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# A Switch-Box Router "BOX-PEELER" and Its Tractable Problems

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**SUMMARY** Given a switch-box, let $C$ be a connection requirement. If there is a polynomial time algorithm (router) to complete $C$, $C$ is said to be tractable by the algorithm. There have been proposed a number of switch-box routers but none that makes clear its tractable problems. We propose a switch-box router, or rather a principle, BOX-PEELER with a simple characterization of a class of tractable problems. BOX-PEELER is developed to be an underlying concept in switch-box routing as LEFT-EDGE method has been in 2-side channel routing.

## 1. Introduction

The concept of channel routing was initiated by Ref. (1) introducing the concept of 2-layer 2-side channels. They proposed an algorithm (router) "LEFT-EDGE" to complete a given connection requirement (problem) with the fewest tracks in polynomial if the net list is consisting of 2-terminal nets and the vertical constraint is empty. After then, a number of highly sophisticated routers have been proposed to compete with more difficult cases. However, LEFT-EDGE has been the only method for which we can recognize its tractable problems, i. e. the problems for which the router guarantees in polynomial time the optimal connections.

A generalized version of routing is the 4-side channel (switch-box) routing, which is the subject of this paper. The most significant difference of the switch-box routing from the 2-side channel routing is in that the switch-box includes the concept of fixed area, without a natural optimization problem such as "to minimize the area". Thus, it leads to a decision problem that if a given problem is completely routable. However, among a number of routers, even a router cannot be found that characterizes its tractable problems.

This paper demonstrates a certain class of tractable problems with a linear time switch-box router. The router is called BOX-PEELER by its manner in routing as it fixes the outmost net one after another. Executing the nets from outside is a popular idea in heuristic routers (e. g. Refs. (2), (3)).

The connection requirements which the router considers are simplest and basic but not trivial. BOX-PEELER is developed to be a conceptual switch-box router as LEFT-EDGE has been in 2-side channel routing. For actual use, it could be developed some method that extracts a maximal sub-netlist that matches BOX-PEELER and executes the rest by a maze router, for example. But it is not mentioned here about those ideas.

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## 2. Definitions

A switch-box is a rectangular grid area bounded by the four walls on which terminals are assigned. Orthogonal grids are called horizontal and vertical tracks and linear wire segments are placed on them. Terminals are on the (end of) tracks and labeled with positive integers. The set of terminals with the same label is called a net and referred to by the label.

The switch-box routing problem is given in terms of net list and design rule. Net list $N$ is the set of nets demanding all the terminals of each net be connected by the set of wire segments which is called the connection of the net.

Our switch-box routing is constrained by ;

[NET CONSTRAINTS]

Each net is assumed to satisfy

SINGLE-TERMINAL-TRACK : There is at most one terminal on a track, and

THROUGH-NET : Each net is consisting of two terminals on opposite walls.

[DESIGN CONSTRAINTS]

NO-KNOCKED-KNEE-AND-MIN : Two wire segments of distinct connections are allowed to cross but not to share corners (knocked knees) or otherwise overlap. Each connection is consisting of two bends and three segments.

A net whose terminals are on the top and bottom walls is called a vertical net and the set of such nets is denoted by $N_v$. Horizontal nets and the set $N_h$ are analogously defined.

In the following, definitions and descriptions are symmetric with respect to "vertical" and "horizontal", and with respect to "right" and "left". Taking this into account, we often give only one of them. For the terms related to "vertical", "horizontal", "right", and "left", we use the letters "v", "h", "r", and "l" (or their capitals), respectively.

A net or its connection is called left-turn if it turns to the left forwarding from one terminal of the net

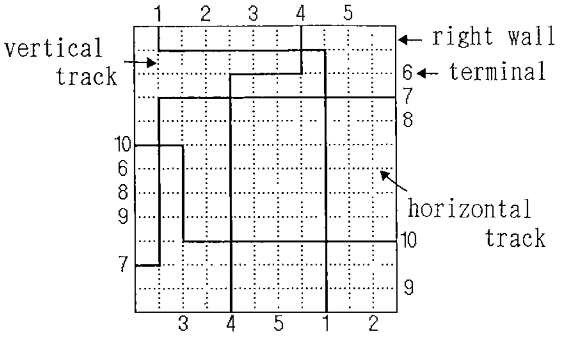


Fig. 1 Definitions of net type.
vertical left-turn net: $N_{vl}=\{1, 2\}$
vertical right-turn net: $N_{vr}=\{3, 4, 5\}$
horizontal left-turn net: $N_{hl}=\{6, 7, 8\}$
horizontal right-turn net: $N_{hr}=\{9, 10\}$

