PAPER

Time Domain Synthesis of Recursive Digital Filters for Finite Interval Response

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SUMMARY A least squares approximation method of recursive digital filters for finite interval response with zero value outside the interval is presented. According to the characteristic of the method, the modified Gauss Method is utilized in iteratively determining design parameters. Convergence, together with the stability of the resulting filter, are guaranteed.

key words: *time domain synthesis, IIR filters, convergence, non-linear optimization*

1. Introduction

Time domain synthesis for IIR filters has received considerable attention(1)−(5) as a synthesis method for determining parameters of recursive system directly when the desired impulse response can be specified. Consider the case when a long length of impulse response is given, then it may be impractical to use a nonrecursive system as an estimating model, since, for a high order system, a large number of multipliers and delay elements are required, and the number of arithmetic computations involved in generating each output of the system is, therefore, unavoidably high. To obtain an economical model, an efficient method that best approximates the specified response for a recursive system is desired.

Apparently, one straightforward way to obtain a good approximation is to find system parameters that minimize the square error between the specified response and the system response, that is

$$\min_{\boldsymbol{a}} \sum_{n} [h_o(n) - h(\boldsymbol{a}, n)]^2, \tag{1}$$

where $h_o(n)$ is the specified impulse response, $h(\boldsymbol{a}, n)$ is the IIR system impulse response and $\boldsymbol{a}$ is the system parameter vector defined by

$$\boldsymbol{a} = \{a_1, a_2, \cdots, a_m\},$$

where $a_i (i=1, 2, \cdots, m)$ is a system parameter.

Due to practical reason, the number of specified response data to be approximated is restricted to be a certain value, say, M which is selected to be large enough to cover the interval where $h_o(n)$ takes significant values. Therefore, the approximation is merely made over the impulse sequence within the interval $[0, M]$. In some applications, such as in synthesis of a low-order IIR filter from a long-length FIR prototype, the response outside the interval is required to be suppressed so that the distortion due to approximation is further reduced to a suitable degree.

Although the previous methods(1)−(3) can also be applied to the problem by placing an amount of zero data next to the specified response, the cost of calculation usually increases in accordance with the number of data and the entire response in time domain is not really taken into consideration. Also, when the Hessian matrix is involved in the estimating procedure,(1) the error in calculation of the matrix may increase with M so that the convergence rate of iterative approximation is affected.

In this paper, we introduce a synthesis method of recursive digital filters for approximating the impulse response whose values outside the specified interval are limited to be zero. Notice that the form of the system response $h(\boldsymbol{a}, n)$ is highly nonlinear, so the suitable optimization method has to be applied. In Sect. 2, it is shown that the modified Gauss Method(1),(6) provides a satisfying solution for the IIR parameter estimation when the particular form of the system function is selected. According to the characteristic of the procedure proposed in Sect. 3, the entire response in time domain is approximated and the Hessian matrix can be determined to be a closed form, independent of the value of M, so that the error in calculation is reduced. Also, the stable system function is always available. Then, the design strategy is described in Sect. 4 and the design examples are given in Sect. 5.

2. The Nonlinear Parameter Estimation Problem

Now we are concerned with the problem of minimizing the error function (1) when the finite interval response is specified. As mentioned above, we also want to suppress the impulse response outside the specified interval $[0, M]$. Then the error function in Eq. (1) becomes

$$f(\boldsymbol{a}) = \sum_{n=0}^{\infty} [h_o(n) - h(\boldsymbol{a}, n)]^2, \tag{2}$$

where $h_o(n) = 0$ for $M+1 \leqq n < \infty$.

Manuscript received August 18, 1992.
Manuscript revised December 7, 1992.

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For the unconstrained nonlinear optimization problem, it is clear that the gradient-based methods[(6),(7)] whose cost function is approximated by the quadratic model are very efficient. These methods are iterative in nature. In each step of iteration, the new value of parameter vector $\boldsymbol{a}$ is determined so that $f(\boldsymbol{a})$ decreases gradually to the minimum value. This can be expressed by

$$\boldsymbol{a}^{k+1}=\boldsymbol{a}^{k}+\alpha\Delta\boldsymbol{a}^{k} \tag{3}$$

and

$$f(\boldsymbol{a}^{k+1})<f(\boldsymbol{a}^{k}),$$

where $\boldsymbol{a}^k$ is the system parameter vector at the kth iteration. $\Delta\boldsymbol{a}^k$ is the direction vector evaluated at $\boldsymbol{a}^k$, and α is a positive scalar. The value of α should be selected so that $f(\boldsymbol{a}^k+\alpha\Delta\boldsymbol{a}^k)$ attains the minimum value for each iterative step.

