E + c_{13}^E) \left(\frac{\partial^2 u_z}{\partial z \partial r} \right) \quad (5.75)$$

$$\rho \frac{\partial^2 u_z}{\partial t^2} = c_{44}^E \left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} \right) + c_{33}^D \frac{\partial^2 u_z}{\partial z^2} + (c_{44}^E + c_{13}^E) \left(\frac{\partial^2 u_r}{\partial r \partial z} + \frac{1}{r} \frac{\partial u_r}{\partial z} \right) \quad (5.76)$$

Assuming that the 2nd and 3rd terms on the right-hand side of the wave equations 5.75 and 5.76 is equal to the constant multiplied by u_r and u_z respectively, the wave equations can be transformed into the following equations respectively.

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\omega^2}{v_r^2} u_r = 0 \quad (5.77)$$

$$\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} + \frac{\omega^2}{v_r^2} u_z = 0 \quad (5.78)$$

However, v_r is a constant for the speed of sound in the radial direction. Using the function of the cylindrical coordinates by the separation of variables method, the general solution is as follows.

$$u_r = [A \cos(\frac{\omega}{v_z} z) + B \sin(\frac{\omega}{v_z} z)] J_1(\frac{\omega}{v_r} r) e^{j\omega t} \quad (5.79)$$

$$u_z = [C \cos(\frac{\omega}{v_z} z) + D \sin(\frac{\omega}{v_z} z)] J_0(\frac{\omega}{v_r} r) e^{j\omega t} \quad (5.80)$$

However, v_z is a constant for the speed of sound in the thickness direction, while A, B, C and D are constants.

The boundary conditions are

$$\begin{aligned} r = 0 &\Rightarrow u_r = 0 : \text{automatically satisfied;} \\ z = 0 &\Rightarrow u_z = 0 : C = 0. \end{aligned}$$

and u_r is equal constant multiplication $\frac{\partial^2 u_z}{\partial z \partial r}$, thus $B = 0$. Therefore the displacement functions u_r and u_z are as follows.

$$u_r = A \cos(\frac{\omega}{v_z} z) J_1(\frac{\omega}{v_r} r) e^{j\omega t} \quad (5.81)$$

$$u_z = D \sin(\frac{\omega}{v_z} z) J_0(\frac{\omega}{v_r} r) e^{j\omega t} \quad (5.82)$$

5.3.1 Speed of Sound v_r and v_z

since

$$\begin{aligned} J'_n(x) &= -\frac{n}{x} J_n(x) + J_{n-1}(x) \\ J'_n(x) &= -\frac{n}{x} J_n(x) - J_{n+1}(x), \end{aligned}$$

5.81 and 5.82 are substituted into 5.75 and 5.76 to obtain the following equation.

$$\frac{\rho\omega^2}{c_{11}^E} - \frac{c_{44}^E}{c_{11}^E} \left(\frac{\omega}{v_z}\right)^2 - \frac{c_{44}^E + c_{13}^E}{c_{11}^E} \frac{\omega}{v_z} \frac{\omega}{v_r} \frac{D}{A} = \left(\frac{\omega}{v_r}\right)^2 \quad (5.83)$$

$$\frac{\rho\omega^2}{c_{44}^E} - \frac{c_{33}^D}{c_{44}^E} \left(\frac{\omega}{v_z}\right)^2 - \frac{c_{44}^E + c_{13}^E}{c_{44}^E} \frac{\omega}{v_z} \frac{\omega}{v_r} \frac{A}{D} = \left(\frac{\omega}{v_r}\right)^2 \quad (5.84)$$

Integrating 5.83 and 5.84 gives the following equation.

$$(c_{44}^E - \rho v_z^2 + c_{11}^E \left(\frac{v_z}{v_r}\right)^2)(c_{44}^E - \rho v_r^2 + c_{33}^D \left(\frac{v_r}{v_z}\right)^2) = (c_{44}^E + c_{13}^E)(c_{44}^E + c_{13}^E) \quad (5.85)$$

From this equation, v_z and v_r can be solved as follows

$$v_z = \sqrt{\frac{\frac{c_{33}^D c_{11}^E - (c_{13}^E)^2}{c_{11}^E + c_{13}^E}}{\rho}}, v_r = \sqrt{\frac{\frac{c_{33}^D c_{11}^E - (c_{13}^E)^2}{c_{33}^D + c_{13}^E}}{\rho}}. \quad (5.86)$$

5.3.2 Equivalent Circuit

Because the mechanical boundary conditions are free at both surfaces ($z = \pm \frac{l}{2}$) in the thickness direction and ($r = a$) in the radial direction, the stress $T_{zz} = 0$ and $T_{rr} = 0$ must always be independent of time t when $z = \pm \frac{l}{2}$, $r = a$ are substituted in the general solution of Equations 5.81 and 5.82. Because stress T_{zz} is a function of strain and flux density, the 3rd and 5th of piezoelectric equations 5.66 lead to

$$T_{zz} = c_{13}^E(S_{rr} + S_{\theta\theta}) + c_{33}^D S_{zz} - \frac{e_{33}}{\varepsilon_{33}^S} D_z. \quad (5.87)$$

To satisfy the boundary conditions $T_{zz} = 0$ and $T_{rr} = 0$ at $z = \pm \frac{l}{2}$, $r = a$, D_z is constant regardless of position, which obtains the two relations

$$\begin{aligned} T_{rr}|_{z=\pm \frac{l}{2}, r=a} &= c_{11}^E A \cos\left(\frac{l\omega}{2v_z}\right) \left(\frac{\omega}{v_r} J_0\left(\frac{\omega}{v_r} a\right) - \frac{1}{a} J_1\left(\frac{\omega}{v_r} a\right)\right) e^{j\omega t} \\ &+ c_{12}^E A \cos\left(\frac{l\omega}{2v_z}\right) \frac{1}{a} J_1\left(\frac{\omega}{v_r} a\right) e^{j\omega t} \\ &+ c_{13}^E \frac{\omega}{v_z} D \cos\left(\frac{l\omega}{2v_z}\right) J_0\left(\frac{\omega}{v_r} a\right) e^{j\omega t} = 0 \end{aligned} \quad (5.88)$$

$$T_{zz}|_{z=\pm \frac{l}{2}, r=a} = (c_{13}^E \frac{\omega}{v_r} A + c_{33}^D \frac{\omega}{v_z} D) \cos\left(\frac{l\omega}{2v_z}\right) J_0\left(\frac{\omega}{v_r} a\right) e^{j\omega t} - \frac{e_{33}}{\varepsilon_{33}^S} D_z|_{r=a} = 0. \quad (5.89)$$

Because of the piezoelectric longitudinal effect, the electric flux density inside the vibrator is constant and the same as the charge density in the electrode at $z = \pm \frac{l}{2}$. Therefore, when the electrical terminals are opened, $D_z = 0$ should be set in the second term of the left side of Equation 5.89. Accordingly, from the Equation 5.89 and $A \neq 0$ and $D \neq 0$, what satisfies

$$\cos\left(\frac{l\omega_a}{2v_z}\right)J_0\left(\frac{\omega_a}{v_r}a\right) = 0 \quad (5.90)$$

corresponds to the anti-resonant angular frequency ω_a of the vibration mode that can be excited.