following NO-KNOCKED-KNEE-AND-MIN. Note that whichever of terminals of a net is the starting one, a net is uniquely defined to be left-turn or right-turn. The set of horizontal left-turn nets is denoted by $N_{hl}$. Thus, net list $N$ is partitioned as

$$N=N_{vl}\cup N_{vr}\cup N_{hl}\cup N_{hr}.$$

We can assume that $N_{vl}\cup N_{vr}\neq\phi$ and $N_{hl}\cup N_{hr}\neq\phi$, since otherwise the problem is that of 2-side channel routing. A net list $N$ is called L-R type if only $N_{vl}$ and $N_{hr}$ of the four are nonempty. See Fig. 1. for these definitions.

A net or terminal is called fixed if its connection is realized, otherwise unfixed. The routing algorithms proposed here go in such a fashion that one net is fixed after another. Following terms are defined at a stage on the way where $N_v^*\subset N_v$ denotes the set of the unfixed vertical nets.

Let $n_v$ be a vertical net. By the net and design constraints, its connection is unique except on which horizontal track its horizontal segment is placed. If horizontal track $t_h$ has an enough empty interval for the horizontal segment of $n_v$ to be put, $t_h$ is said to accept $n_v$. If $t_h$ accepts $n_v$ and has no unfixed terminal, $t_h$ is said to $\phi$-accept $n_v$. If $n_v$ is fixed putting its horizontal segment at $t_h$, it is simply said that net $n_v$ is fixed at $t_h$. If $t_h$ $\phi$-accepts any one member (not necessarily all simultaneously) of $N_v^*$, it is said that $t_h$ $\phi$-accepts $N_v^*$.

Suppose that $N_v^*\neq\phi$. The leftest terminal of nets of $N_v^*$ and the track on which the terminal exists are called the left border terminal and left border track, respectively, of $N_v^*$. See Fig. 2. Bounded by the left and right vertical border tracks, the switch-box area is partitioned into three zones, the left outside, right outside, and inside of $N_v^*$, the last including the border tracks. If $N_v^*=\phi$, we define that all the area is outside of $N_v^*$. A vertical marginal track of $N_v^*$ is a track in the outside of $N_v^*$ that $\phi$-accepts $N_h^*$.

Let a vertical marginal track $t_m$ exist in the left outside of $N_v^*$. If a border terminal of $N_h^*$ is on the left wall, the terminal is called the corner terminal with respect to $t_m$. It may be that there is no corner terminal with respect to a marginal track $t_m$.

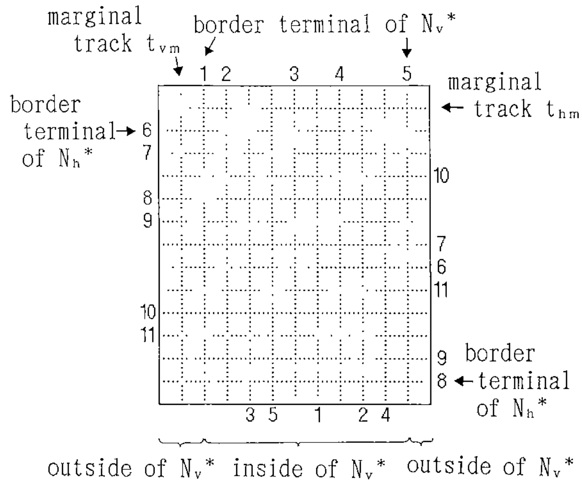


Fig. 2 Definitions of border terminals and tracks, outside and inside zones, marginal tracks, and corner terminals.
$N_v^*=\{1, 2, 3, 4, 5\}$
$N_h^*=\{6, 7, 8, 9, 10, 11\}$
Terminal 6 on the left wall (border terminal of $N_h^*$) is a corner terminal with respect to marginal track $t_{vm}$, but terminal 8 on the right wall is not a corner terminal. Terminals 1 and 5 on the top wall are corner terminals with respect to $t_{hm}$.

Neglecting the horizontal net, suppose that we are going to fix all the vertical nets at the minimum number, let it be $T(N_v)$, of horizontal tracks. As is well-known, one way to get such a routing is LEFT-EDGE. It is sometimes convenient to treat all the horizontal segments on the same track as one segment. We call it a fusion segment. Thus, we say that $N_v$ can be fused into $T(N_v)$ fusion segments.

