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PAPER

Design of High-Order Noise-Shaping FIR Filters for Overload-Free Stable Single- and Multi-Bit Data Converters

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SUMMARY This paper presents optimum and sub-optimal designs of noise-shaping FIR filters for single- and multi-bit data converters. In the designs, only three parameters, the number of taps, oversampling ratio (OSR) and l_1-norm of the filter coefficients are specified, and the in-band peak of the amplitude response is minimized under the specifications. The minimization problem is formulated with the overload-free condition, which guarantees the rigorous stability, and an overload-free converter generates no distortion in any output signals. In the optimum design, the minimization problem is directly and exactly solved, but the sub-optimal method solves this problem by iteratively utilizing the simplex method. The iterative sub-optimal method without the exact optimality is far faster and more efficient than the optimum method. In design examples, optimum and sub-optimal noise-shaping FIR filters for single- and multi-bit data converters are designed, and their optimal performance is revealed. For single-bit data converters with OSR 64, a noise-shaping FIR filter is designed and then shown to achieve a signal to noise and distortion ratio (SNDR) 107.6 [dB] in the band of interest.

key words: *stability, noise-shaping FIR filter, oversampling ΣΔ modulator, optimization*

1. Introduction

Sigma-delta (ΣΔ) data converters have been widely used in both A-D and D-A converters. Figure 1(a) shows the block diagram of a ΣΔ modulator. The topology of a ΣΔ modulator can be rearranged into an equivalent error feedback structure illustrated in Fig. 1(b) [1]. In the error feedback structure, if the loop filter has a finite impulse response (FIR), then the noise transfer function (NTF) is also realized as an FIR filter, which is often more suitable than an infinite impulse response (IIR) filter [2]. The error feedback structure is generally applicable to a digital noise-shaping converter rather than an analog ΣΔ modulator, because a characteristic of the FIR loop filter and subtractor is sensitive to the performance of the noise-shaping.

Multi-bit noise-shaping digital data converters are often applied to D-A converters with the dynamic element matching techniques [3]. Many algorithms to design stable multi-bit noise-shaping data converters have been proposed (e.g. [4]–[6] etc.), but the algorithms are based on optimizing the location of poles and/or zeros of an NTF and may not ensure the optimality in terms of its amplitude response.

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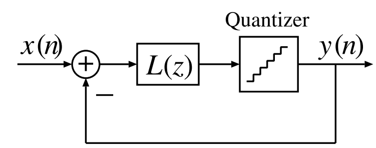
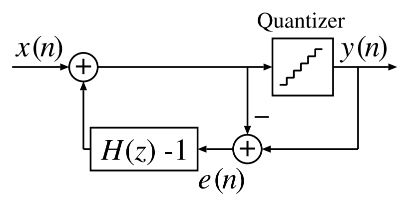


(a) ΣΔ modulator.



(b) Noise-shaping converter with error feedback.

Fig. 1 Oversampling noise-shaping data converters.

As for rigorously stable single-bit data converters, the first-order converters etc. have been known but do not achieve a substantial performance of the noise-shaping with a moderate OSR. On the other side, many papers e.g. [1], [7], [8] have introduced efficient design methods of ΣΔ modulators and their design examples, but the stability is just empirically treated with rules of thumb due to complicated nonlinear behavior of ΣΔ modulators. A number of papers have tackled this stability problem e.g. [1], [9]–[14] but have not successfully solved it for high-order ΣΔ modulators. Therefore it is just known that generally high-order ΣΔ modulators are prone to suffer from the instability so that any papers have never illuminated an existence and achievable performance of rigorously stable higher-order ΣΔ modulators in spite of their potential. We have started to investigate and tackle the above design problem for rigorously stable high-order ΣΔ modulators [15]–[18] and successfully have reduced the OSR without a significant degradation of the performance. In this paper, we focus on design of still higher-order noise-shaping FIR filters and propose two methods to design an NTF with some theoretical optimality for a given design specification. The first method derives the optimum NTF at the price of computing cost. The second method is faster than the first one but does not guarantee the optimality of the designed NTF.

