

論文 / 著書情報
Article / Book Information

題目(和文)	非整数階微分を含む半線形熱方程式の臨界指数における可解性について
Title(English)	On solvability of a time-fractional semilinear heat equation with the critical exponent
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学位種別(和文)	博士論文
Category(English)	Doctoral Thesis
種別(和文)	論文要旨
Type(English)	Summary

論文要旨

THESIS SUMMARY

系・コース： Department of, Graduate major in	数学 数学	系 コース	申請学位（専攻分野）： Academic Degree Requested	博士 Doctor of	(理学)
学生氏名： Student's Name	小島 瑞輝		審査員主査： Chief Examiner	小野寺 有紹	

要旨（英文 800 語程度）
Thesis Summary (approx.800 English Words)

The Fujita equation, a typical example of semilinear parabolic equations, contains a very rich structure relative to its simple form, and has been extensively investigated by many mathematician. When it comes to the solvability, analysis methods based on its scaling property often plays an important role. That is, the solution structure is often depicted by a space-time scaling transformation that makes the equation invariant. We can determine the so-called scale-critical space whose norm is invariant under the scaling transformation, and this critical space implies the threshold for the solvability of the equation. In particular, under the scale critical situation, it is known that the Fujita equation exhibits the global and local-in-time unsolvability. On the other hand, the time-fractional Fujita equation, in which the derivative with respect to time is of fractional order, has a scaling property that is common to the classical Fujita equation. Under the subcritical situation, it is known that they share the same properties on the solvability. However, under the critical situation, the time-fractional problem has the global and local-in-time solvability. The aim of this thesis is to clarify quantitatively the difference in global and local-in-time solvability under the critical situation, as the derivative order with respect to time approaches one. For this purpose, as for the global-in-time solvability, it is observed that the set of initial data that admit global-in-time solutions shrinks to the trivial one, as the derivative order approaches one. More precisely, the critical norm of the initial data must be evaluated from above by a constant which goes to zero as the derivative order approaches one. As for the local-in-time solvability, life span estimate for typical singular initial data is obtained in a way that takes into account the correspondence between the time-fractional and the classical case for the local-in-time solvability. For singular initial data allowing local-in-time solutions for the classical equation, life span does not go to zero, whereas for initial data which do not admit local-in-time solvability of the classical equation, the life span goes to zero, as the derivative approaches one. These results are regarded as a quantitative evaluation for the difference of solvability, in the sense of global and local-in-time respectively. To deduce these results, the first part of this thesis is devoted to give some necessary conditions and sufficient conditions for the solvability of the time-fractional Fujita equation, partially based on joint work with Kotaro Hisa. For the necessary conditions, we refer to the method of ordinary differential inequalities introduced by Laister and Sierżęga. The idea of this method is a reduction of the solvability of the original partial differential equation to a solvability of a nonlinear ordinary differential equation, which can be solved explicitly. On the other hand, we shall obtain sufficient conditions through the fixed point argument
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in appropriate function spaces. Firstly, we choose the uniformly local Lebesgue and Morrey spaces. This method enables us to deduce the optimal sufficient condition except for the Fujita critical case. However, for the Fujita critical case, we have to observe the fine structure of singularities of initial data to give the optimal condition. At this point, we introduce a function space called uniformly local weak Zygmund type space, which is investigated by Ioku, Ishige, and Kawakami recently.

These results obtained above are utilized to deduce the estimates for the norm of initial data allowing the global-in-time solvability, and we can obtain the optimal life span estimate for the typical singular initial data, as desired.

備考：論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。

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