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PAPER

Interval Properties of Lattice Allpass Filters with Applications

Saed SAMADI†, Akinori NISHIHARA† *and* Nobuo FUJII†, *Members*

SUMMARY In practical applications of digital filters it is more realistic to treat multiplier coefficients as finite intervals than restricting them to infinite or very long word-length representations. However, this can not be done if the frequency response performance under interval assumption is difficult to analyze. In this paper, it is proved that stable lattice allpass filters possess bounded continuous phase response when lattice parameters vary in bounded intervals. It is shown that sharp bounds on the interval phase response can be computed easily at an arbitrary frequency using a simple recursive procedure. Application of this property to the problem of finite word-length lattice allpass filter design is also discussed. By formulating this problem as an interval design it is possible to solve it efficiently independent of the number system used to represent multiplier coefficients.

key words: *linear and nonlinear digital filters, digital signal processing*

1. Introduction

Boundary implications for frequency response of FIR and IIR filters were recently investigated by Bose and Kim [1]. According to their results, if the transfer function coefficients are bounded from above and bellow (interval coefficients), bounds on the amplitude response at a given frequency can be computed using certain extreme values of the coefficients. They have also shown that the specific extreme values that should be used at each frequency can be determined easily and the bounds of amplitude response can be computed efficiently.

In this paper we analyze the interval properties of lattice allpass filters assuming that lattice parameters are restricted to bounded closed intervals. Various applications of allpass filters in modern digital signal processing and the importance of lattice structures is our prominent motivation for this study [2]. However we will not limit ourselves to theoretical properties and consider practical design applications too.

The concept of "interval design" has been used in many areas of engineering to ensure that the implemented systems' specifications are immune to undesirable deviation of system parameters from their nominal values. When designing digital filters, we should pay attention to the well known problem of multiplier coefficient quantization. The quantization effect can be incorporated into the design problem in two ways. If the designer is aware of the specific number system in which the multiplier coefficients are to be represented and the allowed number of bits, he or she can adopt a discrete design method that works on the space of finite word-length coefficients only [3]. In this way the results can be realized in the number system without any quantization. We call this method "direct discrete design." However, direct discrete design may be difficult to formulate and solve in some cases and/or become very time consuming for high order filters because of the properties of the finite word-length number system. In such cases it is preferred to formulate the problem as an interval design, where the multiplier coefficients are assumed to be intervals, and determine the intervals such that the interval frequency response satisfies the specifications. An appropriate finite word-length number can be found within each interval. This method is independent of the number system and the problem can be formulated in a continuous manner now.

The organization of this paper is as follows: In Sect. 2, the phase response of lattice allpass filters is described and its interval properties are analyzed. A simple recursive algorithm for computing sharp bounds on the interval phase response is also derived. In Sect. 3, practical applications of the results of Sect. 2 to the finite word-length design of lattice allpass structures is studied and a general interval design formulation is proposed. Section 4 consists of an illustrative example of the interval design and its formulation. Finally, the conclusion is given in Sect. 5.

2. Phase Response of Lattice Allpass Filters

Assume the lattice realization (also known as Gray and Markel lattice) [4] of an Nth order allpass function. Since there are many ways to realize lattice filters, with different types and numbers of multiplier coefficients, we do not specify the structure and illustrate the filter by the block diagram shown in Fig. 1. The letters k_i designate lattice parameters and i takes on integer values from 0 to $N-1$. When we talk about "interval lattice parameters," we mean that $k_i \in I_i \stackrel{\text{def}}{=}$

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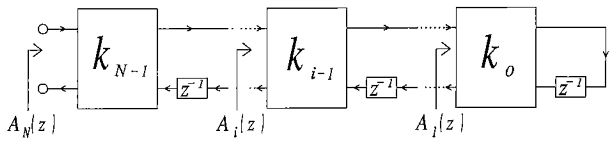

Fig. 1 Block diagram of an Nth order lattice allpass filter.