In general, the form of $\Delta\boldsymbol{a}$ is given by

$$\Delta\boldsymbol{a}=-\boldsymbol{N}^{-1}\boldsymbol{q}, \tag{4}$$

where the Hessian matrix $\boldsymbol{N}$ and the gradient vector $\boldsymbol{q}$ are defined by

$$\boldsymbol{N}=\left[\frac{\partial f(\boldsymbol{a})}{\partial a_i\partial a_j}\right]_{m\times m},$$

$$\boldsymbol{q}=\left[\frac{\partial f(\boldsymbol{a})}{\partial a_i}\right]_{m\times 1},$$

for $a_i,\ a_j\in\boldsymbol{a}$ and $i,j=1,2,\cdots,m$.

It is well known that the Hessian matrix $\boldsymbol{N}$ is not necessarily positive definite for each iteration, consequently, $\boldsymbol{a}^k$ may not converge to the stationary point. For this reason, the various methods[(6),(7)] are developed so that the Hessian matrix is modified to be positive definite and the quadratic convergent property is also preserved. In this paper, the modified Gauss Method is chosen due to the simplicity in computation and the quadratic-like convergence. By using this method, the estimated solution of Hessian matrix can be calculated without evaluating the second partial derivatives of the cost function, which simplifies the realization procedure. And the modified matrix is always positive definite (or at least semidefinite).

3. Calculation of the Direction Vector

According to Eq. (4), in order to determine the direction vector $\Delta\boldsymbol{a}$, the gradient vector $\boldsymbol{q}$ and the modified matrix $\boldsymbol{N}'$ have to be derived. We first show how to calculate the vector $\boldsymbol{q}$. Expanding Eq. (2), we obtain

$$f(\boldsymbol{a})=\sum_{n=0}^{\infty}h_o^2(n)-2\sum_{n=0}^{\infty}h_o(n)h(\boldsymbol{a},n)+\sum_{n=0}^{\infty}h^2(\boldsymbol{a},n). \tag{5}$$

Since the specified impulse response $h_o(n)$ is finite, the derivatives of the related terms can be calculated directly. For the last term of Eq. (5) which is the squared impulse response of the IIR system, the explicit form of the sum is required so as to determine the derivatives of the entire sequence correctly.

From the Parseval's relation[(8)], we know that

$$\sum_{n=0}^{\infty}h^2(\boldsymbol{a},n)=\frac{1}{2\pi j}\oint_c H(\boldsymbol{a},z)H^*(\boldsymbol{a},z)z^{-1}dz, \tag{6}$$

where $H(\boldsymbol{a},z)$ is the z-transform of $h(\boldsymbol{a},n)$, hold. For a stable function, one of the feasible closed contour c is the unit circle. Also, if the sum on the left of Eq. (6) is finite, the integral on the right can be evaluated from the Cauchy residue theorem.

In the light of the above mention, both $H(\boldsymbol{a},z)$ and $h(\boldsymbol{a},n)$ should be described as the functions of poles, that is

$$H_e(\boldsymbol{a},z)=r_0+\sum_{i=1}^{l/2}\left(\frac{r_i+js_i}{1-u_ie^{jv_i}z^{-1}}+\frac{r_i-js_i}{1-u_ie^{-jv_i}z^{-1}}\right),$$

$$h_e(\boldsymbol{a},n)=\sum_{i=1}^{l/2}[(r_i+js_i)(u_ie^{jv_i})^n+(r_i-js_i)(u_ie^{-jv_i})^n]+r_0\delta(n),$$

for l=even order and

$$H_o(\boldsymbol{a},z)=\sum_{i=1}^{(l-1)/2}\left(\frac{r_i+js_i}{1-u_ie^{jv_i}z^{-1}}+\frac{r_i-js_i}{1-u_ie^{-jv_i}z^{-1}}\right)+r_0+\frac{r_{(l+1)/2}}{1-u_{(l+1)/2}z^{-1}},$$

$$h_o(\boldsymbol{a},n)=\sum_{i=1}^{(l-1)/2}[(r_i+js_i)(u_ie^{jv_i})^n+(r_i-js_i)(u_ie^{-jv_i})^n]+r_0\delta(n)+r_{(l+1)/2}u^n_{(l+1)/2}, \tag{7}$$

for l=odd order.

where the parameter vector $\boldsymbol{a}=\{r_0, r_1, s_1, u_1, v_1, \cdots\}$ and $\delta(n)$ is the delta function.