In the case of piezoelectric longitudinal effects, the resonance frequency ω_r cannot be determined from equation 5.89 because the flux density D_z depends on the displacement amplitudes A and D of the vibration mode being excited. Therefore, the electric field distribution E_z and the electric flux density D_z are determined. As shown in Equation 5.71, the charge inside the piezoelectric material does not depend on it, thus by using $\frac{\partial D_z}{\partial z} = 0$, which indicates that the flux density is constant for the variable z , Equation 5.72 can be transformed from the 5th of piezoelectric equations 5.66. Therefore, there is a relationship between the electric field and displacement as in Equation 5.73. Integrating both sides with z leads to

$$E_z = -\frac{e_{33}}{\varepsilon_{33}^S} \frac{\partial u_z}{\partial z} + M, \quad (5.91)$$

with $M(r)$ as the integration constant for variable z . The integration constant $M(r)$ for variable z is determined by the potential difference between the electrodes. As shown in Figure 5.21, the voltage $V = +V_0 e^{j\omega t}$ is given to the bottom electrode ($z = -\frac{l}{2}$), thus the potential difference from $z = -\frac{l}{2}$ to $z = \frac{l}{2}$ is $-V_0 e^{j\omega t}$, which is

$$\begin{aligned} 0 - V_0 e^{j\omega t} &= - \int_{-\frac{l}{2}}^{\frac{l}{2}} E_z dz \\ &= 2\left(\frac{e_{33}}{\varepsilon_{33}^S} D\right) \sin\left(\frac{l\omega}{2v_z}\right) J_0\left(\frac{\omega}{v_r} r\right) e^{j\omega t} - M(r)l \end{aligned} \quad (5.92)$$

using Equations 5.81 and 5.82, noting the direction of integration. By this potential condition, the integration constant $M(r)$ for the variable z is found to be

$$M(r) = \frac{2}{l} \frac{e_{33}}{\varepsilon_{33}^S} D \sin\left(\frac{l\omega}{2v_z}\right) J_0\left(\frac{\omega}{v_r} r\right) e^{j\omega t} + \frac{V_0 e^{j\omega t}}{l} \quad (5.93)$$

By this $M(r)$, the electric field E_z , shown by Equation 5.91, is expressed as

$$\begin{aligned} E_z = & -\frac{e_{33}}{\varepsilon_{33}^S} \frac{\omega}{v_z} D \cos\left(\frac{\omega}{v_z} z\right) J_0\left(\frac{\omega}{v_r} r\right) e^{j\omega t} \\ & + \frac{2}{l} \frac{e_{33}}{\varepsilon_{33}^S} D \sin\left(\frac{l\omega}{2v_z}\right) J_0\left(\frac{\omega}{v_r} r\right) e^{j\omega t} + \frac{V_0 e^{j\omega t}}{l} \end{aligned} \quad (5.94)$$

using the displacement functions $u_z(r, z, t)$ and $u_r(r, z, t)$. The first term on the right side of Equation 5.94 is a function of position dependent on the piezoelectric vibration distribution, while the second term is a value dependent only on the radial r position. This second term has the meaning of a correction term for the anti-electric field in the thickness direction at a certain position in the piezoelectric body.

Furthermore, substituting the 5th of piezoelectric equation 5.66 for the general solution of the strain, u_r , u_z and the electric field distribution E_z , to obtain the flux density D_z , which is

$$D_z = \frac{2}{l} e_{33} D \sin\left(\frac{l\omega}{2v_z}\right) J_0\left(\frac{\omega}{v_r} r\right) e^{j\omega t} + \varepsilon_{33}^S \frac{V_0 e^{j\omega t}}{l} \quad (5.95)$$

This confirms that the flux density D_z has a z -independent form and that the condition $\frac{\partial D_z}{\partial z} = 0$ is satisfied. In Equation 5.95, the second term on the right side corresponds to the charge density in the absence of non-piezoelectric bodies and piezoelectric distortion. On the other hand, the first term on the right side is the flux density component corresponding to the charge supplied by the power source to generate the correction term for the electric field in Equation 5.94.

Having obtained the relationship between the displacement amplitudes A and D of the vibration modes and the flux density $D_z|_{r=a}$, using Equations 5.88 and 5.89,

$$A = -\frac{ac_{13}^E \frac{\omega}{v_z} J_0\left(\frac{\omega}{v_r} a\right)}{c_{11}^E\left(\frac{\omega}{v_r} a\right) J_0\left(\frac{\omega}{v_r} a\right) + (c_{12}^E - c_{11}^E) J_1\left(\frac{\omega}{v_r} a\right)} D \quad (5.96)$$

$$\begin{aligned} & A \left[c_{13}^E \frac{\omega}{v_r} \cos\left(\frac{l\omega}{2v_z}\right) J_0\left(\frac{\omega}{v_r} a\right) \right] \\ & + D \left[c_{33}^D \frac{\omega}{v_z} \cos\left(\frac{l\omega}{2v_z}\right) J_0\left(\frac{\omega}{v_r} a\right) - \frac{e_{33}^2}{\varepsilon_{33}^S} \frac{2}{l} \sin\left(\frac{l\omega}{2v_z}\right) J_0\left(\frac{\omega}{v_r} a\right) \right] = \frac{e_{33} V_0}{l} \end{aligned} \quad (5.97)$$

can be obtained. Thus,

$$A = \frac{\frac{e_{33}}{c_{13}^E} \frac{v_r}{\omega} \frac{1}{\cos(\frac{l\omega}{2v_z}) J_0(\frac{\omega}{v_r} a)} \frac{V_0}{l}}{1 - \left[\frac{c_{11}^E(\frac{\omega}{v_r} a) J_0(\frac{\omega}{v_r} a) + (c_{12}^E - c_{11}^E) J_1(\frac{\omega}{v_r} a)}{\frac{(c_{13}^E)^2}{c_{33}^D}(\frac{\omega}{v_r} a) J_0(\frac{\omega}{v_r} a)} \right] \left(1 - \frac{e_{33}^2}{\varepsilon_{33}^S c_{33}^D} \left(\frac{2v_z}{l\omega} \right) \tan\left(\frac{l\omega}{2v_z}\right) \right)} \quad (5.98)$$

$$D = \frac{\frac{e_{33}}{c_{33}^D} \frac{v_z}{\omega} \frac{1}{\cos(\frac{l\omega}{2v_z}) J_0(\frac{\omega}{v_r} a)} \frac{V_0}{l}}{1 - \frac{e_{33}^2}{\varepsilon_{33}^S c_{33}^D} \left(\frac{2v_z}{l\omega} \right) \tan\left(\frac{l\omega}{2v_z}\right) - \left[\frac{\frac{(c_{13}^E)^2}{c_{33}^D}(\frac{\omega}{v_r} a) J_0(\frac{\omega}{v_r} a)}{c_{11}^E(\frac{\omega}{v_r} a) J_0(\frac{\omega}{v_r} a) + (c_{12}^E - c_{11}^E) J_1(\frac{\omega}{v_r} a)} \right]} \quad (5.99)$$

also can be obtained. Here, the vibration mode excited by the boundary conditions just given corresponds to the resonance frequency ω_r , and $D \rightarrow \infty$ even if the drive voltage amplitude is $V_0 \rightarrow 0$. According to Equation 5.90, $\cos(\frac{l\omega_a}{2v_z}) J_0(\frac{\omega_a}{v_r} a) = 0$ is the conditional equation for the anti-resonant angular frequency ω_a . Therefore,

$$1 - \frac{e_{33}^2}{\varepsilon_{33}^S c_{33}^D} \left(\frac{2v_z}{l\omega_r} \right) \tan\left(\frac{l\omega_r}{2v_z}\right) - \left[\frac{\frac{(c_{13}^E)^2}{c_{33}^D}(\frac{\omega_r}{v_r} a) J_0(\frac{\omega_r}{v_r} a)}{c_{11}^E(\frac{\omega_r}{v_r} a) J_0(\frac{\omega_r}{v_r} a) + (c_{12}^E - c_{11}^E) J_1(\frac{\omega_r}{v_r} a)} \right] = 0 \quad (5.100)$$

is the conditional expression for ω_r . Equation 5.100 can be transformed into

$$\frac{e_{33}^2}{\varepsilon_{33}^S c_{33}^D} = \frac{1 - \left[\frac{\frac{(c_{13}^E)^2}{c_{33}^D}(\frac{\omega_r}{v_r} a) J_0(\frac{\omega_r}{v_r} a)}{c_{11}^E(\frac{\omega_r}{v_r} a) J_0(\frac{\omega_r}{v_r} a) + (c_{12}^E - c_{11}^E) J_1(\frac{\omega_r}{v_r} a)} \right]}{\left(\frac{2v_z}{l\omega_r} \right) \tan\left(\frac{l\omega_r}{2v_z}\right)}. \quad (5.101)$$

Which can be rewritten as [9]

$$\frac{e_{33}^2}{\varepsilon_{33}^S c_{33}^D} = k_t^2 \quad (5.102)$$

therefore,

$$k_t^2 = \frac{1 - \left[\frac{\frac{(c_{13}^E)^2}{c_{33}^D}(\frac{\omega_r}{v_r} a) J_0(\frac{\omega_r}{v_r} a)}{c_{11}^E(\frac{\omega_r}{v_r} a) J_0(\frac{\omega_r}{v_r} a) + (c_{12}^E - c_{11}^E) J_1(\frac{\omega_r}{v_r} a)} \right]}{\left(\frac{2v_z}{l\omega_r} \right) \tan\left(\frac{l\omega_r}{2v_z}\right)}. \quad (5.103)$$

is the formula that satisfies the angular frequency ω_r . However, k_t is the electromechanical coupling coefficient.