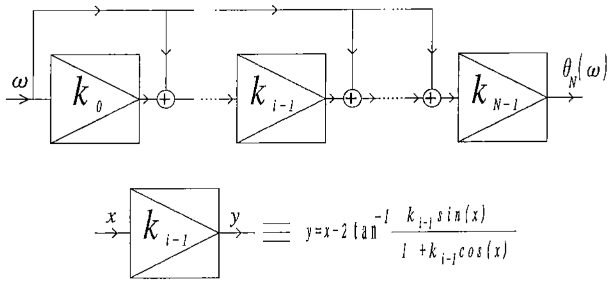

Fig. 2 Block diagram for computing phase functions.

$[\underline{k}_i, \bar{k}_i]$, or in other words, each k_i can have any value within I_i independently. Consequently, the number of combinations of the parameters is infinite as long as we have at least one interval with non-zero width. With signal processing application of these filters in mind, we shall restrict ourselves to intervals that produce stable filters, i.e., $|\underline{k}_i|, |\bar{k}_i|<1$. Analyzing the phase response of filters with interval lattice parameters ("interval lattice filters") is the purpose of this Section.

First we give some preliminary information on the phase response of lattice allpass filters when lattice parameters are fixed. The phase response can be computed using recursion [5]

$$\theta_i(\omega)=\omega+\theta_{i-1}(\omega) -2\tan^{-1}\left[\frac{k_{i-1}\sin(\omega-\theta_{i-1}(\omega))}{1+k_{i-1}\cos(\omega+\theta_{i-1}(\omega))}\right] \quad (1)$$

$$\text{for} \quad i=1, \cdots, N, \quad \theta_0(\omega)=0,$$

where the phase response becomes $-\theta_N(\omega)$. We call $\theta_i(\omega)$ "phase functions" and there exists N of these functions for a filter of order N (excluding $\theta_0(\omega)$). The ith phase function, $\theta_i(\omega)$, is the absolute value of the phase response of an ith order allpass filter with lattice parameters $k_0, \cdots, k_{i-1}$. Figure 2 illustrates recursion (1) as a cascade of N computation blocks each of which computes the phase function of a first order allpass filter with an appropriate lattice parameter as its coefficient.

2. 1 Interval Phase Response

Now we assume that lattice parameters can vary within the intervals. Obviously, we can not determine the exact values of the phase functions under this assumption and it is desirable to analyze their interval behavior. The question is whether interval phase functions are continuous with respect to (w.r.t.) the lattice parameters and there exists a simple way to obtain sharp bounds on them. The following Theorems answer this question.

Theorem 1: If the interval phase function $\theta_{i-1}(\omega)\in[\underline{\theta}_{i-1}(\omega), \bar{\theta}_{i-1}(\omega)]$ is continuous w.r.t. interval lattice parameters $k_0, \cdots, k_{i-2}$ at a fixed ω, then the interval phase function $\theta_i(\omega)$ is continuous w.r.t. $k_0, \cdots, k_{i-1}$ and its minimum and maximum values at ω become

$$\underline{\theta}_i(\omega)=\begin{cases}\theta_i(\omega)\,|_{\theta_{i-1}(\omega)=\underline{\theta}_{i-1}(\omega),\ k_{i-1}=\bar{k}_{i-1}} & \text{for } \left\lfloor\frac{\omega+\underline{\theta}_{i-1}(\omega)}{\pi}\right\rfloor^{\dagger} \text{ even,}\\ \theta_i(\omega)\,|_{\theta_{i-1}(\omega)=\underline{\theta}_{i-1}(\omega),\ k_{i-1}=\underline{k}_{i-1}} & \text{for } \left\lfloor\frac{\omega+\underline{\theta}_{i-1}(\omega)}{\pi}\right\rfloor \text{ odd,}\end{cases}$$

$$\bar{\theta}_i(\omega)=\begin{cases}\theta_i(\omega)\,|_{\theta_{i-1}(\omega)=\bar{\theta}_{i-1}(\omega),\ k_{i-1}=\underline{k}_{i-1}} & \text{for } \left\lfloor\frac{\omega+\bar{\theta}_{i-1}(\omega)}{\pi}\right\rfloor \text{ even,}\\ \theta_i(\omega)\,|_{\theta_{i-1}(\omega)=\bar{\theta}_{i-1}(\omega),\ k_{i-1}=\bar{k}_{i-1}} & \text{for } \left\lfloor\frac{\omega+\bar{\theta}_{i-1}(\omega)}{\pi}\right\rfloor \text{ odd,}\end{cases}$$

where, $\underline{\theta}_i(\omega)\,(\underline{\theta}_{i-1}(\omega))$ and $\bar{\theta}_i(\omega)\,(\bar{\theta}_{i-1}(\omega))$ denote the minimum and maximum values of $\theta_i(\omega)\,(\theta_{i-1}(\omega))$ respectively.