Note that the necessary and sufficient condition for u_i so that the Cauchy residue theorem can be applied to Eq. (6), is

$$\|u_i\|<1, \tag{8}$$

$$\text{for } i=\begin{cases}0,1,\cdots,l/2 & \text{(even) and}\\ 0,1,\cdots,(l-1)/2 & \text{(odd)}.\end{cases}$$

At the kth iteration, the values of u_i^{k+1} can be controlled through selecting the value of α in Eq. (3) such that u_i^{k+1} are always within the desired range. This is not any restriction of the procedure. Since, in Eq. (5), we consider the approximation over the interval $[0,\infty)$, it is clear that the gradient vector cannot be calculated anyway, if any of u_i has its value out of the range in Eq. (8). When the order of system function is suitably selected, the value of a which further reduces $f(\boldsymbol{a})$ in each iteration should be available, since the modified Gauss Method always provides a declined direction and only system parameters which satisfy Eq. (8) can minimize $f(\boldsymbol{a})$ in Eq. (2). Consequently, when $f(\boldsymbol{a})$ cannot be reduced to the acceptable value, a change of initial value or system function's order is recommended.

Using the model functions (7), the specified response is approximated by the recursive system with simple poles, which is usually the case. If the model with multiple poles is found to be more suitable, the form of model functions (7) can be changed accordingly.

Consequently, the partial derivative of Eq. (5) becomes

$$\frac{\partial f(a)}{\partial a_k}=-2\sum_{n=0}^{M}h_o(n)\frac{\partial h(a,\ n)}{\partial a_k}+\frac{\partial\sum_{i=0}^{l}R_i}{\partial a_k}, \quad (9)$$

where R_i is the residue of the integral of Eq. (6) and a_k is the kth element of the parameter vector $\boldsymbol{a}$.

Therefore, the gradient vector $\boldsymbol{q}$ is found to be

$$\boldsymbol{q}^T=\left[\frac{\partial f(\boldsymbol{a})}{\partial r_0},\ \frac{\partial f(\boldsymbol{a})}{\partial r_1},\ \frac{\partial f(\boldsymbol{a})}{\partial s_1},\ \cdots\right]_{1\times m}. \quad (10)$$

Next, consider the calculation of modified Hessian matrix $\boldsymbol{N}'$. The second partial derivative of Eq. (5) is given by

$$\frac{\partial f(\boldsymbol{a})}{\partial a_i\partial a_j}=2\sum_{n=0}^{\infty}\frac{\partial h(\boldsymbol{a},\ n)}{\partial a_i}\frac{\partial h(\boldsymbol{a},\ n)}{\partial a_j}+2\sum_{n=0}^{\infty}[h(\boldsymbol{a},\ n)-h_o(n)]\frac{\partial^2 h(\boldsymbol{a},\ n)}{\partial a_i\partial a_j}, \quad (11)$$

for $a_i,\ a_j\in\boldsymbol{a}$. In the modified Gauss method, the last term of Eq. (11) is neglected, since the value is assumed to be small compared with the first term and becomes zero as $\boldsymbol{a}^k$ gradually converges to the stationary point. Then, Eq. (11) becomes

$$\frac{\partial f(\boldsymbol{a})}{\partial a_i\partial a_j}\approx 2\sum_{n=0}^{\infty}\frac{\partial h(\boldsymbol{a},\ n)}{\partial a_i}\frac{\partial h(\boldsymbol{a},\ n)}{\partial a_j}=t_{ij}, \quad (12)$$

which is only the function of the first partial derivatives. Notice that each t_{ij} can be determined to be an explicit form, if the condition (8) holds. Therefore, we obtain that t_{ij} is independent of n. With the model functions (7), the partial derivative of $h(\boldsymbol{a},\ n)$ by a certain parameter reduces to a simple form because only the terms related to the parameter remain. Hence, the matrix $\boldsymbol{N}'$ is found to be

$$\boldsymbol{N}'=\begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1m} \\ t_{21} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ t_{m1} & \cdots & \cdots & t_{mm}\end{bmatrix}_{m\times m}. \quad (13)$$