In our design methods, to ensure the stability of a noise-shaping converter, the impulse response of an NTF is constrained so as to have its l_1-norm not exceeding a specified level [9], and then directly optimized to minimize the peak amplitude of the NTF in a specified band. Under this stability condition, any overload cannot occur at the quantizer so that the quantization error is strictly bounded to the quantization step size for arbitrary input signal $x(n)$ whose amplitude is upper-bounded to a certain level. This overload-free condition is sufficient for the stability and may be conservative, but, due to its rigorousness and simplicity, the condition can be handled in optimizing an NTF. The design problem can be directly formulated as a linear programming problem, which can be solved with the optimum solution by the simplex method [19]. Our first method exactly solves the linear programming problem and can derive the optimum solution for a specification with three parameters; the number of taps, oversampling ratio (OSR) and l_1-norm of the filter coefficients.

In the above first method, the l_1-norm-based stability condition is rearranged into linear inequalities, but the number of those inequalities exponentially increases with the growth of the order of the NTF. So the above first method becomes computationally intensive especially for design of high order NTFs. In order to investigate an existence of overload-free stable higher order data converters, we also propose another method and analyze its performance. This second method is sub-optimal and does not derive exactly optimum solution. However it can save large memory space and computing cost of the filter design, and besides derive near optimal solutions.

In design examples, a number of optimal noise-shaping FIR filters are designed, and the performance of overload-free stable noise-shaping converters with the optimized NTFs is evaluated. Then the effectiveness of our two design methods is confirmed.

2. Design Methods

2.1 Output Signals and Stability of Oversampling Noise-Shaping Data Converters

Let us write the Fourier Transforms of $x(n)$, $y(n)$ and $e(n)$ in Fig. 1(b) as $X(e^{j\omega})$, $Y(e^{j\omega})$ and $E(e^{j\omega})$, respectively. Then from Fig. 1(b), we obtain

$$Y(e^{j\omega}) = X(e^{j\omega}) + H(e^{j\omega})E(e^{j\omega}). \tag{1}$$

Thus the NTF $H(z)$ needs to substantially attenuate quantization noise $E(z)$ in a band of interest. In the following, the in-band peak amplitude of $|H(e^{j\omega})|$ is minimized. Also, here we briefly define a general deterministic stability as follows; for a specified set of input signals and an initial value of each internal signal, if each internal and output signal of a data converter can be upper-bounded to a certain level for all possible input signals of the specified set, then the converter is defined as a stable system for the specified set of input signals and the initial values.

Here let the impulse response of a T-tap FIR NTF $H(z)$ be $h(n)$ for $n = 0, \cdots, T-1$. From the structure shown in Fig. 1(b), the first sample $h(0)$ is constrained to be 1.0. Then a rigorous sufficient condition for the stability is known [1], [9] and written as

$$\text{any input peak amplitudes} \leq (Q - \|h\|_1 + 1)\Delta, \tag{2}$$

where Q and 2Δ stand for the number of quantization levels and its step size, respectively. Also $\|h\|_1$ denotes l_1-norm of $h(n)$. If any input signals meet Eq. (2) and the initial amplitudes of the delay-elements of $H(z) - 1$ are less than or equal to Δ in magnitude, then the converter cannot cause any overload and thus is rigorously stable [1], [9].

2.2 Amplitude Response of Noise-Shaping FIR Filters

In our design, the amplitude response $|H(e^{j\omega})|$ can be weighted by a specified non-negative real function $W(\omega)$ and then optimized by minimizing the peak of the weighted amplitude response

$$\max_{0\leq\omega\leq\pi} |W(\omega)H(e^{j\omega})| \tag{3}$$

$$= \max_{0\leq\omega\leq\pi} \left| W(\omega) \sum_{n=0}^{T-1} h(n)e^{-jn\omega} \right|, \tag{4}$$

where $h(0)$ is 1.0. The weight function $W(\omega)$ can be specified by taking account of the amplitude response of a filter to suppress out-of-band noise. The weighted amplitude response in (4) is a nonlinear function of the filter coefficients to be optimized. Then we first introduce a grid into ω and approximately rewrite (4) as

$$\max_{1\leq i\leq L} \left| W(\omega_i) \sum_{n=0}^{T-1} h(n)e^{-jn\omega_i} \right|. \tag{5}$$