Proof: Since lattice parameters are less than unity in their absolute values, and ω is fixed, from (1) we have

$$\frac{\partial\theta_i(\omega)}{\partial\theta_{i-1}(\omega)}=\frac{1-k_{i-1}^2}{1+2k_{i-1}\cos(\omega+\theta_{i-1}(\omega))+k_{i-1}^2}>0 \quad (2)$$

and

$$\frac{\partial\theta_i(\omega)}{\partial k_{i-1}}=\frac{-2\sin(\omega+\theta_{i-1}(\omega))}{1+2k_{i-1}\cos(\omega+\theta_{i-1}(\omega))+k_{i-1}^2}\begin{cases}\geqq 0 \text{ for } \lfloor\frac{\omega+\theta_{i-1}(\omega)}{\pi}\rfloor \text{ odd,}\\ \leqq 0 \text{ for } \lfloor\frac{\omega+\theta_{i-1}(\omega)}{\pi}\rfloor \text{ even.}\end{cases} \quad (3)$$

Thus, from (2), (3) and $|k_{i-1}|<1$, we conclude that $\theta_i(\omega)$ is always a continuous increasing function w.r.t. $\theta_{i-1}(\omega)$, and a continuous decreasing, increasing or constant function w.r.t. k_{i-1}.

Now assume that

$$\underline{\theta}_i(\omega)=\theta_i(\omega)\,|_{\theta_{i-1}(\omega)=\hat{\theta}_{i-1}(\omega),\ k_{i-1}=\hat{k}_{i-1}},$$

where, $\hat{k}_{i-1}\in[\underline{k}_{i-1}, \bar{k}_{i-1}]$ and $\hat{\theta}_{i-1}(\omega)\in[\underline{\theta}_{i-1}(\omega), \bar{\theta}_{i-1}(\omega)]$. First we state that $\hat{\theta}_{i-1}(\omega)$ is equal to $\underline{\theta}_{i-1}(\omega)$. This can be proved by noting that if $\hat{\theta}_{i-1}(\omega)>\underline{\theta}_{i-1}(\omega)$, from (2) we have $\theta_i(\omega)<\underline{\theta}_i(\omega)$ at $\theta_{i-1}(\omega)=\underline{\theta}_{i-1}(\omega)$, regardless of the value of $\hat{k}_{i-1}$. But this

† $\lfloor x \rfloor$ denotes the largest integer smaller than x.

contradicts the definition of $\underline{\theta}_i(\omega)$. Consequently,

$$\hat{\theta}_{i-1}(\omega)=\underline{\theta}_{i-1}(\omega).$$

Now that the value of $\hat{\theta}_{i-1}(\omega)$ is determined we show how it affects $\hat{k}_{i-1}$. If $\lfloor\frac{\omega+\underline{\theta}_{i-1}(\omega)}{\pi}\rfloor$ is even, $\theta_i(\omega)$ becomes a decreasing or constant function with respect to k_{i-1}. In the case of decreasing function, it is clear that $\hat{k}_{i-1}=\bar{k}_{i-1}$. In the constant case, the value of k_{i-1} does not make any difference and we may again adopt $\hat{k}_{i-1}=\bar{k}_{i-1}$.

The remaining parts can be proved similarly. □

Theorem 2: An Nth order, stable, interval lattice filter possesses a continuous phase response w.r.t. parameters $k_0, \cdots, k_{N-1}$ whose maximum and minimum values at a fixed frequency can be computed recursively.

