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Authors	Saed Samadi, Nishihara Akinori
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The World Of Flatness

Saed Samadi* and Akinori Nishihara**
*Concordia University, **Tokyo Institute of Technology

***Abstract*— In the published literature on circuit theory and signal processing, the maximally flat FIR filters are commonly attributed to O. Herrmann. It is the purpose of this paper to elucidate the status of these filters before and after Herrmann's paper. Our survey shows that both the formula and the shape of the magnitude response of these filters had been known to actuaries and mathematicians before the publication of Herrmann's paper. We provide a broad outline of the contributions made by actuaries and mathematicians on this class and other related FIR filters. Some recent developments and extensions are also reviewed.**

I. Introduction

An actuary[1], as defined by the Oxford English Dictionary, is "an official in an insurance office, whose duty ... is to compile statistical tables of mortality, and estimate therefrom the necessary rates of premium, etc.; or one whose profession ... is to solve for insurance companies or the public, all monetary questions that involve a consideration of the separate or combined effect of interest and probability, in connexion with the duration of human life, the average proportion of losses due to fire or other accidents, etc." Actuaries use an operation traditionally referred to as "graduation" in their data processing tasks in order to remove irregularities from data. A popular method of graduation consists in "replacing each term of a sequence by weighted mean of a number of terms, so as to give an opportunity for neighboring errors of opposite signs to balance one another" [1]. In engineering terminology, this method of graduation is known as digital filtering with an FIR filter in order to remove noise and smooth data. For a reader who is familiar with digital filtering through its roots in engineering, it may be interesting to know that actuaries have been aware of such digital filtering methods from the late 19th century. Actuaries and mathematicians interested in actuarial sciences have been pioneers in using digital filtering techniques to suppress errors and smooth data. Moreover, they were among the first who discovered a special class of filters called today maximally flat FIR filters.

In this contribution, we provide a brief introduction to the convolution-based graduation techniques developed by actuaries and mathematicians. The main emphasis is on the connection between these methods and the maximally flat FIR filters. We see that T.N.E. Greville, in a 1966 paper, developed the first formula for maximally flat FIR filters from a general formula for data smoothing discovered in the early 20th century by W.F. Sheppard. We also review I.J. Schoenberg's 1946 proof of the equivalence of time domain conditions for exact reproduction of a polynomial signal and the frequency-domain flatness conditions at the origin. We then review some recent developments on maximally flat FIR filters which have been of special interest to the authors.

The organization of the remainder of this paper is as follows. In Section 2, we provide a more precise introduction to what the actuaries traditionally refer to as graduation by linear compounding or moving averages and introduce the two features a "good" graduation technique is usually expected to possess. One of the features is the "exactness" of the technique with respect to the polynomial component of the data. This topic is dealt with in Section 3, where we quote a proof, originally developed by Schoenberg, of the fact that exactness with respect to polynomials is equivalent to the flatness of the frequency response at the origin. In Section 4, we see how the second feature, the "smoothness" of the graduation results, is treated by Sheppard and quote his general formula. Sheppard's formula, given in the form of a transfer function defining a three-parameter family of smoothing digital filters, is not known among the members of the engineering

[1] For a modern description of the profession visit: http://www.beanactuary.org.

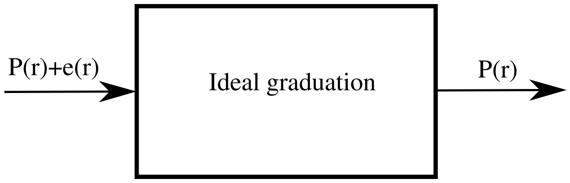

Fig. 1. Ideal graduation of a polynomial with additive noise as the observed data $u_r = P(r) + e(r)$.

community. We then see, following Greville, that Sehppard's formula, in its limiting form, results in what is called a maximally flat lowpass FIR filter. In Section 5, we provide a review of some recent developments in the design of various FIR maximally flat filters with a focus on explicit expressions for the transfer function.

II. Graduation by convolution

In the actuarial literature, graduation is defined as an operation performed in order to secure, from an irregular set of observed data, a corresponding smooth set of values consistent in a general way with the observed values. The sequence of observed data is usually denoted by u_r, and the symbol v_r is customarily reserved for the graduated values. A well-known method of graduation among mathematicians and actuaries uses a weighted moving average, i. e., an FIR digital filter, where the crude values u_r and the graduated sequence are related as

$$v_r = \sum_{l=-n}^{n} \rho_l \, u_{r-l}. \tag{1}$$

The above convolution is sometimes referred to as a linear smoothing formula [2]. This method of data smoothing has also been called graduation by linear compounding [3]. "Smoothness" and "fit" are two main requirements the weights ρ_l should fulfill in order to achieve an acceptable graduation. The term fit refers to the consistency of v_r with the observed data u_r, whereas smoothness is a desirable quality of the graduated values. A common practice among the actuarial mathematicians, as a means to ensure a reasonable fit to the observed data, is to require that the formula be exact for the degree j, or in other words, for every polynomial $P(x)$ of degree j or less, the relation

$$P(r) = \sum_{l=-n}^{n} \rho_l \, P(r-l) \tag{2}$$

be satisfied. To ensure the smoothness of the graduated values, a measure of the form

$$R_m^2 = \frac{\sum_{l=-n}^{n} (\Delta^m \rho_l)^2}{\binom{2m}{m}} \tag{3}$$

is employed [2], [3], where the difference operator Δ^m is defined recursively by

$$\begin{aligned} \Delta^0 \rho_l =& \rho_l, \\ \Delta^1 \rho_l =& \rho_{l+1} - \rho_l, \\ \Delta^m \rho_l =& \Delta(\Delta^{m-1} \rho_l), \end{aligned} \tag{4}$$

for integer values of m. In many graduation formulas, the coefficients ρ_l are determined in a manner so as to minimize R_m, which is called the smoothing coefficient of order m [2], for a given m. The rationale behind the minimization of R_m becomes clear if we model the observations u_r as

$$u_r = w_r + e_r, \tag{5}$$

where w_r is the "true" value and e_r is the error or noise term. It is further assumed that each e_r is randomly and independently distributed according to

$$E[e_n] = 0, \qquad E[e_n e_m] = \begin{cases} \sigma^2 & n = m; \\ 0 & n \neq m. \end{cases} \tag{6}$$