It seems troublesome, at the first glance, that all the elements of $\boldsymbol{N}'$ have to be calculated into explicit forms, especially in the case of high order system. In fact, consider Eq. (7), we see that the partial derivative of $h(\boldsymbol{a},\ n)$ by a parameter, say, r_i has the same form of solution with the derivative by another parameter, r_j, which differs in subscript and also that $\boldsymbol{N}'$ is a symmetric matrix. Therefore, the number of the patterns of explicit forms which really need to be calculated is reduced considerably. Let T_{ij} be a matrix of the form

$$T_{ij}=\begin{bmatrix} \Delta r_ir_j & \Delta r_is_j & \Delta r_iu_j & \Delta r_iv_j \\ \Delta s_ir_j & \Delta s_is_j & \Delta s_iu_j & \Delta s_iv_j \\ \Delta u_ir_j & \Delta u_is_j & \Delta u_iu_j & \Delta u_iv_j \\ \Delta v_ir_j & \Delta v_is_j & \Delta v_iu_j & \Delta v_iv_j \end{bmatrix}, \quad (14)$$

where $\Delta a_ia_j=\sum_{n=0}^{\infty}\partial h(\boldsymbol{a},\ n)/\partial a_i\cdot\partial h(\boldsymbol{a},\ n)/\partial a_j$. Then, $\boldsymbol{N}'$ can be expressed in the partitioned form as

$$\boldsymbol{N}'=\begin{bmatrix} T_{11} & T_{12} & T_{13} & \cdots & T_{1p} \\ T_{21} & T_{22} & \cdots & \cdots & \cdots \\ T_{31} & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ T_{p1} & \cdots & \cdots & \cdots & T_{pp}\end{bmatrix}_{m\times m}. \quad (15)$$

It can easily be shown that the number of the different patterns within $\boldsymbol{N}'$ is always 26, for any dimension of $\boldsymbol{N}'$. In the previous method,[1] where each t_{ij} is calculated from determining $\partial h(\boldsymbol{a},\ n)/\partial a_i$ and $\partial h(\boldsymbol{a},\ n)/\partial a_j$ for each n in Eq. (12), we see that the cost of calculation of the matrix $\boldsymbol{N}'$ increases according to the value of n. In our case, t_{ij} is independent of n and can be calculated directly by substituting all parameters in each iterating step into the explicit form. Therefore, our method provides a more convenient way to determine $\boldsymbol{N}'$ in the iterative manner. And, since $\boldsymbol{N}'$ has the explicit form, the reduction of error in calculating $\boldsymbol{N}'$ in each iteration can be expected.

When the second derivative term of Eq. (11) is relatively large at the solution (so-called large residual problem), the rate of convergence of the modified Gauss method may be poor, due to the inappropriate Hessian approximation, as claimed in Ref. (7). In our concerned case, it can be shown that the model functions introduced in Eq. (7) can help improving the situation.

According to the characteristic of the model functions in Eq. (7), it is clear that the second derivative terms of Eq. (11) for all elements of T_{ij} in Eq. (14), *where* $i \neq j$, are naturally zero. So we obtain that, using the modified Gauss Method, only the elements of T_{ij}, *where* $i=j$, are changed in our approximation, compared with the original N matrix. This result, which defines the characteristic of the method, can be stated that all the eigenvalues of the Hessian matrix are adjusted through changing the values of elements along the diagonal region so that the modified matrix is positive definite. Since the modified Hessian matrix N' is very similar to the original one, it can be expected that the quadratic-like convergent property is well preserved, including in the large residual case.