Vertical and horizontal densities $D_v$ and $D_h$ are defined conventionally as follows. For a vertical line $l$ that cuts the switch-box, $D_v(l)$ is the number of nets whose one terminal is on $l$ or two terminals are in different sides of $l$. $D_v$ is the maximum of $D_v(l)$ over distinct $l$. By the design constraint, they are given by

$$D_v=|N_h|+T(N_v),\quad D_h=|N_v|+T(N_h).$$

## 3. Routing Rule and Lemma

Our algorithm always follows;

[BASIC RULE]

(1) The nets are fixed one after another.

(2) A net is fixed at a track that $\phi$-accepts it.

BASIC RULE makes the following three lemmas hold. Since the proof are trivial by definition, they are omitted.

[Lemma 1] Let $t_m$ be a vertical marginal track in the left outside of $N_v^*$. For any $n_h\in N_h^*$, let $t_h$ be the

horizontal track that contains a terminal of $n_h$ in the left outside. Then, after fixing $n_h$ at $t_m$, $t_h$ $\phi$-accepts $N_v^*$.
[Lemma 2] Let $t_m$ be a vertical marginal track. Assuming that there is a corner terminal with respect to $t_m$, let it be a terminal of $n_h \in N_h^*$ and at the horizontal track $t_h$. ($t_h$ is a border track of $N_h^*$.) Then, after fixing $n_h$ at $t_m$, $t_h$ is a horizontal marginal track for currently unfixed net set $N_h^* - \{n_h\}$.
[Lemma 3] Let $t_m$ be a vertical marginal track. Suppose that $N_h^*$ consists only of the same type nets. Then one of two border terminals of $N_h^*$ is the corner terminal with respect to $t_m$.

## 4. Basic Router, BOX-PEELER I

The class of problems we are going to concern are characterized by the conditions on cardinalities, types of nets, distribution of marginal tracks, and density. They are respectively described under [CARD], [TYPE], [MARG], and [DENS].

We provide five routers BOX-PEELER I~V. This section is devoted to describe the most basic router BOX-PEELER I for studying the concept of "peeling the box" which is common in all the BOX-PEELER's.

Note that to define an algorithm is to define the order of the nets to be processed and assign the track at which each net is fixed, since each net is to be routed with three line segments.

If all the nets of $N$ are able to be fixed, it is said that $N$ is completely routable.

[THEOREM 1] Net list $N$ subject to the following conditions is completely routable by the router BOX-PEELER I.
[CARD]: $|N_v| = |N_h|$.
[TYPE]: The type is either of L-L, or L-R, or R-L, or R-R.
[MARG]: There is at least one marginal track.
<Router: BOX-PEELER I>
Let $t_m$ be a given marginal track.
$N_v^* \leftarrow N_v$, $N_h^* \leftarrow N_h$.
Apply the following subroutine for $N_v^* \cup N_h^*$, and $t_m$, while $N_v^* \cup N_h^* \neq \phi$. Else stop and get a solution.
<PEEL-THE-BOX [$N_v^* \cup N_h^*$, $t_m$]>
Find a corner terminal with respect to $t_m$ and let it be at track $t_c$. Let its net be $n_c$ and its the other terminal be at track $t_c'$. Fix $n_c$ at $t_m$. Return with $N_v^* \cup N_h^* \leftarrow N_v^* \cup N_h^* - \{n_c\}$, and $t_m \leftarrow t_c$. (END)
(Proof) According to Lemma 3, there is a corner terminal with respect to any marginal track at each stage. Lemma 2 says that a marginal track exists in each stage when a corner terminal's net is fixed at a marginal track. Routing continues to fix with respect to the horizontal nets and vertical nets alternately. Since the number of vertical nets equals that of the horizontal nets, BOX-PEELER I runs until the net list is empty. □

Example 1: Given a switch-box routing problem SB1 as shown in Fig. 3, we can apply BOX-PEELER I since $N$ is R-R type and there is one vertical marginal track $t_m$. The corner terminal with respect to $t_m$ is terminal 1 on the left wall. Thus we start with fixing net 1 at $t_m$. Next we find net 2 being the second to be processed. It will be nets 3, 4, ⋯, 10 to follow. The result is shown in Fig. 4. (END)

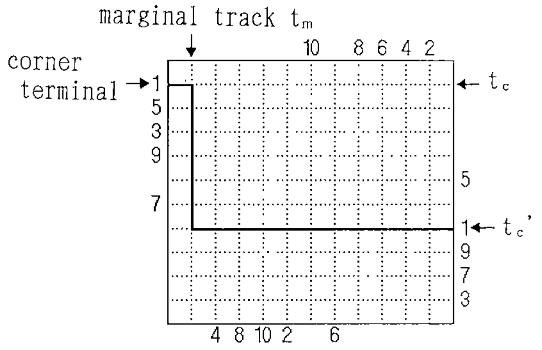

Fig. 3 Routing problem SB1. After one time application of PEEL-THE-BOX, net 1 has been fixed at $t_m$ and $t_c$ becomes a marginal track of the next stage.