Proof: The phase response is $-\theta_N(\omega)$ and all we have to do is to prove the Theorem for the phase function $\theta_N(\omega)$. Applying Theorem 1 from $i=1$ to $i=N$ recursively we can easily compute sharp upper and lower bounds on the Nth order interval phase function and simultaneously prove its continuity. Note that for $i=1$, Theorem 1 holds because of the boundary condition $\underline{\theta}_0(\omega)=\bar{\theta}_0(\omega)=\theta_0(\omega)=0$. □

2.2 Procedure for Computing Bounds of the Interval Phase Response

Theorem 2 proposes a recursive algorithm to compute $\underline{\theta}_N(\omega)$ and $\bar{\theta}_N(\omega)$ as summarized in the following procedure.

$\underline{\theta}_0(\omega) \leftarrow 0$, $\bar{\theta}_0(\omega) \leftarrow 0$

i ← 1

while(i<=N)

{

$\theta_{i-1}(\omega) \leftarrow \underline{\theta}_{i-1}(\omega)$

if $\text{mod}(\frac{\omega+\underline{\theta}_{i-1}(\omega)}{\pi},2)=0$ then $k_{i-1} \leftarrow \bar{k}_{i-1}$

else $k_{i-1} \leftarrow \underline{k}_{i-1}$

calculate $\theta_i(\omega)$ from equation (1)

$\underline{\theta}_i(\omega) \leftarrow \theta_i(\omega)$

$\theta_{i-1}(\omega) \leftarrow \bar{\theta}_{i-1}(\omega)$

if $\text{mod}(\frac{\omega+\bar{\theta}_{i-1}(\omega)}{\pi},2)=0$ then $k_{i-1} \leftarrow \underline{k}_{i-1}$

else $k_{i-1} \leftarrow \bar{k}_{i-1}$

calculate $\theta_i(\omega)$ from equation (1)

$\bar{\theta}_i(\omega) \leftarrow \theta_i(\omega)$

i ← i+1

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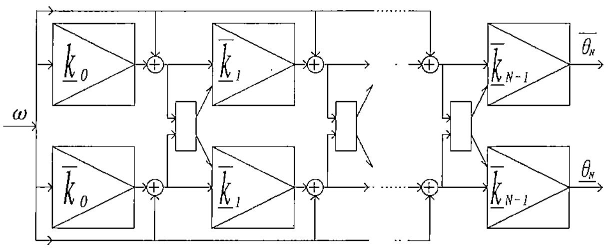

Fig. 3 Block diagram for computing extreme values of interval phase functions.

The computation process can be illustrated by the block diagram shown in Fig. 3, where $\underline{\bar{k}}_i$ denotes the maximum or minimum value of the interval parameters. The small blocks located between adjacent phase computation blocks are responsible for controlling the lattice parameter that should be used in the next two phase computing block. This is done based on conditions given in Theorem 1. The two leftmost blocks do not need to be controlled because $\partial\theta_1(\omega)/\partial k_0 \leqq 0$ for $\omega \in [0, \pi]$ and we can always use $\underline{k}_0$ for computing $\bar{\theta}_1(\omega)$ and $\bar{k}_0$ for computing $\underline{\theta}_1(\omega)$. From this Figure we can observe that the computation cost is approximately the same as computing $2N$ first order phase functions.

3. Practical Applications of Interval Lattice

Here we consider the application of the interval approach described in Sect. 1, to finite word-length design of lattice allpass filters. The gist of this approach is that if the interval phase response is such that its extrema satisfy the specifications' tolerance at all frequencies, then any choice of lattice parameters within corresponding intervals will meet the specifications. Fortunately, the phase response of interval lattice allpass filters possess sharp upper and lower bounds that can be computed easily. Thus, formulating the problem as an interval design is possible and easy. It is important to note that this is not the case for most of other allpass structures where sharp bounds on the interval phase response are difficult to obtain.