4. The Realization Procedure

Once the explicit forms of $\boldsymbol{q}$ in Eq. (10) and N' in Eq. (15) are determined, the direction vector $\Delta\boldsymbol{a}$ in Eq. (4) can be calculated simply by substituting all parameter values of each iterating step into $\boldsymbol{q}$ and N'. Then, one next determines the suitable value of α in Eq. (3) so that the new parameter vector further reduces the error of Eq. (2). Some inventive methods for determining α, proposed in Refs. (6) and (7), are commonly applied. The iterative procedure continues until the specified conditions are satisfied. Usually, the iteration comes to the end when the error of the approximation becomes less than a preassigned value or, especially in the large residual problem, when the parameter vector reaches a stationary point. In the former case, the normalized error criterion used in measuring the quality of the approximation is defined by

$$f_N(\boldsymbol{a}) = \frac{f(\boldsymbol{a})}{\sum_{n=0}^{M} h_o^2(n)}. \tag{16}$$

In the latter, the parameter vector is said to have converged to a stationary point when the norm of gradient vector $\Delta\boldsymbol{a}$ is such that

$$\Delta^T\boldsymbol{a} \cdot \Delta\boldsymbol{a} < d, \tag{17}$$

where d is a small positive scalar.

5. Numerical Examples

Example 1: The impulse response of the FIR lowpass filter in Table 1 was approximated by the 4th order IIR model function in Eq. (7).

The initial value was made by the method suggested in Ref. (9). In each step, we simply selected the value of α to be 1, 1/2, 1/4, 1/8, $\cdots$ respectively, until the error of Eq. (2) was improved. The selected α might not be the best available value but our purpose here was to measure the convergence in this somewhat general case. The iteration was halted when $d < 10^{-7}$.

At the 14th iteration, d was found to be 5.8874×10^{-8}, and the $f_N(\boldsymbol{a})$ in Eq. (16) was 0.004634961. The total error outside the specified interval was reduced to 0.000240628. The convergence was second order in the first period and became superlinear convergence in the end.

The resulting system parameters are shown in Table 2, and the impulse response and amplitude characteristic of the IIR system and the FIR prototype

Table 1 Impulse response of Example 1.

n	$h_o(n)$	n	$h_o(n)$
1	-0.042975336	9	-0.087593136
2	0.000013938	10	-0.042709632
3	0.148910390	11	0.047426369
4	0.297110607	12	0.042961831
5	0.353754004	13	0.000000000
6	0.267330229	14	-0.023276534
7	0.087062827	15	0.000002178
8	-0.052122033		

Table 2 Parameters of the IIR function of Example 1.

i	r_i	s_i	u_i	v_i
0	0.5336197	-	-	-
1	0.1062341	0.2261560	0.7929802	1.0609526
2	-0.3945317	-1.2238856	0.5510295	0.4749205

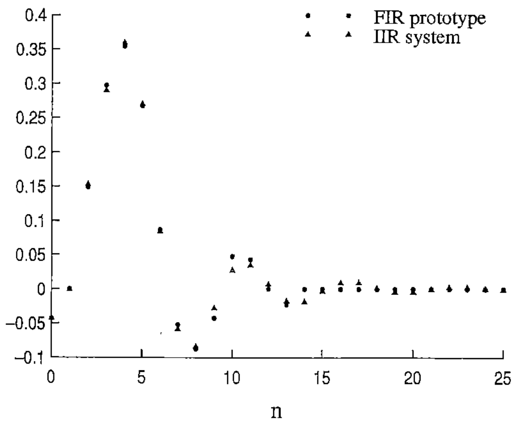

Fig. 1 Comparison between the impulse response of the FIR prototype and the designed IIR function of Example 1.

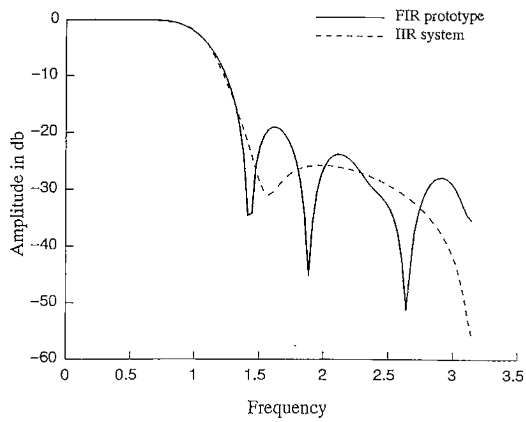

Fig. 2 Comparison between the amplitude characteristic of the FIR prototype and the designed IIR function of Example 1.

are compared in Fig. 1 and Fig. 2, respectively.