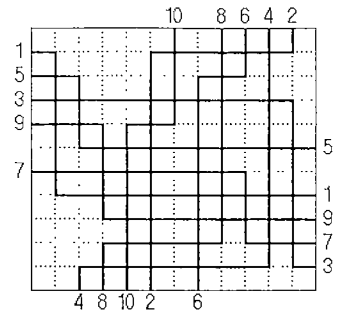

Fig. 4 Application of BOX-PEELER I to SB1.

After fixing a corner terminal's net at a marginal track in subroutine PEEL-THE-BOX, it is all right to consider that the resulting switch-box is the one obtained by discarding net $n_c$, and shrinking $t_m$ and $t_c'$, and considering $t_c$ as a marginal track. In the process, $|N_v^*|$ and $|N_h^*|$ are equal or differ by 1 depending on evenness or oddness of the times of applications of the subroutine. At the following stage, the marginal track $\phi$-accepts the no smaller set. The key issue is the balance of $|N_v^*|$ and $|N_h^*|$ within 1. Therefore, we have the following.

COROLLARY 1: Net list $N$ subject to the following conditions is completely routable by the router BOX-PEELER I.
[CARD]: $|N_v| - |N_h| = \pm 1$.
[TYPE]: The type is either of L-L, or L-R, or R-L, or R-R.
[MARG]: There is at least one marginal track that $\phi$-accepts the no smaller set. (END)

Often it is not necessary to provide one marginal track to fix a net. In fact, it is easy to see that SB1 can be routed without using the marginal track which cannot be obtained as far as we apply PEEL-THE-BOX. However it is also true that there are cases for which the router provides critical solutions.

## 5. BOX-PEELERS for [CARD] Constraint Problems

BOX-PEELER I is generalized to be applicable to the problems without [TYPE] constraints. The main idea is to partition a given switch-box into three sub-switch-boxes such that each satisfies the conditions of THEOREM 1.

[THEOREM 2] Net list $N$ subject to the following conditions is completely routable by BOX-PEELER II.

[CARD] : $|N_v|=|N_h|$.

[MARG] : There are at least three marginal tracks.

⟨Router : BOX-PEELER II⟩

Without loss of generality, we assume that $|N_{vr}|$ is not less than any of $|N_{vl}|$, $|N_{hr}|$, and $|N_{hl}|$.

⟨Step 1⟩ Partition the switch-box into three subswitch-boxes $SB^1$, $SB^2$, and $SB^3$ with net lists $N^1$, $N^2$, and $N^3$ such that

$SB^1$ : $N^1=N^1_{vr}\cup N^1_{hr}$, where $N^1_{hr}=N_{hr}$ and $N^1_{vr}$ is any subset of $N_{vr}$ satisfying $|N^1_{vr}|=|N^1_{hr}|$. A marginal track is contained.

$SB^2$ : $N^2=N^2_{vl}\cup N^2_{hl}$, where $N^2_{vl}=N_{vl}$ and $N^2_{hl}$ is any subset of $N_{hl}$ satisfying $|N^2_{hl}|=|N^2_{vl}|$. A marginal track is contained.

$SB^3$ : $N^3=N^3_{vr}\cup N^3_{hl}$, all the nets not contained in the above. A marginal track is contained.

⟨Step 2⟩ Apply BOX-PEELER I to each sub-switch-box and superimpose the results. (END)

(Proof) From [CARD] condition and the assumption.

$$k=|N_{vr}|-|N_{hr}|$$
$$=|N_{hl}|-|N_{vl}|\geqq 0.$$

Then it is obvious that all three sub-netlists are consistently defined and each satisfies the conditions of THEOREM 1. Thus each sub-switch-box is completely routable. By BASIC RULE that each net in each subproblem is fixed at the $\phi$-acceptable track, these three resultant routings can be superimposed without violating DESIGN CONSTRAINT. □

Example 2 : Given a switch-box routing problem SB2 in Fig. 5, we can apply BOX-PEELER II. The result is shown in the figure. (END)

The next algorithm BOX-PEELER III is another variation of BOX-PEELER I.

[THEOREM 3] Net list $N$ subject to the following conditions is completely routable by BOX-PEELER III.

[CARD] : $|N_v|=|N_h|$.

[MARG] : There is at least one horizontal marginal track and one vertical marginal track.

⟨Router : BOX-PEELER III⟩

Without loss of generality, we assume that $a=|N_{vr}|$ is not less than any of $b=|N_{vl}|$, $|N_{hr}|$, and $c=|N_{hl}|$ and the given vertical marginal track is in the left outside of $N_v$.