Return to the derivation of admittance to obtain the electromechanical equivalent circuit. The displacement amplitudes D allow the general solution for the flux density D_z to be

$$D_z = \left(1 + \frac{k_t^2 \left(\frac{2v_z}{l\omega}\right) \tan\left(\frac{l\omega}{2v_z}\right) \frac{J_0\left(\frac{\omega}{v_r}a\right)}{J_0\left(\frac{\omega}{v_r}a\right)}}{1 - k_t^2 \left(\frac{2v_z}{l\omega}\right) \tan\left(\frac{l\omega}{2v_z}\right) - \left[\frac{\frac{(c_{13}^E)^2}{c_{33}^E} \left(\frac{\omega}{v_r}a\right) J_0\left(\frac{\omega}{v_r}a\right)}{c_{11}^E \left(\frac{\omega}{v_r}a\right) J_0\left(\frac{\omega}{v_r}a\right) + (c_{12}^E - c_{11}^E) J_1\left(\frac{\omega}{v_r}a\right)}\right]}\right) \frac{\varepsilon_{33}^S V_0 e^{j\omega t}}{l}. \quad (5.104)$$

The current i supplied by the power supply can be area differentiated by the time derivative of the flux density equation, which is

$$\begin{aligned} i &= \int_0^a 2\pi r \frac{\partial D_z}{\partial t} dr \\ &= j\omega \varepsilon_{33}^S \frac{a^2 \pi}{l} V_0 e^{j\omega t} + j\omega \frac{\varepsilon_{33}^S k_t^2 \left(\frac{2v_z}{l\omega}\right) \tan\left(\frac{l\omega}{2v_z}\right) \frac{J_1\left(\frac{\omega}{v_r}a\right)}{\left(\frac{\omega}{v_r}a\right) J_0\left(\frac{\omega}{v_r}a\right)} \frac{2a^2 \pi}{l} V_0 e^{j\omega t}}{1 - k_t^2 \left(\frac{2v_z}{l\omega}\right) \tan\left(\frac{l\omega}{2v_z}\right) - \left[\frac{\frac{(c_{13}^E)^2}{c_{33}^E} \left(\frac{\omega}{v_r}a\right) J_0\left(\frac{\omega}{v_r}a\right)}{c_{11}^E \left(\frac{\omega}{v_r}a\right) J_0\left(\frac{\omega}{v_r}a\right) + (c_{12}^E - c_{11}^E) J_1\left(\frac{\omega}{v_r}a\right)}\right]}, \end{aligned} \quad (5.105)$$

and thus the admittance $Y(= \frac{i}{V})$ is

$$\begin{aligned} Y &= \frac{i}{V_0 e^{j\omega t}} = j\omega \varepsilon_{33}^S \frac{a^2 \pi}{l} + j\omega \frac{\varepsilon_{33}^S k_t^2 \left(\frac{2v_z}{l\omega}\right) \tan\left(\frac{l\omega}{2v_z}\right) \frac{J_1\left(\frac{\omega}{v_r}a\right)}{\left(\frac{\omega}{v_r}a\right) J_0\left(\frac{\omega}{v_r}a\right)} \frac{2a^2 \pi}{l}}{1 - k_t^2 \left(\frac{2v_z}{l\omega}\right) \tan\left(\frac{l\omega}{2v_z}\right) - \left[\frac{\frac{(c_{13}^E)^2}{c_{33}^E} \left(\frac{\omega}{v_r}a\right) J_0\left(\frac{\omega}{v_r}a\right)}{c_{11}^E \left(\frac{\omega}{v_r}a\right) J_0\left(\frac{\omega}{v_r}a\right) + (c_{12}^E - c_{11}^E) J_1\left(\frac{\omega}{v_r}a\right)}\right]} \\ &= j\omega C_d + \frac{1}{\frac{1}{-j\omega \alpha^2 C_d} + \frac{1}{Y_{zm}} + \frac{1}{(-Y_{rm})}}. \end{aligned} \quad (5.106)$$

However, the braking capacity $C_d = \varepsilon_{33}^S \frac{a^2 \pi}{l}$, the capacity $C_{zr} = 2 \frac{J_1\left(\frac{\omega}{v_r}a\right)}{\left(\frac{\omega}{v_r}a\right) J_0\left(\frac{\omega}{v_r}a\right)} C_d$ the dynamic admittance

$$\begin{aligned} Y_{zm} &= j\omega 2C_d k_t^2 \left(\frac{2v_z}{l\omega}\right) \tan\left(\frac{l\omega}{2v_z}\right) \frac{J_1\left(\frac{\omega}{v_r}a\right)}{\left(\frac{\omega}{v_r}a\right) J_0\left(\frac{\omega}{v_r}a\right)}, \text{ and} \\ Y_{rm} &= j\omega 2C_d k_t^2 \left(\frac{2v_z}{l\omega}\right) \tan\left(\frac{l\omega}{2v_z}\right) \frac{J_1\left(\frac{\omega}{v_r}a\right)}{\left(\frac{\omega}{v_r}a\right) J_0\left(\frac{\omega}{v_r}a\right)} \frac{c_{11}^E \left(\frac{\omega}{v_r}a\right) J_0\left(\frac{\omega}{v_r}a\right) + (c_{12}^E - c_{11}^E) J_1\left(\frac{\omega}{v_r}a\right)}{\frac{(c_{13}^E)^2}{c_{33}^E} \left(\frac{\omega}{v_r}a\right) J_0\left(\frac{\omega}{v_r}a\right)}. \end{aligned}$$

By the admittance formula 5.106, in an equivalent circuit with capacitor $-C_{zr}$ with negative capacitance and dynamic admittance Y_{zm} and Y_{rm} connected in series. The braking capacitance C_d and $-C_{zr}$, Y_{zm} , Y_{rm} series circuit connected in parallel. The overall shape is shown in Figure 5.22.

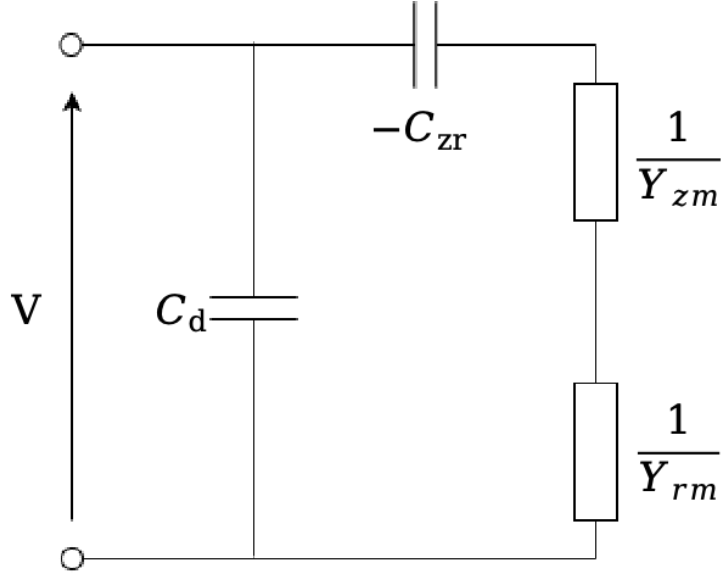


Figure 5.22: Thickness-vibration-mode Equivalent circuit (I)

In a normal electrical circuit, the capacitance of the capacitor is always positive, whereas in the equivalent circuit for the thickness vibration mode, a $-C_{zr}$ with a negative sign is connected in series with the dynamic admittance Y_{zm} , Y_{rm} , which represents mechanical vibration. This corresponds to the correction of the influence of the anti-electric field generated by the piezoelectric vibration on the potential difference between the electrodes, as in the calculation of the electric field distribution inside the piezoelectric vibrator. With a normal capacitor, this would result in a voltage drop, but this $-C_{zr}$ with a negative sign works in the opposite direction, pushing the voltage up, resulting in the vibrator being driven with a voltage amplitude higher than the input voltage.