Example 2: Next, a truncated version of the ideal lowpass filter's impulse response was realized by the 4th order IIR function.

$$h_o(n)=\begin{cases}0 & n<0,\ n>150\\ \dfrac{\omega_c}{2\pi} & n=0\\ \dfrac{\sin(n\omega_c)}{2n\pi} & 0<n\leqq 150\end{cases} \tag{18}$$

The cutoff frequency ω_c was chosen to be 0.2π and then, we followed the same design procedure as in Example 1. At the 22nd iteration, d was found to be 4.8743×10^{-9}, and the $f_N(\boldsymbol{a})$ in Eq. (16) was 0.001756285. The method converged in a quadratic-like manner, as in the above example. The resulting system parameters are shown in Table 3, and the impulse response of the IIR system and the prototype response are compared in Fig. 3.

The case when the cutoff frequency ω_c was chosen

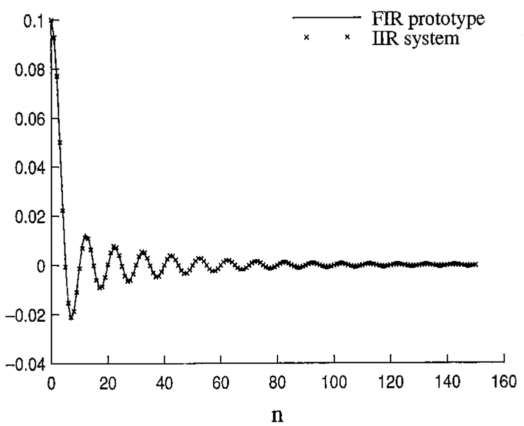

Fig. 3 Comparison between the impulse response of the FIR prototype and the designed IIR function of Example 2 (ω=0.2).

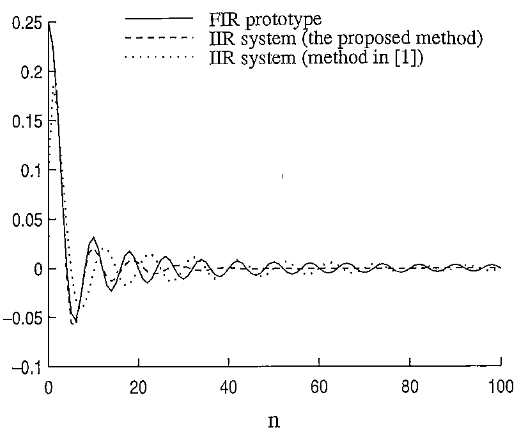

Fig. 4 Comparison of the impulse response of the FIR prototype, the IIR function of Ref. (1) and the designed IIR function (ω=0.25).

to be 0.25π is also shown in Fig. 4. The impulse response of the set of optimum coefficients of the 4th order IIR filter designed by the method in Ref. (1) was also plotted, together with the impulse response of IIR filter designed by our proposed method and the prototype impulse response. Here, the range of n in Eq. (18) was $0\leqq n\leqq 254$ and the 4th order IIR function was used in realization. The resulting system parameters are shown in Table 4. It can be seen that the IIR filter designed by our proposed method gives a response which is well fit to the prototype response in the first period. The $f_N(\boldsymbol{a})$ of our method was 0.0390536, where the value was 0.245968 in the method.[1]

Example 3: Then, the convergence in the large residue case was measured. The impulse response of the test function, shown in Table 5, was approximated by the 4th order IIR function in Eq. (7).

The same design procedure was applied again and the result was given in Table 6.

Here, $\|\boldsymbol{h}\|$ is the Euclidian norm of vector $\boldsymbol{h}$ defined by

$$\boldsymbol{h}=\boldsymbol{a}^k-\boldsymbol{a}^*, \tag{19}$$

where $\boldsymbol{a}^*$ is the parameter vector at the stationary point (or the parameter vector at the last iteration).