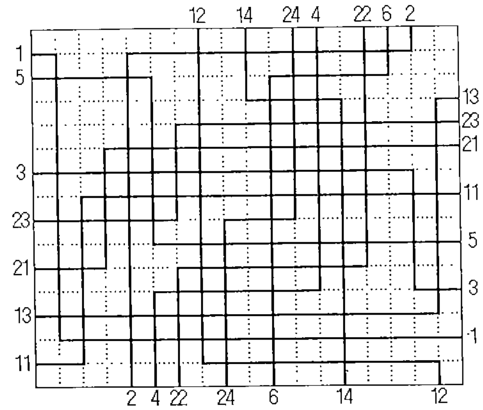


Fig. 5 Application of BOX-PEELER II to SB2.
$N^1=\{1, 2, 3, 4, 5, 6\}$
$N^2=\{11, 12, 13, 14\}$
$N^3=\{21, 22, 23, 24\}$

⟨Step 1⟩ • Let $t_{mv}$ and $t_{mh}$ be vertical and horizontal marginal tracks, respectively.

• Let $N^*_{vl}\leftarrow N_{vl}$, $N^*_{vr}\leftarrow N_{vr}$, $N^*_{hl}\leftarrow N_{hl}$, and $N^*_{hr}\leftarrow N_{hr}$.

• Let $N^*_{vs}\leftarrow N_{vr}$.

• Prepare three spaces $SB^1$, $SB^2$, and $SB^3$ which will be completed as sub-switch-boxes when

$SB^1$ : L-R type. The number of vertical nets and that of horizontal nets are equally $b$.

$SB^2$ : R-L type. The number of vertical nets and that of horizontal nets are equally $c$.

$SB^3$ : R-R type. The number of vertical nets and that of horizontal nets are equally $k=a-c$.

/* It is not decided in advance which nets each sub-switch-box contains. This is a difference from BOX-PEELER II. */

⟨Step 2⟩ • Let $N^*_{vt}\subset N^*_{vs}$ be the set of nets that have the lower terminals in the left outside of $N^*_{vl}$.

• Let $n$ be the maximum even integer not greater than any of the numbers of vertical unfixed nets of $SB^3$ and $|N^*_{vt}|$.

• Let $N^*_{vs}\leftarrow N^*_{vs}-N^*_{vt}$.

• Apply PEEL-THE-BOX to $SB^3$ $2n$ times with

Input : $N^*_{vt}\cup N^*_{hr}$, and $t_{mv}$,

Output : $N^*_{vt}\cup N^*_{hr}$, and $t_{mv}$.

⟨Step 3⟩ • If $SB^1$ is completed ($N^*_{vl}=\phi$), then go to Step 4, else apply PEEL-THE-BOX to $SB^1$ 2 times with

Input : $N^*_{vl}\cup N^*_{hr}$, and $t_{mv}$,

Output : $N^*_{vl}\cup N^*_{hr}$, and $t_{mv}$.

• Go to Step 2.

⟨Step 4⟩ • If $k$ is even, apply BOX-PEELER I to $SB^2$ with

Input : $N^*_{vr}\cup N^*_{hl}$, and $t_{mh}$,

else apply PEEL-THE-BOX to one horizontal unfixed net of $SB^3$ with

Input : $N^*_{vr}\cup N^*_{hr}$, and $t_{mv}$,

and apply BOX-PEELER I to one vertical unfixed net of $SB^3$ and $SB^2$ with

Input: $N^*_{vr} \cup N^*_{hl}$, and $t_{mh}$. (END)

(Proof) It is obvious that the switch-box is completed if $SB^1$, $SB^2$, and $SB^3$ are completed.

First, we show that the nets of $SB^1$ and $SB^3$ are fixed in Steps 2 and 3. Initially there is a marginal track with respect to $N^*_{vl}$ that $\phi$-accepts $N^*_{hr}$. In Step 2, suppose that the nets of $SB^3$ have been fixed and there is a marginal track with respect to $N^*_{vl}$ that $\phi$-accepts $N^*_{hr}$ in the left outside of $N^*_{vl}$. From the selection of $N^*_{vt}$, it is also a marginal track with respect to $N^*_{vt}$. Then there is a marginal track with respect to $N^*_{vl}$ that $\phi$-accepts $N^*_{hr}$ in the left outside of $N^*_{vl}$ when PEEL-THE-BOX applies multiple of 4 times. In Step 3, there is a marginal track in the left outside of $N^*_{vl}$ when PEEL-THE-BOX applies 2 times. That is, at each step, the condition that there is a marginal track with respect to $N^*_{vl}$ that $\phi$-accepts $N^*_{hr}$ in the left outside is maintained. Therefore Step 2 and 3 run consistently.