Returning to equation 5.106, C_{zr} , Y_{zm} and Y_{rm} have coefficients $\alpha = \sqrt{\frac{J_1(\frac{\omega}{v_r}a)}{(\frac{\omega}{v_r}a)J_0(\frac{\omega}{v_r}a)}}$ and $\beta = \sqrt{\frac{\frac{(c_{13}^E)^2}{c_{33}^E}(\frac{\omega}{v_r}a)J_0(\frac{\omega}{v_r}a)}{c_{11}^E(\frac{\omega}{v_r}a)J_0(\frac{\omega}{v_r}a) + (c_{12}^E - c_{11}^E)J_1(\frac{\omega}{v_r}a)}}$ that vary with angular frequency ω , and if these coefficients are extracted and expressed in terms of ideal transformers α and β in the equivalent circuit, they can be obtained as shown in Figure 5.23 in the equivalent circuit of the concentrated constant system.

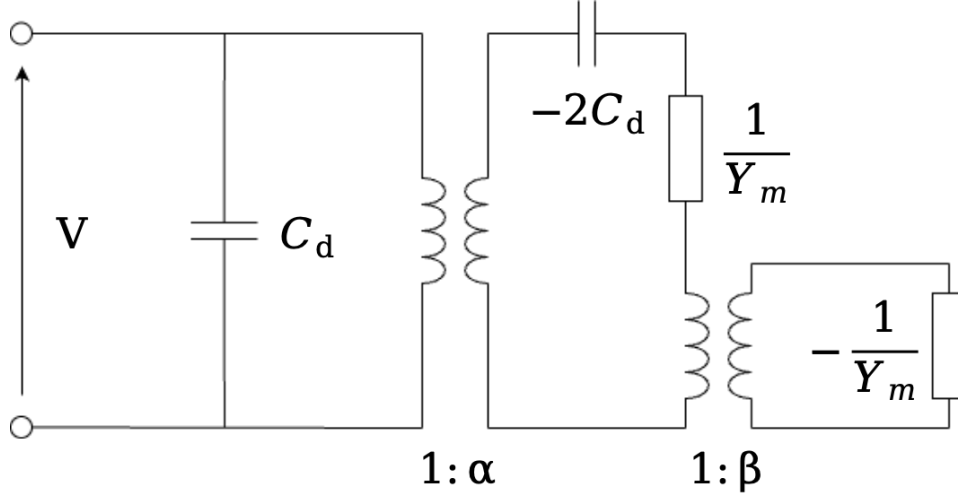


Figure 5.23: Thickness-vibration-mode Equivalent circuit (II)

As thickness vibration differs from pure longitudinal vibration in that not only longitudinal but also radial vibration is considered, the input power is converted into kinetic energy in longitudinal and radial vibration respectively, and the ideal transformer α can represent the proportional coefficient of the power converted into the longitudinal component of thickness vibration. Similarly, as radial vibration is also considered, ideal transformer β can represent the proportionality coefficient of the longitudinal vibration due to radial vibration compared to the pure longitudinal vibration excited by the input power. Therefore, $Y_m = j\omega 2C_d k_t^2 \left(\frac{2v_z}{l\omega}\right) \tan\left(\frac{l\omega}{2v_z}\right)$ represents the pure longitudinal vibration excited by the input power, the combination of Y_{zm} and Y_{rm} represent the longitudinal component of the thickness vibration.

The equivalent circuit of the thickness vibration has been derived from the new piezoelectric equation, the next section investigates the admittance characteristics of a PZTc204 disc-shaped vibrator of 10 mm diameter and 1 mm thickness at no load, using the equivalent circuit for thickness vibrations.

5.3.3 Admittance Characteristics without External Load

In this section, the admittance characteristics for a diameter of 10 mm and a thickness of 1 mm will be calculated based on the equivalent circuit without external load derived in the previous section using the PZTc204(FUJI CERAMICS) material parameters mentioned in Table 5.1.

The equation 5.106 for the admittance Y can be transformed into

$$Y = j\omega C_d \left(\frac{1}{\frac{1-\beta^2-k_t^2(\frac{2v_z}{l\omega})\tan(\frac{l\omega}{2v_z})}{1-\beta^2+(2\alpha^2-1)k_t^2(\frac{2v_z}{l\omega})\tan(\frac{l\omega}{2v_z})}} \right) \quad (5.107)$$

In the resonance state, when the current amplitude is ∞ for a constant input amplitude voltage, which means that $Y \rightarrow \infty$, the denominator in Equation 5.107 is

$$\frac{1 - \beta^2 - k_t^2(\frac{2v_z}{l\omega})\tan(\frac{l\omega}{2v_z})}{1 - \beta^2 + (2\alpha^2 - 1)k_t^2(\frac{2v_z}{l\omega})\tan(\frac{l\omega}{2v_z})} = 0. \quad (5.108)$$

And transforming Equation 5.108 gives the same result as Equation 5.103, as follows

$$k_t^2 = \frac{1 - \left[\frac{\frac{(c_{13}^E)^2}{c_{33}^D}(\frac{\omega_r}{v_r}a)J_0(\frac{\omega_r}{v_r}a)}{c_{11}^E(\frac{\omega_r}{v_r}a)J_0(\frac{\omega_r}{v_r}a) + (c_{12}^E - c_{11}^E)J_1(\frac{\omega_r}{v_r}a)} \right]}{(\frac{2v_z}{l\omega_r})\tan(\frac{l\omega_r}{2v_z})}. \quad (5.109)$$

Thus, the electromechanical coupling coefficient k_t is included in the equation for ω_r , and it can be seen that the resonance angular frequency varies with piezoelectricity.

On the other hand, the anti-resonance angular frequency ω_a is when the impedance is ∞ and no current flows. which means that $Y \rightarrow 0$, thus from Equation 5.107,

$$\cos(\frac{l\omega_a}{2v_z})J_0(\frac{\omega_a}{v_r}a) = 0 \quad (5.110)$$

holds.

Returning to the dynamic admittance Y_m in Equation 5.106, by Laurent expansion of $\tan(\frac{l\omega}{2v_z})$, it can be expressed in a circuit with LC series components corresponding to each vibration mode component connected in parallel. Laurent expansion of Y_m allows from

$$\tan(\frac{l\omega}{2v_z}) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2 \frac{\pi^2}{8} \frac{2v_z}{\omega l} - \frac{\omega l}{4v_z}} \quad (5.111)$$

to be

$$Y_m = A^2 \sum_{n=1}^{\infty} \frac{1}{j\omega L_{2n-1} + \frac{1}{j\omega C_{2n-1}}}. \quad (5.112)$$

However, $A = \frac{4a^2\pi\epsilon_{33}}{l}$ is the force coefficient, $M = a^2\pi l\rho$ is the actual mass of the vibrator, $L_{2n-1} = M \frac{c_{33}^D(c_{11}^E+c_{13}^E)}{c_{33}^D c_{11}^E - (c_{13}^E)^2}$ is the equivalent mass and $C_{2n-1} = \frac{l}{(2n-1)^2 a^2 \pi^3 c_{33}^D}$ is the equivalent compliance.