The weighted amplitude response is evaluated at a set of specified frequencies ω_i for $i = 1, \cdots, L$. Equation (5) is still nonlinear due to the amplitude function. Note that the amplitude of an arbitrary complex number $|r|e^{j\alpha}$ can be obtained by calculating the maximum

$$\max_{0\leq\theta<2\pi} \text{Re}\left\{(|r|e^{j\alpha})e^{j\theta}\right\} = |r|. \tag{6}$$

The above real part is a function of θ, and a grid is further introduced into θ so that $|r|$ can be approximately obtained as the maximum of the real part defined over the grid. By this technique, further we approximately rewrite (5) as

$$\max_{\substack{1\leq i\leq L\\ 1\leq k\leq M}} W(\omega_i) \sum_{n=0}^{T-1} h(n)\cos(n\omega_i - 2\pi k/M), \tag{7}$$

where L and M are the numbers of sample points of ω and θ, respectively. The above technique is used in the complex Chebyshev approximation for FIR filter design [20].

2.3 Optimum Solution

In the proposed design method, the peak amplitude (7) is minimized under a specified set of T, $\|h\|_1$, M, ω_i and $W(\omega_i)$ for $i = 1, \cdots, L$. Then the optimization problem can be formulated as

$$\begin{aligned}
&\text{Minimize} && \max_{\substack{1\le i\le L\\1\le k\le M}} W(\omega_i)\sum_{n=0}^{T-1} h(n)\cos\left(n\omega_i - \frac{2\pi k}{M}\right)\\
&\text{Subject to} && h(0) = 1,\\
& && \|h\|_1 = \sum_{n=0}^{T-1} |h(n)| \le c,\\
& && \text{and} \quad \sum_{n=0}^{T-1} h(n) = 0.
\end{aligned} \tag{8}$$

In the above formulation, c is specified as a constant. The last constraint is to suppress the amplitude around DC in order to somewhat improve SNDR in a band of interest, but the constraint can be optional. The objective function to be minimized has the *max* operation, which is a kind of nonlinear function, although, by introducing an additional variable, it can be handled as linear functions [19]. The constraint on l_1-norm is defined by using the nonlinear amplitude function but can be equivalently rearranged into 2^{T-2} inequalities (Note $h(0) = 1$). However through this rearrangement, the number of inequality constraints exponentially increases with the growth of T, but a today's personal computer enables us to handle it for $T \le 20$–30.

2.4 Sub-Optimal Solution

The simplex method for the optimum design generally requires large memory space and/or computing cost. In the following design examples, computer simulations for the optimum design are demonstrated for at most $T = 29$ from this reason. In this section, we propose a fast and efficient algorithm to design sub-optimal noise-shaping FIR filters. If $\|h\|_1$ is restricted, optimum FIR filters are usually sparse. Also generally, even if we truncate the impulse response of an NTF for simplicity, its amplitude response is not significantly degraded. So we give some priority to optimizations of $h(n)$ for smaller n in order to reduce memory space and computing cost of the optimization. First, the impulse response $h(0), \cdots, h(T-1)$ of an NTF is decomposed into several blocks. For a specified T_b, the m-th block contains $h(mT_b), \cdots, h(mT_b+T_b-1)$ for $m = 0, \cdots, \lfloor T/T_b\rfloor - 2$, where $\lfloor T/T_b\rfloor$ stands for the maximum integer not exceeding T/T_b. Note that the last block contains only $h(T_b\lfloor T/T_b\rfloor - T_b), \cdots, h(T-1)$. Then in the first stage, the algorithm optimizes the coefficients of only the first block and minimizes the in-band peak error of the amplitude response of a noise-shaping T_b-tap FIR filter having the first T_b coefficients. Then the algorithm estimates tap positions having nonzero coefficients only in the first block. In the second stage, the algorithm truncates all the coefficients of the third through the last blocks and simultaneously optimizes the coefficients of the second block and at the estimated tap positions in the first block. Then if the FIR filter is sparse, even for large T_b, most of coefficients of the first block should be zero, which can reduce the number of coefficients to be optimized in the first block. Through this optimization, the algorithm estimates tap positions having nonzero coefficients in the second block, and only the estimated tap positions in the first and second blocks are taken into account in the third stage optimization. In the same way, the algorithm assumes tap positions having nonzero coefficients in the first $(m-1)$ blocks and then estimates tap positions having nonzero coefficients in the m-th block by simultaneously optimizing all the coefficients of the m-th block and at the assumed tap positions. By optimizing the coefficients of all the blocks, we get a sub-optimal noise-shaping FIR filter for T_b. The algorithm further optimizes filters for other T_b's less than a specified positive integer T_{max} and selects the best filter as the solution. The above proposed algorithm is completely shown below:

1. Set $T_b := 1$ and a set of designed filters $\Psi_{filt} := \phi$.
2. Set a block index $m := 0$ and introduce a set $\Psi_p := \phi$.
3. If $m < \lfloor T/T_b\rfloor - 1$, then add non-negative integers $\{mT_b, \cdots, mT_b + T_b - 1\}$ into new elements of Ψ_p. Otherwise, if $m = \lfloor T/T_b\rfloor - 1$, add $\{mT_b, \cdots, T-1\}$ into Ψ_p.
4. Solve the following minimax problem by the simplex method:
$$\begin{aligned}
&\text{Minimize} && \max_{\substack{1\le i\le L\\1\le k\le M}} W(\omega_i)\sum_{n\in\Psi_p} h(n)\cos\left(n\omega_i - \frac{2\pi k}{M}\right)\\
&\text{Subject to} && h(0) = 1,\\
& && \|h\|_1 = \sum_{n\in\Psi_p} |h(n)| \le c,\\
& && \text{and} \quad \sum_{n\in\Psi_p} h(n) = 0.
\end{aligned} \tag{9}$$
Let the optimum solution be $\tilde{h}(n)$ for $n \in \Psi_p$ and set $\tilde{h}(n) := 0$ for $n \notin \Psi_p$.
5. If $m = \lfloor T/T_b\rfloor - 1$, then go to Step 6. Otherwise, if $m < \lfloor T/T_b\rfloor - 1$, find all integers $\tilde{n}$ such that $\tilde{h}(\tilde{n}) = 0$ for $mT_b \le \tilde{n} < (m+1)T_b$, and remove those integers from Ψ_p. Then increment m by $m := m + 1$, and go to Step 3.
6. Let the designed filter for current T_b be an element of Ψ_{filt}, and increment T_b by $T_b := T_b + 1$. Then if $T_b > T_{max}$, the program selects the filter with the minimum in-band peak amplitude among the designed filters of Ψ_{filt} as the solution and stops. Otherwise go to Step 2.

We also show an example of optimizing a 12-tap FIR NTF for a specified T_{max} by the above proposed algorithm in Fig. 2. Figure 2 first explains the optimization with $T_b = 4$ for example, but the optimization is actually carried out for each T_b less than or equal to T_{max}. Then among those optimized NTFs, the NTF having the minimum peak of the weighted amplitude response is selected as a solution.

3. Numerical Examples

To design a number of optimum and sub-optimal high-order

○ Decompose the impulse response $h(n)$, $n = 0, \cdots, 11$ of the NTF into sub-blocks. Each sub-block has $T_b(= 4)$ elements as

$\{h(0), h(1), h(2), h(3)\}$, $\{h(4), h(5), h(6), h(7)\}$, and $\{h(8), h(9), h(10), h(11)\}$.