Next, we show that all the nets of $SB^3$ are fixed at Step 2 if $k$ is even, and all the nets of $SB^3$ except one vertical and one horizontal nets are fixed at Step 2 if $k$ is odd. In other words, $2\lfloor k/2 \rfloor$ vertical nets are fixed as the nets of $SB^3$. A net of $N_{vr}$ is a candidate to be a net of $SB^3$ if the net in either side of it is the net of $N_{vr}$ when all the nets of $N_{vl}$ and $N_{vr}$ are arranged in line along with the horizontal coordinates of the terminal of $N_{vl}$ on the upper wall and the terminal of $N_{vr}$ on the bottom wall. Let $a_i$ be the number of nets of $N_{vr}$ such that their lower terminals are between the upper terminals of $N_{vl}$. Then

$$a = \sum_{i=0}^{b} a_i.$$

The number of candidates of $SB^3$ is

$$\sum_{i=0}^{b} 2\lfloor a_i/2 \rfloor \geqq a - b - 1 \geqq a - c - 1 = k - 1.$$

The first equality holds if all $a_i'$ s are odd. If $k$ is odd, then at least one of $a_i'$ s is even. The second inequality holds, because $a \geqq c \geqq b$ from the assumption. Therefore

$$\sum_{i=0}^{b} 2\lfloor a_i/2 \rfloor \geqq 2\lfloor k/2 \rfloor.$$

Then it is possible that enough number of nets are fixed as the nets of $SB^3$.

In Step 4, when the number of nets of $SB^3$ is even, it is possible to fix $SB^2$ according to THEOREM 1. In case the number of $SB^3$ is odd, $t_{mv}$ obviously $\phi$-accepts the horizontal unfixed net of $SB^3$, and the vertical unfixed net of $SB^3$ is of the same type of vertical nets of $SB^2$. Then from COROLLARY 1 it is possible to fix them all. □

Example 3: Given a switch-box routing problem SB3 shown in Fig. 6, we can apply BOX-PEELER III. The numbers of vertical nets of $N^1$, $N^2$, and $N^3$, which are equal to those of horizontal nets, are 2, 3, and 2, respectively. Initially, $N^*_{vt} = \{11\}$. But no net is fixed at Step 2, because $n=0$. Thus, fix the nets 1 and 2 at Step 3 as the nets of $SB^1$. Now, $N^*_{vt} = \{22, 24\}$. Fix 21, 22, 23, and 24 at Step 2 as $SB^3$. Next fix the nets 3 and 4 at Step 3 as $SB^1$. Finally fix the nets 11, 12, 13, and 14 at Step 4 as $SB^2$. The result is shown in the figure. (END)

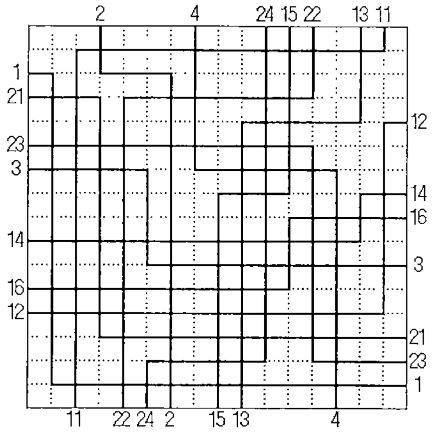


Fig. 6 Application of BOX-PEELER III to SB3.
$N^1 = \{1, 2, 3, 4\}$
$N^2 = \{11, 12, 13, 14, 15, 16\}$
$N^3 = \{21, 22, 23, 24\}$

## 6. BOX-PEELER for [TYPE] Constraint Problems

Preceding BOX-PEELER's are to use one track for one net, thus tending to be inefficient. Following routers try to pack as many nets in a track.

[THEOREM 4] Net list $N$ subject to the following conditions is completely routable by BOX-PEELER IV.

[TYPE]: Either L-L, or L-R, or R-L, or R-R.

[MARG]: There is at least one marginal track.

[DENS]: $n \geqq D_v$, $m \geqq D_h$.

$n$: the number of horizontal tracks

$m$: the number of vertical tracks

⟨Router: BOX-PEELER IV⟩

⟨Step 1⟩ • Apply LEFT-EDGE to $N_v$ and $N_h$ on the tracks which have no terminals. Fuse the nets that are fixed on the same track as a fusion net. Let two end terminals of a fusion net be the terminals of the leftmost and rightmost (topmost and bottommost) terminals of all terminals of component nets.