The equivalent mass L_{2n-1} is independent of the mode order, whereas the equivalent compliance C_{2n-1} depends on the mode order and decreases at a rate of $(2n-1)^2$ as the mode order increases. The equivalent spring constant is the reciprocal of C_{2n-1} , which means that the stiffness increases as the mode order increases.

Considering only the first-order mode, the equivalent circuit is an $L_1 C_1$ series circuit through transformer A connected in series with a capacitor $-2C_d$ with negative capacitance. The dynamic admittance Y_m can be converted to

$$Ym = A^2 \frac{1}{j\omega L_1 + \frac{1}{j\omega C_1}}, \quad (5.113)$$

However, $L_1 = M \frac{c_{33}^D(c_{11}^E+c_{13}^E)}{c_{33}^D c_{11}^E - (c_{13}^E)^2}$ and $C_1 = \frac{l}{a^2 \pi^3 c_{33}^D}$.

To obtain the full admittance characteristic of PZTc204 disc-shaped vibrator, the resistance R_m of the vibration loss and the resistance R_d of the dielectric loss must be determined in the electromechanical equivalent circuit.

Because the mechanical quality factor Q_m and the Mechanical series angular resonance frequency $\omega_s = \frac{1}{\sqrt{LC}}$ are known, R_m can be obtained from

$$R_m = \frac{\omega_s L}{Q_m}. \quad (5.114)$$

Because the induced loss $\tan\delta = 0.0025$ can be found from Table 5.1, the braking capacitance C_d and the resistance Rd in parallel can be obtained from

$$R_d = \frac{1}{\tan\delta\omega C_d} \quad (5.115)$$

Finally, the electromechanical equivalent circuit is obtained as shown in Figure 5.4.

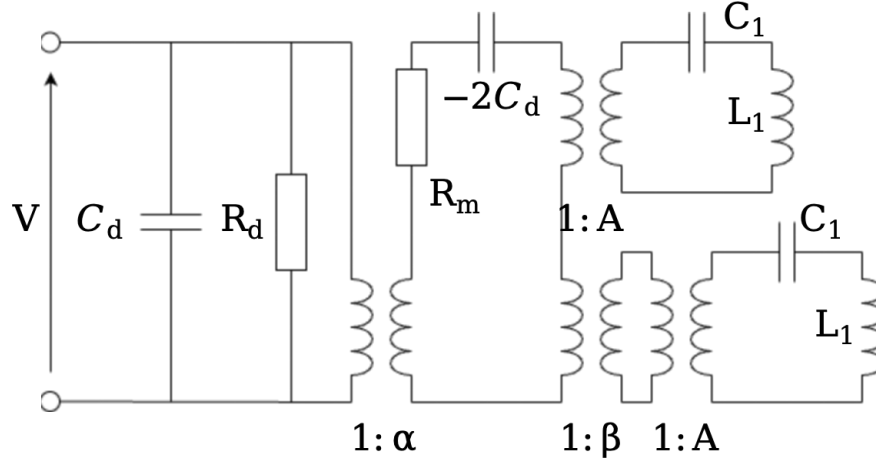


Figure 5.24: Equivalent Circuit of the Thickness Vibration with losses

Combining the known material parameters with the equation for admittance Y shows that the elastic modulus s_{13}^E is missing, thus s_{13}^E must first be determined. The resonance angular frequency ω_r can be found if the frequency factor N_t and the thickness l of the vibrator are known. The value of s_{13}^E can be solved for using equation 5.109. However, during the calculation, $e_{33} = d_{33}c_{33}^E$, $\varepsilon_{33}^S = \varepsilon_{33}^T - d_{33}\overline{e_{33}}$.

From this electromechanical equivalent circuit the admittance characteristics can be calculated as shown in Fig. 5.25. The resonance frequency is 2.29 MHz and the conductance at the resonance frequency is 1060.2 mS. The admittance characteristics of the actuator using the PZTc204 disc-shaped vibrator with a diameter of 10 mm and a thickness of 1 mm, measured in air, are shown in Fig. 5.26, with a resonance frequency of 2.13 MHz and a conductance at the resonance frequency of 85.8 mS.

As far as the resonance frequency is concerned, the difference between the calculated and measured results is reduced to 0.16 MHz compared to the calculated results considering only longitudinal vibrations.

Although the calculated results of the admittance characteristics are quite different from the measured results, compared with the calculated results in Figure 5.5, the calculated results of the admittance characteristics after considering the radial vibration are found that the resonance frequencies of the thickness vibration are distributed around the resonance frequencies of the radial vibration, which are 1.27 MHz, 1.61 MHz, 1.95 MHz, 2.63 MHz, and 2.97 MHz, respectively. The results of the admittance measurements also show that there are multiple radial resonance frequencies represented by multiple peaks near the resonance frequency of the thickness vibration.

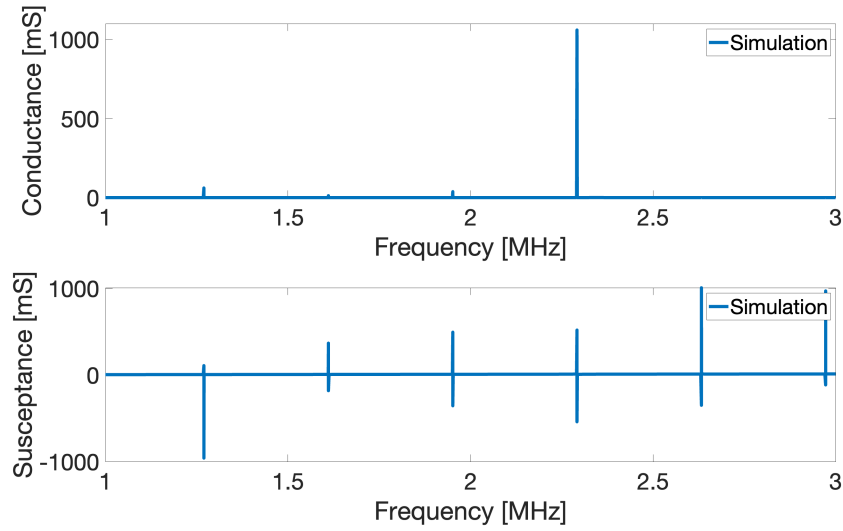


Figure 5.25: Measurement result for the Admittance of the Actuator using PZTc204 Disc-shaped 10mm Diameter 1mm Thickness Transducer without loads (Resonance Frequency: 2.29 MHz; Conductance: 1060.2 mS)

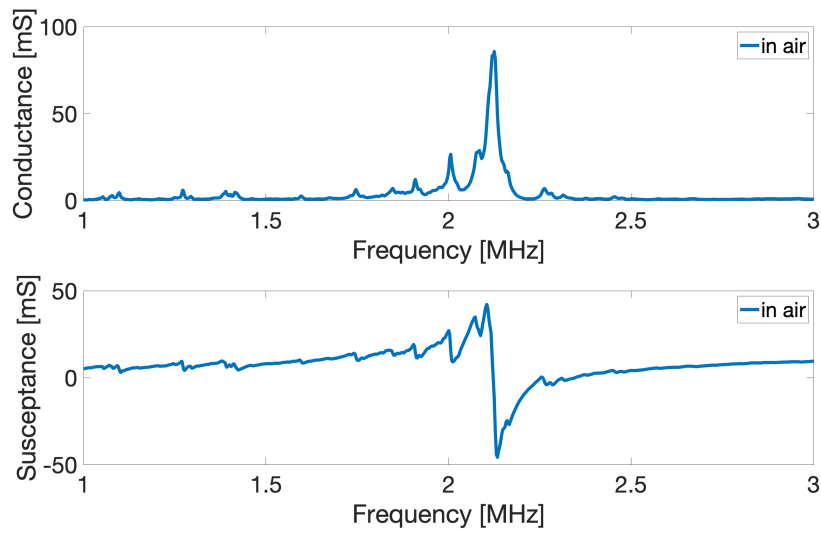


Figure 5.26: Measurement result for the Admittance of the Actuator using PZTc204 Disc-shaped 10mm Diameter 1mm Thickness Transducer in Air (Resonance Frequency: 2.13 MHz; Conductance: 85.8 mS)

As the calculations are for a disc-shaped vibrator with free end surfaces, it will be investigated whether the results can be improved by adding loads to both end surfaces. In order to verify this, it is necessary to create an electromechanical equivalent circuit (Mason equivalent circuit), which can deal with arbitrary boundary conditions, and the Mason equivalent circuit for thickness vibration was investigated in the next section.

