

論文 / 著書情報
Article / Book Information

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Type(English)	Summary

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論文要旨
THESIS SUMMARY

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要旨 (英文 800 語程度)
Thesis Summary (approx.800 English Words)

Dijkgraaf and Witten studied how to construct toy models of TQFT from the viewpoint of the group homology. In their study, for a given finite group G, they introduced a topological invariant of connected oriented closed 3-manifolds. This invariant is called the Dijkgraaf-Witten (DW) invariant. In previous studies, the DW invariants were examined in the context of combinatorics, and hence, the difficulties in constructing TQFTs from the DW invariant are well-known. For example, how to extend the DW invariants to connected oriented compact 3-manifolds with boundary is one of such difficulties. Moreover, unless G is abelian or nilpotent, procedures for calculating the DW invariants are often complicated, and there are few quantitative computational examples. Precisely, there is no computational result in the case where both a finite group G and the fundamental group of a 3-manifold are non-nilpotent. This thesis consists of two chapters as follows. In Chapter 1, we approach the problem of calculating the DW invariant of a non-nilpotent group G. To address this problem, we focus on K-homology rather than the ordinary homology theory. As a generalization, we can define orientation classes of any generalized homology as an analogy of the orientation class in ordinary homology. Thus, thanks to these orientation classes, it is possible to extend the definition of TQFTs, including the DW invariants, to generalized homologies under certain assumptions. Chapter 1 studies the DW invariant in terms of K-homology, which is called the KDW invariant. The computation of the KDW invariant often seems difficult. To solve this difficulty, based on Knapp's work, we quantitatively examine the KDW invariant from viewpoints of the representation theory. More precisely, through Knapp's work, the computation of the KDW invariant without 2-torsion follows from some computations of induced representations between the representation rings. In application, we compute the KDW invariants without 2-torsion in the following three cases: (i) some lens spaces with $G = \mathbb{Z}/k$, (ii) the lens spaces with $G = \mathrm{PSL}_2(\mathbb{F}_p)$, (iii) some Brieskorn homology spheres with $G = \mathrm{PSL}_2(\mathbb{F}_p)$. These are the first computational results for the DW invariant in generalized homology theory. Moreover, we should remark that, in (iii), the fundamental groups and the finite groups are not nilpotent. On the other hand, Chapter 2 considers the extension of the definition of the DW invariant to 3-manifolds with boundary. In the original study, Dijkgraaf and Witten reformulated the DW invariants in a combinatorial way and implied that this reformulation provides an extension of the DW invariants for 3-manifolds with boundary. However, it is known that the reformulated DW invariant is not a topological invariant for 3-manifolds with boundary. In order to solve the problem of extending the DW invariants, there have been several studies on generalizations of the DW invariant applicable to compact oriented 3-manifolds with boundary. In such a study, for the case of $G = \mathrm{SL}_2(\mathbb{C})$, Zickert focused on the relative group homology to define an invariant of 3-manifolds with boundary. Analogously, when $G = \mathrm{SL}_2(\mathbb{F}_p)$ for some prime p, Karuo defined a reduction of the DW invariant as an invariant of knots in the 3-sphere. He suggested a procedure of computing the reduced DW invariants via ideal triangulations of knot complements; however, to ensure that Karuo's approach, many aspects of the procedure need to be verified. In our study, we focus on the case $G = \mathrm{SL}_2(\mathbb{F}_q)$ and define the parabolic DW invariant as an invariant of links. Here, q is a power of a prime p. Roughly speaking, the definition of the parabolic DW invariant is an analogue of the DW invariant of the p-fold cyclic covering space of the 3-sphere branched over a link. The definition of the parabolic DW invariant is relatively simple, and it turns out to be a generalization of the above reduced DW invariant defined by Karuo. Moreover, we compute the parabolic DW invariants of several links whose double branched covering spaces are homeomorphic to some lens spaces. In application, we compute the parabolic DW invariants of the $(2, 2m)$-torus link and the $2m$-twist knot for all $m \in \mathbb{N}$. Moreover, an approach of using quandle theory and techniques with a relation to branched coverings, we establish a procedure for recovering the reduced DW invariant from the parabolic DW invariant. This procedure is based on some quandle colorings of a link diagram. Therefore, in contrast to Karuo's computation via ideal triangulations, our procedure provides easier computations of the reduced DW invariant based on only link diagrams. As a corollary, when $7 \leq p \leq 37$, we compute the reduced DW invariants of the prime knots with seven or fewer crossings.

備考：論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。
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