⟨Step 2⟩ • Apply PEEL-THE-BOX with respect to the fusion nets until either $N^*_v$ or $N^*_h$ is empty.

Input: $N^*_v \cup N^*_h$, and a marginal track.

Output: $N^*_v \cup N^*_h$, and a marginal track.

/* Fixing a fusion net is to fix each component net. */

⟨Step 3⟩ • Fix unfixed fusion nets in any order at arbitrary tracks that $\phi$-accept them.

(Proof) In Step 1, a fusion net is left-turn or right-turn if the component nets are all left-turn or right-turn, respectively. Therefore if the original switch-box is a type then the switch-box with respect to the fusion nets is the same type. According to a similar argument for the previous algorithm, Step 2 runs without a contradic-

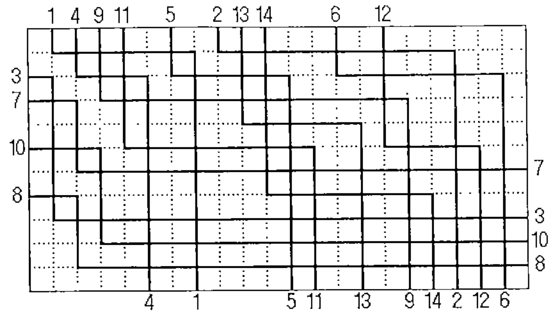


Fig. 7 Application of BOX-PEELER IV to SB4.
$N_v=\{1, 2, 4, 5, 6, 9, 11, 12, 13, 14\}$
$N_h=\{3, 7, 8, 10\}$

tion until either $N_h^*$ or $N_v^*$ is empty.

The problem is whether there are enough number of tracks that $\phi$-accept all the unfixed nets in Step 3. We check how many tracks $\phi$-accept unfixed nets.

Suppose that all the nets of $N_h$ were fixed in Step 2. All the nets are fixed at marginal tracks in Step 2. It in turn produces as many number of tracks that $\phi$-accept $N_v^*$ by Lemma 1. A track that $\phi$-accepts $N_v^*$ also $\phi$-accepts $N_v^{**}\subset N_v^*$ before nets of $N_v^*-N_v^{**}$ are fixed at the track. Let $X$ be the number of horizontal tracks that have no terminal and $Y$ be the number of horizontal tracks at which nets have been fixed in Step 2. Then at the end of Step 2, there are $|N_h|+X-Y$ tracks that $\phi$-accept $N_v^*$. The number of necessary tracks for unfixed vertical nets is $T(N_v)-Y$. By the density condition

$$n=2|N_h|+X\geqq T(N_v)+|N_h|=D_h.$$

Thus, we have

$$|N_h|+X-Y\geqq T(N_v)-Y.$$

This shows that there are enough number of empty tracks at Step 3. □

Example 4: Given a switch-box routing problem SB4 in Fig. 7, we can apply BOX-PEELER IV. The fusion nets are $\{1, 2\}$, $\{4, 5, 6\}$, $\{11, 12\}$, $\{7, 8\}$ and single nets. First, net 1 and 2 are fixed since there is a given horizontal marginal track. And nets 3, 4, ⋯, 10 are fixed at Step 2. Then $N_h^*=\phi$. At Step 3, net 11, 12, 13, and 14 are fixed at tracks that $\phi$-accept them. The result is shown in Fig. 7. (END)

## 7. BOX-PEELERS for Minimal-Constraint Problems

BOX-PEELER IV is generalized to be more critical in condition [DENS].

[THEOREM 5] Net list $N$ subject to the following conditions is completely routable by BOX-PEELER V.

[MARG]: There are at least two marginal tracks.

[DENS]: Either both of (1) and (2) or both of (α) and (β) is satisfied.

(1) $n\geqq 2|N_h|+\max(T(N_{vr})-|N_{hr}|, \delta_{RR})+\max(T(N_{vl})-|N_{hl}|, \delta_{LL})$

(2) $m\geqq 2|N_v|+\max(T(N_{hr})-|N_{vr}|, 1-\delta_{RR})+\max(T(N_{hl})-|N_{vl}|, 1-\delta_{LL})$

(α) $n\geqq 2|N_h|+\max(T(N_{vr})-|N_{hl}|, \delta_{RL})+\max(T(N_{vl})-|N_{hr}|, \delta_{LR})$

(β) $m\geqq 2|N_v|+\max(T(N_{hl})-|N_{vr}|, 1-\delta_{RL})+\max(T(N_{hr})-|N_{vl}|, 1-\delta_{LR})$

where, $\delta$ is defined as:

If included

two horizontal marginal tracks: All $\delta$'s are 1.

one horizontal marginal track: One of $\delta_{RR}$, $\delta_{LL}$, one of $\delta_{RL}$, $\delta_{LR}$ are 1 and the others are 0.

no horizontal marginal track: All $\delta$'s are 0.