At the 30th iteration, the $f_N(\boldsymbol{a})$ was 0.808807, while d was found to be 1.2586640×10^{-6}, which showed that the parameter vector had already come closed to the stationary point, while the error of the

Table 3 Parameters of the IIR function of Example 2. (ω=0.2)

i	r_i	s_i	u_i	v_i
0	0.0172993	-	-	-
1	0.0007854	-0.008405	0.96711808	0.62633964
2	0.0405649	-0.0501148	0.69267335	0.4864221

Table 4 Parameters of the IIR function of Example 2. (ω=0.25)

i	r_i	s_i	u_i	v_i
0	0.0922536	-	-	-
1	-0.0158283	0.0251032	0.90193063	0.57803498
2	0.0947015	-0.1584670	0.78390658	0.65121177

Table 5 Impulse response of Example 3.

n	$h_o(n)$	n	$h_o(n)$
1	0.00721048331	13	0.31500080461
2	0	14	0
3	0.01362148952	15	0.09647795372
4	0	16	0
5	0.02629043395	17	0.04859472811
6	0	18	0
7	0.04859472811	19	0.02629043395
8	0	20	0
9	0.09647795372	21	0.01362148952
10	0	22	0
11	0.31500080461	23	0.00721048331
12	0.50000000000		

Table 6 Convergence of parameter vector in Example 3.

k	$\|h^k\|$	$\frac{\|h^{k+1}\|}{\|h^k\|}$	$\frac{\|h^{k+1}\|}{\|h^k\|^2}$	$f(a)$
1	0.412581383	0.826538699	2.003334935	0.476681083
2	0.341014480	0.794200708	2.328935438	0.470986101
3	0.270833941	1.139132377	4.206017788	0.466218073
4	0.308515712	1.007933665	3.267041598	0.428205672
5	0.310963372	1.024371798	3.294187962	0.420638886
6	0.318542109	0.978391640	3.071467198	0.410924697
7	0.311658936	0.969543591	3.110912207	0.410135927
8	0.302166925	0.967910671	3.203231694	0.409717838
9	0.292470591	0.979486741	3.349009339	0.409419368
10	0.286471066	0.965305839	3.369645151	0.409092292
11	0.276532193	0.974601074	3.524367499	0.408718464
12	0.269508572	0.957575939	3.553044451	0.408294621
13	0.258074924	0.968879812	3.754257851	0.407745695
14	0.250043584	0.947213092	3.788191941	0.407147281
15	0.236844557	0.962435236	4.063573374	0.406297931
16	0.227947547	0.933298832	4.094357864	0.405397438
17	0.212743179	0.927905755	4.361623985	0.404039066
18	0.197405620	0.924548895	4.683498328	0.402063515
19	0.182511148	0.903059865	4.947970968	0.399642786
20	0.164818493	0.795837760	4.828570770	0.392827831
21	0.131168780	0.863577857	6.583714907	0.389104524
22	0.113274454	0.350202287	3.091626343	0.386116066
23	0.039668973	0.962918004	24.27383236	0.383240378
24	0.038197968	0.935449865	24.48951876	0.383221803
25	0.035732284	0.218949407	6.127495374	0.383206206
26	0.007823562	0.790362857	101.0233965	0.383114439
27	0.006183453	0.528627764	85.49070331	0.383102914
28	0.003268745	0.343221003	105.0008482	0.383097387
29	0.001121902	0.000000000	0.000000000	0.383092934
30	0.000000000	-	-	0.383089128

approximation was slightly reduced. The total error outside the specified interval was reduced to 0.0110922. Although the norm of error was still large at the stationary point which is usually the case for large residue problem, it is clear that the method converged in a quadratic-like manner, as can be expected.

6. Conclusion

The least squares approximation method of recursive digital filters for finite interval response has been presented. Due to the characteristic of the modified Hessian method, the convergence is always guaranteed. It has been shown that when the model functions are appropriately selected, a close approximation of Hessian matrix can simply be obtained, so a quadratic-like convergence is to be expected. The method always provides a stable system function and the entire response in time domain is taken into consideration while the cost of calculation is comparable to the former design methods (Refs. (1)-(3)). It is expected that the proposed method can be well applied to a variety of problems.

Acknowledgment

The authors would like to thank Dr. T. Yoshida and Prof. N. Fujii of Tokyo Institute of Technology for their useful discussions and comments.

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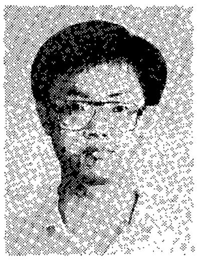

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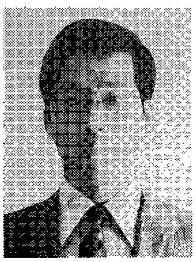

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