There are two major different types of lattice structures. Structures using lattice parameters as multiplier coefficients and normalized structures that use both lattice parameters, k_i, and their complementary terms, $k_i'=\sqrt{1-k_i^2}$. Both of these structures can be designed easily through the interval approach. An interesting case in which the interval design can be very useful is designing normalized lattice allpass filters for CORDIC implementation [6]. In CORDIC implementation the rotation angle $\phi_i=\sin^{-1} k_i$ is approximated by a sequence of elementary angles, $a_i(j)$, as

$$\phi_i = \sum_{j=0}^{n-1} \mu_i(j)\, a_i(j),$$

where $\mu_i(j)=1$ or -1, and n is the number of rotations. Rotation through ϕ_i is performed by n iterative rotations through $\mu_i(j)\, a_i(j)$. Elementary angles $a_i(j)$ should satisfy

$$a_i(j) = \tan^{-1}(2^{-s_i(j)}), \quad \text{for} \quad j=0, \cdots, n-1, \qquad (4)$$

where $s_i(j)$ is an integer. A finite word-length design consists of finding appropriate values for n, $s_i(j)$ and $\mu_i(j)$. Fulfilling this task through a direct discrete design is not only difficult but also suffers from unavoidable numerical errors that arise when computing the values of $a_i(j)$ in (4). In an interval design approach any value of k_i within I_i is acceptable and we are free to choose any n, $s_i(j)$ and $\mu_i(j)$ as long as $\sin \phi_i (=k_i)$ belongs to this interval.

Another option in implementing the normalized structure is realizing multiplier coefficients, k_i and k_i', directly. If the available word-length is long enough, we may have more than one choice for the quantized value $Q[k_i] \in [\underline{k}_i, \bar{k}_i]$ and the problem is finding the best of them. This can be solved by finding the $Q[k_i]$ that minimizes

$$e_i \overset{\text{def}}{=} 1 - Q[k_i]^2 - Q[k_i']^2$$

within the word-length constraints. Once the quantization scheme Q and the word-length are determined a simple search method would be enough.

3.1 Formulation of Interval Phase Approximation Problem

Let us first define our design parameters that specify the intervals I_i. These are

$$c_i = \frac{\bar{k}_i + \underline{k}_i}{2},$$

that denote intervals' centers and

$$w_i = \frac{\bar{k}_i - \underline{k}_i}{2}$$

denoting half of their widths. The following patterns of treating c_i and w_i are of practical use.

- c_i: unknown; w_i: constant.
- c_i: constant; w_i: unknown.

If an infinite word-length design is already provided and the designer wants to find the tolerance of the phase response after applying quantization schemes, the second pattern can be employed. The problem here is finding w_i such that the interval phase response does not violate the specifications.

For an interval design, the first pattern is applicable but the problem is the choice of the interval widths which are constant during the design. Of course if we are aware of the available word-length and the number system for representing multiplier coefficients, w_i can be chosen such that at least one finite word-length solution exists in each interval. Otherwise, we should adopt a reasonable width. The problem for the first pattern can be stated as bellow.

Find N and c_i for $i=0, \cdots, N-1$ such that $\max\{|c_i - w_i|, |c_i + w_i|\} < 1$† and $\forall \omega \in S$,

$$\forall\, k_i \in I_i = [c_i - w_i,\ c_i + w_i],$$
$$|\theta_N(\omega) - \theta_D(\omega)| \leqq \varepsilon(\omega), \qquad (5)$$

or equivalently,

$$\max\{|\underline{\theta}_N(\omega) - \theta_D(\omega)|,\ |\bar{\theta}_N(\omega) - \theta_D(\omega)|\} \leqq \varepsilon(\omega), \qquad (6)$$

where S is the union set of approximation regions, $\theta_D(\omega)$ the desired or ideal value of the phase function and $\varepsilon(\omega)$ the approximation tolerance. The equivalence between (5) and (6) can be verified easily by noting that the bounds are sharp and the phase function is continuous. In this paper we define

$$f(\omega) \overset{\text{def}}{=} \max\{|\underline{\theta}_N(\omega) - \theta_D(\omega)|,\ |\bar{\theta}_N(\omega) - \theta_D(\omega)|\} - \varepsilon(\omega)$$

$$E(c_0, \cdots, c_{N-1}) \overset{\text{def}}{=} \int_S (\max\{f(\omega), 0\})^p d\omega \qquad (7)$$

where p is an integer, and reduce the above problem to finding the solution of equation

$$E(c_0, \cdots, c_{N-1}) = 0 \qquad (8)$$

using an optimization procedure. To determine an optimum N we can start from an optimistic guess and increment it until (8) is satisfied. The c_i of an optimized lower order lattice can be used as initial values for corresponding parameters in the order incremented lattice.