⇓

Stage 1: Choose coefficients to be nonzero in the first sub-block.
Assuming $h(4) \sim h(11) = 0$, optimize the first sub-block. Suppose that coefficients $h(1)$ and $h(3)$ have zero values as a result of this optimization. Then those coefficients are kept zero-valued in the following stages.

⇓

Stage 2: Choose coefficients to be nonzero in the second sub-block.
Assuming $h(1), h(3), h(8) \sim h(11) = 0$, optimize $h(0)$, $h(2)$ and the second sub-block. Suppose that coefficients $h(4) \sim h(6)$ have zero values as a result of this optimization. Then those coefficients are kept zero-valued in the following stages.

⇓

Stage 3: Choose coefficients to be nonzero in the third sub-block.
Assuming $h(1), h(3), h(4) \sim h(6) = 0$, optimize $h(0)$, $h(2)$, $h(7)$ and the third sub-block.

⇓

Then get the NTF for current $T_b(= 4)$.

⇓

Select the best NTF for each $T_b(\leq T_{max})$.

Fig. 2 Example of optimizing a 12-tap FIR NTF for a specified T_{max} by the proposed sub-optimal algorithm.

noise-shaping FIR filters and investigate their performance, we have developed a program for the proposed algorithms by C language. Figure 3 explains the design specification in the frequency domain. The proposed design algorithms could handle an arbitrary weight function $W(\omega)$, but, in this paper, by letting $W(\omega)$ be 1 and 0 for $|\omega| \leq \pi/\text{OSR}$ and $|\omega| > \pi/\text{OSR}$, respectively, only the in-band peak amplitude in $|\omega| \leq \pi/\text{OSR}$ is minimized for specified three parameters; l_1-norm $||h||_1$, the number of taps T and OSR in order to reveal the performance limit of the noise-shaping for a specified set of the three parameters. We first specify allowable maximum $||h||_1$ as 8, which can be used for design of overload-free stable nine-level data converters according to Eq. (2). For NTFs having $||h||_1 \leq 8$, four kinds of OSRs 8,

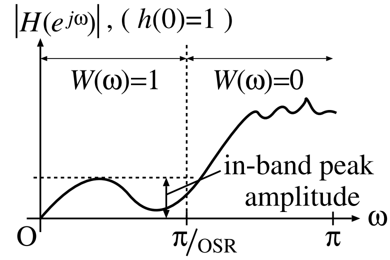
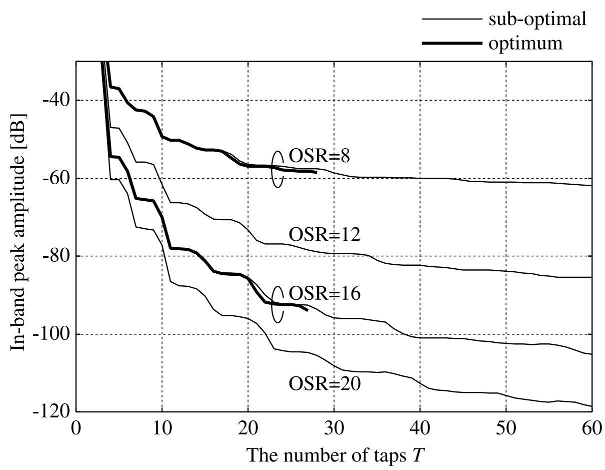


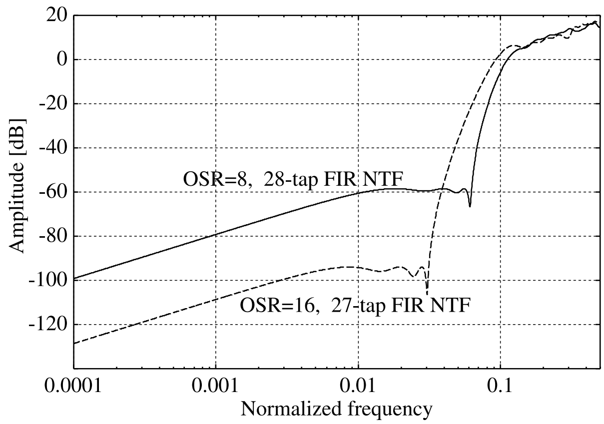
Fig. 3 Design specification in the frequency domain. The peak amplitude in $|\omega| \leq \pi/\text{OSR}$ is minimized.