5.3.4 Mason Equivalent Circuit

Finding electromechanical equivalent circuits for centralised constant systems is constrained by the fact that the boundary conditions are fixed. In this section, Mason equivalent circuit is represented for the thickness vibration that can accommodate arbitrary boundary conditions.

Take the inward positive force in the z -axis direction of the vibrator and define each parameter as F_1 , F_2 and F_3 , respectively, as shown in Fig. 5.27.

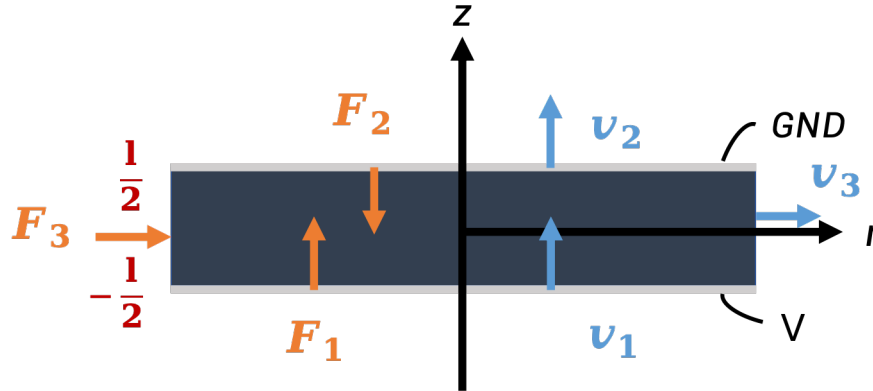


Figure 5.27: Definition of each parameter of the transfer matrix of thickness vibrations.

From the wave equation for thickness vibration, the general solution for displacement is

$$u_r = A \cos\left(\frac{\omega}{v_z} z\right) J_1\left(\frac{\omega}{v_r} r\right) e^{j\omega t} \quad (5.116)$$

$$u_z = [C \cos\left(\frac{\omega}{v_z} z\right) + D \sin\left(\frac{\omega}{v_z} z\right)] J_0\left(\frac{\omega}{v_r} r\right) e^{j\omega t} \quad (5.117)$$

However, $v_z = \sqrt{\frac{c_{33}^D c_{11}^E - (c_{13}^E)^2}{c_{11}^E + c_{13}^E \rho}}$, $v_r = \sqrt{\frac{c_{33}^D c_{11}^E - (c_{13}^E)^2}{c_{33}^D + c_{13}^E \rho}}$. Because no boundary conditions such as freedom at both sides are given here, $C = 0$ is not used and

even functions are included. The electric field in this case agrees with the case of boundary conditions such as two sides free, since the even functions disappear when the potential is calculated in the same way as in Equation 5.92. Thus, from Equation 5.91, the electric field is

$$E_z = -\frac{e_{33}}{\varepsilon_{33}^S} \frac{\omega}{v_z} (-C \sin(\frac{\omega}{v_z} z) + D \cos(\frac{\omega}{v_z} z)) J_0(\frac{\omega}{v_r} r) e^{j\omega t} + \frac{2 e_{33}}{l \varepsilon_{33}^S} D \sin(\frac{l\omega}{2v_z}) J_0(\frac{\omega}{v_r} r) e^{j\omega t} + \frac{V_0 e^{j\omega t}}{l} \quad (5.118)$$

In this derivation process, the mechanical boundary conditions of the vibrator are not included, thus it holds for all boundary conditions and angular frequencies ω . By substituting the general solution of the displacement and the expression for the electric field into the piezoelectric equation T_{zz} and T_{rr} , the stress T_{zz} , T_{rr} are

$$T_{rr} = c_{11}^E A \cos(\frac{\omega}{v_z} z) (\frac{\omega}{v_r} J_0(\frac{\omega}{v_r} r) - \frac{1}{r} J_1(\frac{\omega}{v_r} r)) e^{j\omega t} + c_{12}^E A \cos(\frac{\omega}{v_z} z) \frac{1}{r} J_1(\frac{\omega}{v_r} r) e^{j\omega t} - c_{13}^E \frac{\omega}{v_z} C \sin(\frac{\omega}{v_z} z) J_0(\frac{\omega}{v_r} r) e^{j\omega t} + c_{13}^E \frac{\omega}{v_z} D \cos(\frac{\omega}{v_z} z) J_0(\frac{\omega}{v_r} r) e^{j\omega t}, \quad (5.119)$$

$$T_{zz} = (c_{13}^E \frac{\omega}{v_r} A + c_{33}^D \frac{\omega}{v_z} D) \cos(\frac{\omega}{v_z} z) J_0(\frac{\omega}{v_r} r) e^{j\omega t} - c_{33}^D \frac{\omega}{v_z} C \sin(\frac{\omega}{v_z} z) J_0(\frac{\omega}{v_r} r) e^{j\omega t} - \frac{e_{33}^2}{\varepsilon_{33}^S} \frac{2}{l} D \sin(\frac{l\omega}{2v_z}) J_0(\frac{\omega}{v_r} r) e^{j\omega t} - e_{33} \frac{V_0 e^{j\omega t}}{l}. \quad (5.120)$$

Noting that the stress is positively tensile, whereas the force is positively compressive, F_1 at $z = -\frac{l}{2}$ is

$$F_1 = -2\pi \int_0^a r T_{zz}|_{z=-\frac{l}{2}} dr = [(c_{13}^E \frac{\omega}{v_r} A + c_{33}^D \frac{\omega}{v_z} D) \cos(\frac{l\omega}{2v_z}) + c_{33}^D \frac{\omega}{v_z} C \sin(\frac{l\omega}{2v_z}) - \frac{e_{33}^2}{\varepsilon_{33}^S} \frac{2}{l} D \sin(\frac{l\omega}{2v_z})] (-2\pi (\frac{v_r}{\omega}) a J_1(\frac{\omega}{v_r} a) e^{j\omega t}) + e_{33} \frac{V_0 e^{j\omega t}}{l} a^2 \pi. \quad (5.121)$$

and F_2 at $z = \frac{l}{2}$ is

$$\begin{aligned}
F_2 &= -2\pi \int_0^a r T_{zz}|_{z=\frac{l}{2}} dr \\
&= [(c_{13}^E \frac{\omega}{v_r} A + c_{33}^D \frac{\omega}{v_z} D) \cos(\frac{l\omega}{2v_z}) - c_{33}^D \frac{\omega}{v_z} C \sin(\frac{l\omega}{2v_z}) - \frac{e_{33}^2}{\epsilon_{33}^S} \frac{2}{l} D \sin(\frac{l\omega}{2v_z})] \\
&\quad (-2\pi (\frac{v_r}{\omega}) a J_1(\frac{\omega}{v_r} a) e^{j\omega t}) + e_{33} \frac{V_0 e^{j\omega t}}{l} a^2 \pi.
\end{aligned} \tag{5.122}$$

and F_3 at $r = a$ is

$$\begin{aligned}
F_3 &= -2a\pi \int_{-\frac{l}{2}}^{\frac{l}{2}} T_{rr}|_{r=a} dz \\
&= [c_{11}^E A (\frac{\omega}{v_r} J_0(\frac{\omega}{v_r} a) - \frac{1}{a} J_1(\frac{\omega}{v_r} a)) + c_{12}^E A \frac{1}{a} J_1(\frac{\omega}{v_r} a) + c_{13}^E \frac{\omega}{v_z} D J_0(\frac{\omega}{v_r} a)] \\
&\quad (-4a\pi \frac{v_z}{\omega} \sin(\frac{l\omega}{2v_z}) e^{j\omega t}).
\end{aligned} \tag{5.123}$$