Designing finite word-length allpass filters becomes more convenient by using the interval approach because of the following two basic reasons. Firstly, the problem becomes independent of the number system used for representing multiplier coefficients. That is, the designer does not need to devise different algorithms for different number systems and the complexity of number system does not affect the design. Once an algorithm for interval design is available a finite word-length solution for any number system, e.g. fixed or floating point binary, etc., and any type of structure, e.g. normalized structure with CORDIC algorithm, etc., can be easily obtained. Secondly, by formulating the problem as an interval design the discrete optimization problem of finite word-length design becomes a continuous design problem, and can be solved in the continuous space of c_i parameters.

† max $\{a, b\}$ gives the larger value of a and b.

This is generally an advantage, specially when the number system is complex.

4. Illustrative Design Example

Here we consider the design of a lattice allpass filter whose phase response approximates a linear phase within a given tolerance in prescribed frequency bands. These filters were proposed in [7] to realize complementary filter pairs with approximately linear phase. As an example, consider the case when a lowpass filter is required. The transfer function is of the form

$$H(z)=\frac{A_N(z)+z^{-(N+d)}}{2},$$

where $d=1$ or $d=-1$. The gain is computed as

$$|H(\omega)|=\left|\cos\frac{\theta_N(\omega)-(N+d)\omega}{2}\right|,$$

and the phase tolerance $\varepsilon(\omega)$ can be found from the gain specifications:

- Maximum Passband Attenuation $=a_p$ dB.
- Minimum Stopband Attenuation $=a_s$ dB.

Thus, in interval formulation stated in Sect. 3, we set

$$S=S_p\cup S_s$$

$$\theta_D(\omega)=\begin{cases}(N+d)\omega, \text{ for } \omega\in S_p\\(N+d)\omega+\pi, \text{ for } \omega\in S_s\end{cases}$$

$$\varepsilon(\omega)=\begin{cases}2\cos^{-1}10^{\frac{-a_p}{20}} \text{ for } \omega\in S_p,\\\left|\pi-2\cos^{-1}10^{\frac{-a_s}{20}}\right| \text{ for } \omega\in S_s,\end{cases}$$

where $S_p=[0,\ \omega_p]$ and $S_s=[\omega_s,\ \pi]$.

As an example we designed a lowpass filter with $d=-1$ and spectral requirements

- $\omega_p=0.55\pi$, $\omega_s=0.62\pi$
- $a_p=0.5$ dB, $a_s=30$ dB.

Assuming 8-bit fixed-point binary representation of lattice parameters, we should make the interval widths wide enough such that at least one 8-bit quantized number is included in each. Therefore a realistic value for the width parameters is $w_i=2^{-8-1}$. Equation (8) was solved using an easy-to-implement search method called "flexible polyhedron method" [8]. This method is based on successive alterations of a simplex in variable space and may be modified easily to avoid unstable filters. For initial estimates of interval centers we used a simple iterative method [9]. To amplify large errors we set $p=10$. The design procedure of this example consists of the following steps.

1. Adopt a reasonable value for N, filter's length, and w_i, the width parameters.
2. Find initial values of k_i $(i=0, 1, \cdots, N-1)$ using the simple iterative method introduced in [9]. These initial values will not satisfy the design specifications generally.
3. Set $c_i\leftarrow k_i$ for $i=0, 1, \cdots, N-1$.
4. Find the solution to equation $E(c_0, \cdots, c_{N-1})=0$. If no solution found set $N\leftarrow N+1$, and then $c_{N-1}\leftarrow 0$ and repeat this step.