12, 16 and 20 are specified, and the NTFs are designed by the proposed sub-optimal algorithm for a wide range of T. Also we design optimum NTFs by the proposed optimum algorithm for only OSRs 8 and 16. The both numbers of sample points, L and M, which are defined in Eq. (7), are given as 100, and the grid of L frequency points is introduced only into $|\omega| \leq \pi/\text{OSR}$. In the sub-optimal algorithm, we use $T_{max} = 5$. The C program is carried out on Pentium IV 2.8 GHz. Figure 4 depicts the minimized in-band peak amplitudes of the designed FIR filters. The computing time has been measured for each design and is 5 hours for the optimum 27-tap FIR filter with OSR 16 but 14 seconds for the sub-optimal 27-tap FIR filter with OSR 16. The sub-optimal algorithm consumes only 63 seconds even for the 60-tap FIR filter with OSR 16. We find that the proposed sub-optimal algorithm can design near optimum filters from Fig. 4(a) and is far more efficient than the optimum algorithm. Also it is demonstrated that, with optimized higher order noise-shaping FIR filters, nine-level data converters even for OSR 16 can achieve the in-band noise-suppression of more than 105.3 [dB]. Note that the optimization is performed for every possible integer in the ranges of T. Through these simulations, we find that each optimal FIR filter that is designed with a restricted $||h||_1$ is sparse, i.e., most of coefficients are zero. Therefore since a large increase of T may not significantly reduce the peak amplitude, the minimized peak illustrated in Fig. 4(a) is decreased with the growth of T but not in strict monotone. Figure 4(b) illustrates the amplitude response of the two optimum NTFs; the 28-tap FIR filter for OSR 8 and the 27-tap FIR for OSR 16.

To compare the proposed optimum design, we refer to the minimum phase 11-tap FIR filter [1], [4], which has $||h||_1 = 3.7$ and the peak amplitude of -95.73dB in $|\omega| \leq \pi/128$. The FIR filter with $||h||_1 = 3.7$ can be used in a 3-level data converter [4]. We design the optimum FIR filters for $T = 1, \cdots, 11$ under the same condition on $||h||_1$ and OSR and then find that the optimum 8-tap FIR filter has the amplitude less than -97dB in $|\omega| \leq \pi/128$. This is based on the fact that the design algorithm [1], [4] does not directly minimize an in-band peak amplitude and optimize a filter directly for given three parameters; $||h||_1$, OSR and the number of taps. The coefficients optimized by our algorithm are; $h(0) = 1$, $h(1) = -1.700333$, $h(3) = 0.653712$, $h(4) = 0.196287$, $h(7) = -0.149667$, and $h(n) = 0$ for the other n, and the number of nonzero coefficients happens to

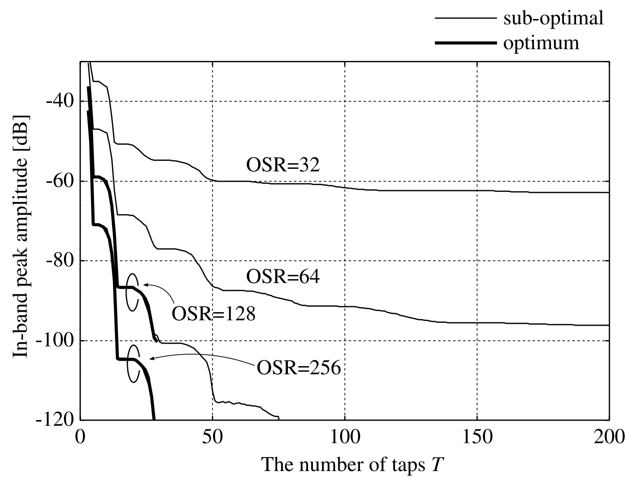


(a) In-band peak amplitude minimized in the optimum and sub-optimal designs.