<Router: BOX-PEELER V>

<Step 1> Partition the switch-box into two subswitch-boxes $SB^1$, $SB^2$ such that they are R-R and L-L type, respectively if both of (1) and (2) are satisfied. They are R-L and L-R type if both of (α) and (β) are satisfied.

<Step 2> Apply BOX-PEELER IV to each sub-switch-box and superimpose the results. (END)

(Proof) [DENS] shows that each sub-switch-box satisfies [DENS] and [MARG] of THEOREM 4.

(1) guarantees that in case each sub-switch-box is of R-R or L-L type, each has enough number of horizontal tracks to fix the nets and contains a proper horizontal marginal track.

(1) is transformed to

$$n\geqq(2|N_{hr}|+\max(T(N_{vr})-|N_{hr}|, \delta_{RR})+(2|N_{hl}|+\max(T(N_{vl})-|N_{hl}|, \delta_{LL})) \quad (1')$$

The first term of right hand side of (1′) shows that the R-R type sub-switch-box satisfies [DENS] of THEOREM 4. That is, for horizontal tracks $n_{RR}$ and vertical density $D_{vRR}$ of the R-R type sub-switch-box, it holds

$$n_{RR}\geqq(2|N_{hr}|+\max(T(N_{vr})-|N_{hr}|, \delta_{RR}))\geqq T(N_{vr})+|N_{hr}|=D_{vRR}.$$

Moreover, $\delta$ guarantees that the given marginal tracks are assigned to each sub-switch-box. In other words, if $\delta_{RR}=1$, then a horizontal marginal track is included in the R-R type sub-switch-box, else, that is, $1-\delta_{RR}=1$, a vertical marginal track is included.

Therefore, [DENS] and [MARG] of THEOREM 4 are satisfied. Thus, each sub-switch-box is completely routable and the results are able to be superimposed. □

Example 5: Given a switch-box routing problem SB5 in Fig. 8, we can apply BOX-PEELER V. Here,

$$n=12, \quad m=19,$$

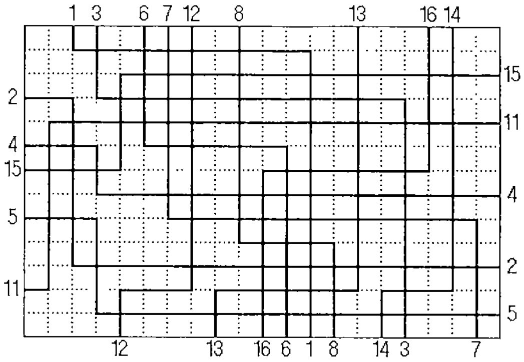


Fig. 8 Application of BOX-PEELER V to SB5.
$N^1=\{1, 2, 3, 4, 5, 6, 7, 8\}$
$N^2=\{11, 12, 13, 14, 15, 16\}$

$$|N_{vl}|=5,\quad |N_{vr}|=4,\quad |N_{hl}|=2,\quad |N_{hr}|=3,$$

$$T(N_{vl})=5,\quad T(N_{vr})=2,\quad T(N_{hl})=2,\quad T(N_{hr})=2.$$

Then ( 1 ) of [DENS] is not satisfied because

$$n \geqq 13 \geqq 10+\max(2-3, \delta_{RR})+\max(5-2, \delta_{LL}).$$

While ( $\alpha$ ) and ( $\beta$ ) are satisfied. $N$ is partitioned into $N^1$ and $N^2$ that are R-L type and L-R type, respectively. The result is shown in the figure. (END)

## 8. Concluding Remarks

The concept of switch-box is so essential everywhere in channel routing that there have been proposed a number of switch-box routers. But they are all heuristics and, as a consequence, we have no knowledge what connection problems are completely routable. It seems that to study them is not meaningful because usually the switch-box routing is a consequence of 2-side channel routings and therefore, to control the switch-box connection problems is no other than to try a globally optimum routing. Still, however, we find it worth from an observation of LEFT-EDGE in 2-side channel routing. It has been a background in channel routing because, so we believe, it is the only method that makes its tractable problems clear.

The routing algorithm in this paper introduced a routing principle PEEL-THE-BOX and studied the tractable problems. To show that the concept could be a potential guideline for practical problems, we need a systematic way to extract maximal tractable subproblems from a given problem.

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