The CPU time for this example was less than 1 minute on a SONY RISC machine. It turned out that the minimum required order was $N=8$ and parameter intervals were found as shown in Table 1. After finding the interval centers, we simply truncated $\underline{k}_i=c_i-w_i$ or $\bar{k}_i=c_i+w_i$ to 8 bits and obtained the ith finite word-length lattice parameter. Figures 4 and 5 show the gain characteristics with lattice parameters of 8 bit

Table 1 Intervals and 8-bit-length parameters.

I_0	[0.158507875 , 0.162414125]	k_0	0.160156250
I_1	[0.572600875 , 0.576507125]	k_1	0.574218750
I_2	[-0.204780515 ,-0.200874265]	k_2	-0.203125000
I_3	[-0.168205821 ,-0.164299571]	k_3	-0.167968750
I_4	[0.115449061 , 0.119355311]	k_4	0.117187500
I_5	[0.044076297 , 0.047982547]	k_5	0.046875000
I_6	[-0.123735315 ,-0.119829065]	k_6	-0.121093750
I_7	[-0.097998615 ,-0.094092365]	k_7	-0.097656250

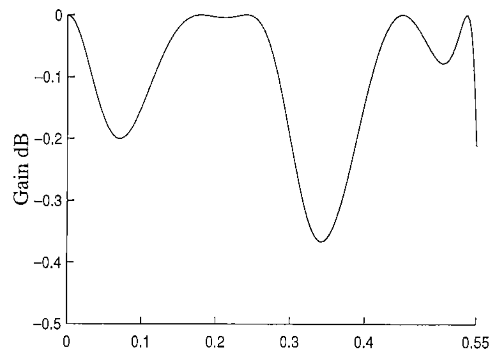

Fig. 4 Gain versus $\frac{\omega}{\pi}$ in passband for 8-bit parameters of Table 1.

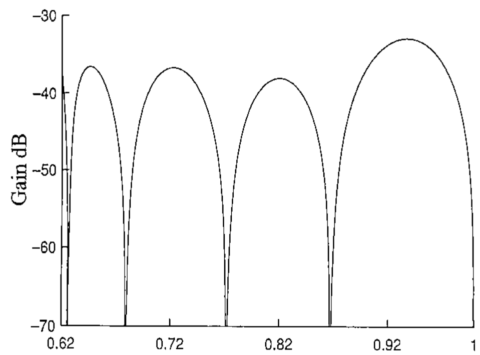

Fig. 5 Gain versus $\frac{\omega}{\pi}$ in stopband for 8-bit parameters of Table 1.

length.

5. Conclusions

In interval design of digital filters we trade filter's order for more flexibility in choosing multiplier coefficients; the wider the intervals, the higher the order. However, if the width parameters are small enough the result will not differ much from the infinite word-length design. It should be noted that in interval design of finite word-length filters, by the term "word-length" we mean the minimum possible word-length. We are free to adopt longer word-lengths as long as the lattice parameters are within the intervals.

In conclusion, we proved that interval lattice filters give rise to continuous and bounded phase responses and derived a simple recursive algorithm for computing sharp bounds on the phase response. This algorithm needs $2N$ first order phase computations to compute the maximum and minimum values of the phase function of an Nth order lattice allpass filter simultaneously. We showed that this property can be employed in interval design of lattice allpass filters as a flexible and efficient approach to the finite word-length design of all types of lattice structures. The formulation of the design problem as an interval design was explained and was reduced to the solution of a nonlinear equation in continuous domain. In this way, the problem can be solved independent of the number system used for representing multiplier coefficients. As an illustrative example we applied our method to the interval design of approximately linear phase lattice allpass filters and solved the problem with a simple search algorithm. This clearly shows that using the interval approach, in addition to the above mentioned merits, we do not have to employ complicated discrete optimization procedures and a general continuous algorithm would suffice.

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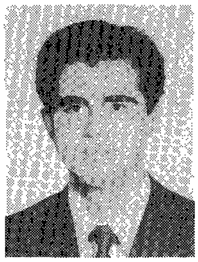

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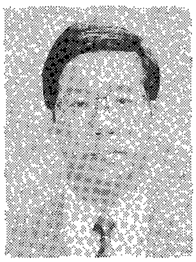

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