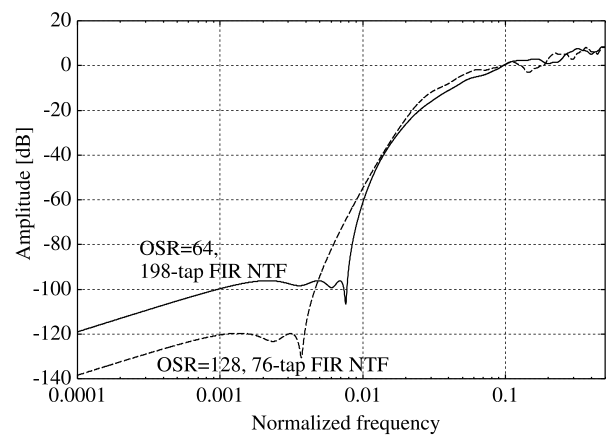


(b) Optimum amplitude response.

Fig. 4 Performance of the designed optimum and sub-optimal NTFs under $\|h\|_1 \leq 8$, which are applicable to overload-free stable nine-level converters.



(a) In-band peak amplitude minimized in the optimum and sub-optimal designs.



(b) Sub-optimal amplitude response. The 198-tap FIR NTF has only nine nonzero coefficients.

Fig. 5 Performance of the designed optimum and sub-optimal NTFs under $\|h\|_1 \leq 2.75$, which are applicable to two-level (single-bit) converters.

be five only.

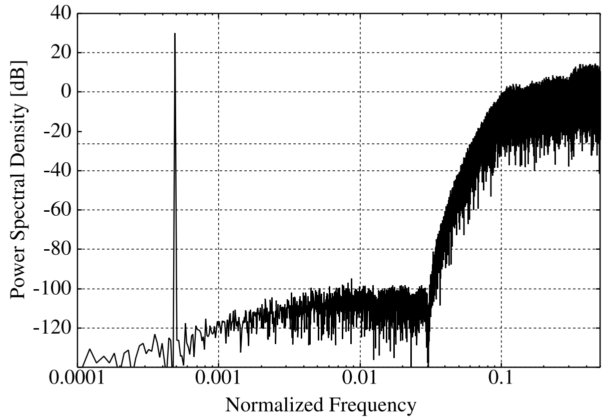
Next for four kinds of OSRs 32, 64, 128 and 256, NTFs having $\|h\|_1 \leq 2.75$ are designed by using the proposed sub-optimal algorithm. These designed NTFs can be embedded in overload-free stable two-level (single-bit) data converters. The proposed optimum algorithm is used to design only for OSR 128 and 256 for reference. Figure 5(a) shows the minimized in-band peak amplitudes and tells us an existence of FIR NTFs having in-band peak amplitudes less than -96.2 [dB] even for the OSR 64. Figure 5(b) illustrates the amplitude response of two designed sub-optimal NTFs. The sub-optimal algorithm has consumed only 96 seconds even for designing the 198-tap FIR NTF with the OSR 64. Also the filter coefficients of the designed 76-tap FIR NTF for OSR 128 are listed in Table 1. The 76-tap filter has only seven nonzero coefficients as shown in Table 1, and the number of nonzero coefficients of the 198-tap filter is only nine. Generally a tight restriction on $\|h\|_1$ significantly reduces the number of nonzero coefficients so that the designed filters with $\|h\|_1 < 2.75$ are very sparse. Even so the proposed sub-optimal algorithm can accurately and simply estimate

Table 1 The filter coefficients $h(n)$ of the 76-tap FIR filter.