On the other hand, the vibration velocities v_1 , v_2 and v_3 on surfaces are

$$\begin{aligned}
v_1 &= 2\pi \int_0^a r (\frac{\partial u_z}{\partial t}|_{z=-\frac{l}{2}}) dr = j\omega [C \cos(\frac{\omega l}{2v_z}) - D \sin(\frac{\omega l}{2v_z})] (2\pi \frac{v_r}{\omega} a J_1(\frac{\omega}{v_r} a)) e^{j\omega t} \\
v_2 &= 2\pi \int_0^a r (\frac{\partial u_z}{\partial t}|_{z=\frac{l}{2}}) dr = j\omega [C \cos(\frac{\omega l}{2v_z}) + D \sin(\frac{\omega l}{2v_z})] (2\pi \frac{v_r}{\omega} a J_1(\frac{\omega}{v_r} a)) e^{j\omega t} \\
v_3 &= 2a\pi \int_{-\frac{l}{2}}^{\frac{l}{2}} (\frac{\partial u_r}{\partial t}|_{r=a}) dz = j\omega A J_1(\frac{\omega}{v_r} a) (4a\pi \frac{v_z}{\omega} \sin(\frac{l\omega}{2v_z})) e^{j\omega t}.
\end{aligned} \tag{5.124}$$

because the general solution of the displacements in Equations 5.116 and 5.117 can be differentiated with time t .

Substituting Equation 5.124 into Equations 5.121, 5.122, 5.123 and $V = V_0 e^{j\omega t}$, gives

$$F_1 = v_3 Z_3 + v_1 Z_1 + (v_1 - v_2) Z_{12} + \frac{1}{j\omega(-C_d a^2 \pi)} A^2 (v_1 - v_2) + AV \tag{5.125}$$

$$F_2 = v_3 Z_3 - v_2 Z_1 + (v_1 - v_2) Z_{12} + \frac{1}{j\omega(-C_d a^2 \pi)} A^2 (v_1 - v_2) + AV \quad (5.126)$$

$$F_3 = v_3 \frac{c_{11}^E(\frac{\omega}{v_r} a) J_0(\frac{\omega}{v_r} a) + (c_{12}^E - c_{11}^E) J_1(\frac{\omega}{v_r} a)}{-j\omega a J_1(\frac{\omega}{v_r} a)} + (v_1 - v_2) \left[-j \frac{c_{13}^E}{v_r} \frac{J_0(\frac{\omega}{v_r} a)}{J_1(\frac{\omega}{v_r} a)} \right] \quad (5.127)$$

However, $Z_3 = \frac{c_{13}^E}{-j2v_z \tan(\frac{l\omega}{2v_z})}$, $Z_1 = j \frac{c_{33}^D}{v_z} \tan(\frac{l\omega}{2v_z})$, $Z_{12} = -j \frac{c_{33}^D}{v_{vz}} \frac{1}{\sin(\frac{l\omega}{v_z})}$, $C_d = \frac{\epsilon_{33}^S a^2 \pi}{l}$, $C_{dm} = a^2 \pi C_d$, $A = \frac{e_{33} a^2 \pi}{l}$, $V = V_0 e^{j\omega t}$.

Substituting Equation 5.127 into Equations 5.125 and 5.125 to integrate gives the Mason equivalent circuit of the thickness vibration shown in Figure 5.28.

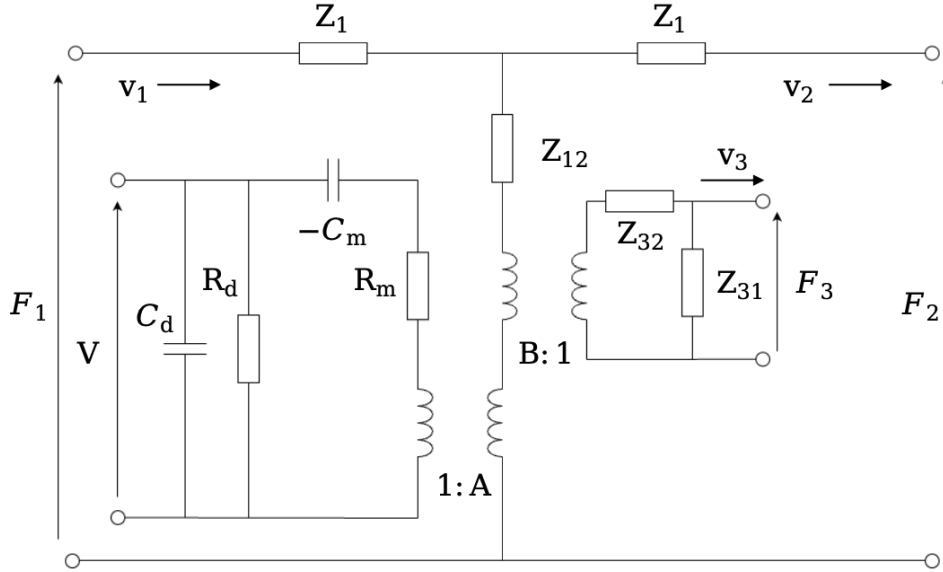


Figure 5.28: Mason Equivalent Circuit of the Thickness Vibration with losses.

$$\text{However, } B = \frac{c_{13}^E(\frac{\omega}{v_r} a) J_0(\frac{\omega}{v_r} a)}{c_{11}^E(\frac{\omega}{v_r} a) J_0(\frac{\omega}{v_r} a) + (c_{12}^E - c_{11}^E) J_1(\frac{\omega}{v_r} a)}, \quad Z_{31} = \frac{c_{11}^E(\frac{\omega}{v_r} a) J_0(\frac{\omega}{v_r} a) + (c_{12}^E - c_{11}^E) J_1(\frac{\omega}{v_r} a)}{-j\omega a J_1(\frac{\omega}{v_r} a)},$$

$$Z_{32} = \frac{c_{13}^E}{-j2v_z \tan(\frac{l\omega}{2v_z})}.$$

In this calculation process, the stress was determined from the force component and the relationship with the vibration velocity was determined, thus the part of the braking capacitance C_d from the power supply side is not represented. The braking capacitance enters in parallel with the input voltage,

as can be seen from the consideration in the concentrated constant system, and if the losses are considered in the same way as in the concentrated constant system, the equivalent circuit for is shown in Figure 5.28. The dielectric loss R_d and vibration loss R_m are calculated in a similar way to the equivalent circuit of a concentrated constant system.

The vibration loss R_m can be calculated from the Mason equivalent circuit short-circuited at both ends (free state at both end faces). Moreover, the Mason's equivalent circuit after short-circuiting at both ends can be converted to the equivalent circuit of the centralised fixed number system, thus R_m and R_d are the same as in the equivalent circuit of the centralised fixed number system. [10]

Because the obtained three-port Mason equivalent circuit can correspond to arbitrary boundary conditions, Theoretically, by adding loads at the ports of the Mason equivalent circuit, the corresponding admittance characteristics as well as the vibrationa velocities can be obtained.

5.3.5 Conclusion

Combining the discussion of longitudinal and radial vibration in the above two sections, it can be learnt that thickness vibration is a combination of longitudinal and radial vibration. Therefore, in this subsection, the electromechanical equivalent circuit of thickness vibration is discussed. The electrical-mechanical equivalent circuit of thickness vibration is obtained by solving the piezoelectric equation with parameters in the longitudinal and radial directions, as shown in Figure 5.28. However, it is not possible to accurately find the conductivity and vibration characteristics of the thickness vibration.

5.4 Conclusion

1. Longitudinal vibration:

In this section, thickness vibration is considered through an equivalent circuit model of longitudinal vibration. In the case of PZTc204, a disc-shaped vibrator with a diameter of 10 mm and a thickness of 1 mm was used to compare the calculation results of the equivalent circuit model of longitudinal vibration with the measured results, and it was concluded that the equivalent circuit model of longitudinal vibration cannot accurately represent the thickness vibration characteristics.

1.1 Calculation results for admittance characteristics:

- In air: resonance frequency f_s : 1.42 MHz; Conductance at resonance frequency: 2041.1 mS.