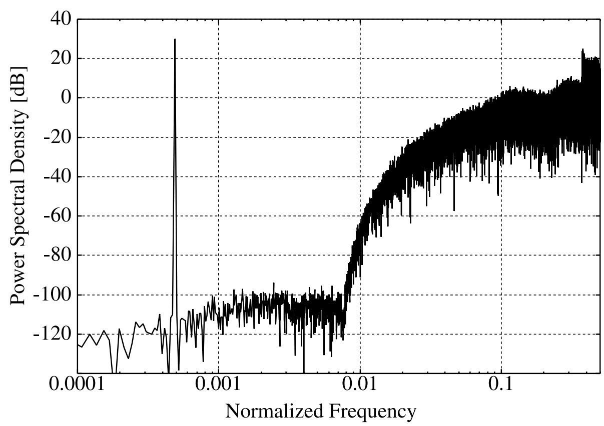
$h(0)$	=	1 ,
$h(1)$	=	−1.2569202121,
$h(8)$	=	0.3486386070,
$h(20)$	=	−0.1086639146,
$h(44)$	=	0.0236642133,
$h(64)$	=	−0.0094158731,
$h(75)$	=	0.0026971796,
for the other n, $h(n)$	=	0

a few tap positions having nonzero coefficients and design near optimum filters as shown in Fig. 5(a). For example the optimum 29-tap filter for OSR 128 has an in-band peak amplitude of -100.42 [dB], and the sub-optimal 29-tap filter for OSR 128 has a peak amplitude of -98.87 [dB], which is very close to the optimum amplitude.

Next for an input sinusoid $0.25\Delta \sin(n\pi/1024)$ to a single-bit data converter with the designed sub-optimal 198-tap FIR filter for OSR 64 and a nine-level data converter with the designed optimum 27-tap filter for OSR 16, assuming the perfect elimination of out-of-band noise, we have



(a) Output power spectral density of the nine-level converter with the designed optimum 27-tap FIR filter for OSR 16.



(b) Output power spectral density of the single-bit converter with the designed sub-optimal 198-tap FIR filter for OSR 64.

Fig. 6 Output power spectral densities of the designed rigorously stable two data converters for an input sinusoid $0.25\Delta\sin(n\pi/1024)$, $n = 0, \cdots, 2^{16}-1$.

measured the in-band SNDR in simulation. Then it has been confirmed that the single-bit converter performs a noise-shaping with a high SNDR 107.6 [dB] and that the nine-level converter achieves an SNDR 103.4 [dB]. The output power spectral densities of the designed two data converters are shown in Figs. 6(a) and (b). From the figures, we see that the designed converters sharply shape the noise spectra as specified with the stability.

It is well-known that generally high-order ΣΔ modulators are prone to instability. However from the above simulation results, we find that this is not true to our overload-free stable high-order ΣΔ modulators designed by the proposed methods, which ensure that higher-order modulators perform more superior noise-shaping for arbitrary input signals, though their peak amplitude is constrained. The overload-free condition for the stability may be conservative in improving the performance of the noise-shaping but strictly bounds the quantization error of each output symbol of a modulator to the minimum level, which can, symbol by symbol, enhance an SNDR of an output sequence.

4. Conclusion

This paper presents two methods to design noise-shaping FIR filters for overload-free stable single- and multi-bit data converters. In the methods, the amplitude response of a noise-shaping FIR filter can be weighted by an arbitrarily specified weight function, and the out-of-band amplitude can also be shaped. The paper shows that the design problem can be approximately formulated as a complex minimax problem. The first method exactly solves this problem and directly derives the optimum solution at the price of computing cost. In the second method, the impulse response of an NTF is decomposed into several blocks, and coefficients of each block are optimized by assuming tap positions having nonzero coefficients in the other several blocks. Then the number of nonzero coefficients to be optimized is significantly reduced, since almost all filter coefficients of an NTF having a limited $||h||_1$ are zero. Then the first method requires large memory space/computing cost, but in the sub-optimal second method the required memory space can be significantly saved. In the numerical examples, many optimum and sub-optimal noise-shaping FIR filters have been designed by the proposed methods so that the optimum performance of the overload-free stable converters has been revealed for the given design parameters; OSR, $||h||_1$ and filter order.

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