- In water: resonant frequency f_s : 1.42 MHz; Conductance at resonance frequency: 121.0 mS.

1.2 Calculation results for vibration velocity distribution:

- In air ($5V_{0p}$): Shape: straight line; Maximum vibration velocity: 2147.5 mm/s.

- In water ($5V_{0p}$): Shape: straight line; Maximum vibration velocity: 156.1 mm/s.

From the above, it was found that the equivalent circuit model considering only longitudinal vibration cannot accurately represent thickness vibration. As shown in Figure 5.1, in addition to longitudinal (z) vibration, there is also radial (r) and rotational (θ) vibration. From the vibration distribution measurements presented in Chapter 2, it is known that the vibration distribution is centrosymmetrical, thus the vibration in the direction of rotation (θ) can be neglected, leaving only the vibration in the radial direction (r).

2. Radial vibration:

In this section, the displacement function $u_r(r, t)$ of the radially symmetric extension vibration of a disc-shaped vibrator is derived using the piezoelectric equation considering only the parameters in the radial direction. After analysing this displacement function u_r and converting it to longitudinal displacement, it is concluded that the symmetric extension vibration in the radial direction does have the possibility of transforming the piston vibration in the longitudinal direction into a thickness vibration. In other words, the thickness vibration may be a combination of the symmetric extension vibration in the radial direction and the piston vibration in the longitudinal direction.

3. Thickness vibration:

The Mason electrical-mechanical equivalent circuit of thickness vibration is obtained by solving the piezoelectric equation with parameters in the lon-

gitudinal and radial directions, as shown in Figure 5.28. Theoretically, by adding loads at the ports of the Mason equivalent circuit, the corresponding admittance characteristics as well as the vibrational velocities can be obtained. However, there are several problems:

firstly, due to the complexity of the equation, the trigonometric functions are intertwined with the Bessel functions, which tends to lead to non-convergence of the calculation results.

Secondly, since there are many equivalence relationships between the parameters, breaking one equivalence relationship will lead to a change in the relationship between many parameters when all parameters are not taken into account.

Third, it is difficult to quantify the forces in the radial direction.

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Chapter 6

CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

This paper, entitled [Study on underwater actuators using thickness vibration piezoelectric elements] and consisting of six chapters, investigates the differences in the properties of an underwater actuator with a disk-shaped thickness vibration piezoelectric element as a result of changes in the material and shape of the thickness vibration piezoelectric element, and summarises research into the relationship between acoustic radiation force and propulsion force.

In Chapter 1 [Introduction], the miniature underwater actuators that have been studied are described, the advantages of underwater ultrasonic actuators are investigated, and the necessity of understanding phenomena such as the principle of propulsion force generation and the quantitative relationship between the two is stated for practical application. The paper states that the objective of the research is to obtain the basic data necessary for optimising the element configuration through prototype fabrication, operation measurement and analysis of three types of underwater ultrasonic actuators.

In Chapter 2 [Miniature Underwater Actuator using a PZT Disk-shaped Transducer], an actuator using a PZT disk-shaped transducer with a diameter of 10 mm, a thickness of 1 mm and a resonance frequency of 2.13 MHz is fabricated to investigate the operating characteristics and the mechanism of propulsion force generation. As operating characteristics, a static propulsion force of 1mN and a no-loaded speed of 150 mm/s were obtained for the underwater ultrasonic actuator at a driving power of 4.6 W. The propulsion force was calculated from the results of measurements of the vibration

velocity characteristics of the PZT disk-shaped transducer and the acoustic radiation characteristics into the water, and from the theory of the acoustic radiation force generated at the solid/liquid boundary.

In Chapter 3 [Miniature Underwater Actuator using $(Bi, Na, Ba)(Ti, Mn)O_3$ Disk-shaped transducer], an actuator with a $(Bi, Na, Ba)(Ti, Mn)O_3$ disk-shaped transducer of 8.4 mm diameter, 0.5 mm thickness and resonance frequency of 4.22 MHz is fabricated and the operating characteristics are measured to show that the disk-shaped transducers that is applied to underwater ultrasonic actuators can be constructed from lead-free piezoelectric material. As operating characteristics, a static propulsion force of 3.9 mN and a no-loaded speed of 60 mm/s were obtained for the underwater BNBTM actuator. It also shows that the underwater BNBTM actuator can be driven with a small driving power requirement of 0.4 W for a propulsion force of 1 mN. The results of calculating the propulsion force from the theory of acoustic radiation force at the solid/liquid boundary, which is already known, show that the calculated value is smaller than the measured value and that further investigation is required.

In Chapter 4 [Miniature Underwater Actuator using a PZT Ring Transducer], an actuator using a PZT ring transducer with an outer diameter of 5 mm, an inner diameter of 3.2 mm, a thickness of 1.4 mm and resonance frequency of 1.09 MHz is fabricated and the operating characteristics are investigated. The PZT ring actuator's static propulsion force of 1 mN and no-load speed of 25 mm/s have been obtained as operating characteristics. The driving power for 1 mN of propulsion force is 4.2 W, which is shown to be almost the same as the value in Chapter 2.

In Chapter 5 [Equivalent Circuit of Disk-shaped Transducer], the coupling of radial vibration to the excitation of the thickness mode results in a complex distribution of vibration on the vibration surface, which is determined by analysis, and an electroacoustic equivalent circuit describing the vibration state is investigated. It is stated that an understanding of the coupling of the two vibration modes is necessary and that further study is required to construct an equivalent model.

In Chapter 6 [Conclusions and Future], the results of this study are summarised and future prospects are discussed.

In summary, this paper is an experimental and analytical study of an underwater actuator using a thickness vibration piezoelectric elements, to investigate modelling problems and to obtain basic data for the construction of an optimum system. Because this paper is the first systematic discussion of a new type of underwater ultrasonic actuator, it contributes to the realisation of underwater ultrasonic actuator applications.

6.2 Future Work

Future work is centred on these three topics:

1. The electrical-mechanical equivalent circuit model by considering two directional parameters in the longitudinal and radial directions.

Although the Mason equivalent circuit for thickness vibrations is derived from the piezoelectric equation, several problems remain:

- How to simplify the Mason equivalence circuit.
- How to quantify the force in the radial direction.
- Is it possible to build a model dedicated to ultrasonic underwater actuators by incorporating Propulsion calculations into the equivalence circuit.
- Are there other better models available.
- Is it possible to improve the model for finite element simulation to achieve numerical matching.

2. In order to explore the propulsion principle of the ultrasonic underwater actuator, it is necessary to quantify its Propulsion force. In this study, the propulsion characteristics are investigated by experimentally measuring sound pressure to calculate the acoustic radiation force.

Although the sound pressure has been successfully measured to calculate the propulsion force, it does not match the actual thrust characteristics. The following issues can be considered in the future:

- The calculation needs the value of the sound speed in the PZT transducer, which is calculated from the equivalent circuit model of longitudinal vibration, thus the value of the sound speed is different from the thickness vibration mode, which leads to the deviation of the calculation result.
- Because the propulsion force is too small, the measured value will be deviated due to the external influence during the measurement.
- Due to the lack of consideration of the viscosity, which causes the calculated values to be larger than measured values.
- Exploring the possibility of alternative propulsion theories more suitable for ultrasonic underwater actuators.

3. To take advantage of the miniaturisation of the ultrasonic underwater actuator.

Although three different ultrasonic underwater actuators were designed and evaluated in this study. However, all of them are not sufficient for practical applications and there are still areas that need to be improved.

- As environmental protection and safety are the issues of the future, the use of non-lead materials for ultrasonic underwater actuators should be promoted.
- Further size reduction to highlight the advantages of ultrasonic underwater actuators.

- For the exploration of application scenarios, field simulation experiments are conducted. For example, in-pipe drives.
- How to reduce the impact of wires on ultrasonic underwater actuators.
- Improvement of the structure of the ultrasonic underwater actuator to enable it to maintain a moving attitude in the water.
- How to improve the high driving efficiency of ultrasonic underwater actuators.

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