The variance of the error contained in $\Delta^m u_r$ is computed as

$$E[(\Delta^m e_r)^2] = \sigma^2 \binom{2m}{m}. \tag{7}$$

On the other hand, the corresponding variance after graduation is given by

$$E[(\Delta^m \sum_{l=-n}^{n} \rho_l e_{r-l})^2] = \sigma^2 \sum_{l=-n}^{n} (\Delta^m \rho_l)^2. \tag{8}$$

Thus, in a graduation formula with a minimized R_m, the variance of the mth difference of the error term is reduced as much as possible. Obviously, the minimization should result in a value for R_m that is less than unity.

As an illuminating example of the impact of the minimization of the smoothness measure on the graduated sequence, let us consider the case where $m = 0$. The minimization of R_0^2 is equivalent to that of $\sum_{l=-n}^{n} \rho_l^2$. If the weighting coefficients ρ_l simultaneously satisfy the exactness conditions (2), and the observations are modeled as

$$u_r = P(r) + e(r), \tag{9}$$

where $P(r)$ is a polynomial of degree j, or less, and the error term satisfies (6), then

$$v_r = P(r) + \sum_{l=-n}^{n} \rho_l \, e_{r-l}. \tag{10}$$

It follows that

$$E[v_r] = P(r), \tag{11}$$

and, therefore,

$$E[(v_r - P(r))^2] = \sum_{l=-n}^{n} \rho_l^2. \tag{12}$$

Thus by choosing the weights to minimize R_0^2, we are actually minimizing the mean square error in the graduated sequence and the resulting operation is identical to smoothing by least squares. Ideally, the error after graduation would be zero as depicted in Fig. 1.

III. Exactness = Flatness

The conditions, given by (2), for the exact reproduction of polynomials by a graduation formula were studied by I.J. Schoenberg, a mathematician with significant contributions to the spline theory, in connection with his "characteristic function" $\phi(u)$. We follow his treatment of this topic as presented in [4] for the symmetric smoothing formulas, i. e., those for which

$$\rho_l = \rho_{-l}.$$

Schoenberg's treatment starts by considering a smoothing formula with a general infinite-length weighting sequence

$$v_r = \sum_{l=-\infty}^{\infty} u_l \rho_{r-l}. \tag{13}$$

In the following, we see how he transformed the exactness conditions to the equivalent relations on what we call the frequency response function today.

A. Exactness Conditions in Explicit Form

Schoenberg wrote down the exactness conditions for the basic monomials $1, x, x^2, \ldots, x^j$, and derived the relations

$$\sum_{l=-\infty}^{\infty} l^s \rho_{n-l} = n^s, \quad s = 0, 1, \ldots, j. \tag{14}$$

The reader is noted to an equivalent form of (14) given by

$$\sum_{l=-\infty}^{\infty} l^s \, \rho_l = \begin{cases} 1 & s = 0; \\ 0 & s = 1, \ldots, j. \end{cases} \tag{15}$$

employed by some other authors.

B. Power Moments

As an insightful remark, we discuss an interesting byproduct of the imposition of the exactness conditions. It can be shown that for an exact smoothing formula, the power moments of u_l are preserved up to the order j, i. e.,

$$\sum_{l=-\infty}^{\infty} l^s \, u_l = \sum_{l=-\infty}^{\infty} l^s \, v_l. \tag{16}$$

This fact can be easily established by writing

$$\begin{aligned} \sum_{l=-\infty}^{\infty} l^s \, v_l &= \sum_{l=-\infty}^{\infty} l^s \sum_{l'=-\infty}^{\infty} u_{l'} \, \rho_{l-l'} \\ &= \sum_{l'=-\infty}^{\infty} u_{l'} \sum_{l=-\infty}^{\infty} l^s \, \rho_{l-l'} \\ &= \sum_{l'=-\infty}^{\infty} u_{l'} \sum_{l=-\infty}^{\infty} l^s \, \rho_{l'-l} \\ &= \sum_{l'=-\infty}^{\infty} u_{l'} l'^s. \end{aligned} \tag{17}$$

C. Characteristic Function

Schoenberg's defined the characteristic function for the graduation coefficients as

$$\begin{aligned} \phi(u) &= \sum_{l=-\infty}^{\infty} \rho_l e^{\mathbf{i} l u} \\ &= \rho_0 + 2\rho_1 \cos u + 2\rho_2 \cos 2u + \cdots , \end{aligned} \tag{18}$$

where $\mathbf{i} = \sqrt{-1}$. In the modern signal processing terminology, (18) is called the frequency response of the system defined by (13). Now, assume that the weighting sequence tends to zero exponentially as stated by the inequality

$$\rho_l \leq Ae^{-B|l|}. \tag{19}$$

This tendency ensures that the characteristic function is regular in a strip of the complex u-plane. We can then write

$$\phi(u)e^{\mathbf{i}nu} = \sum_{l=-\infty}^{\infty} \rho_{l-n} e^{\mathbf{i}lu}. \tag{20}$$

By expanding the left side of the above expression in u around the origin, we obtain

$$\begin{aligned}&(1 + \frac{u^2}{2!}\phi''(0) + \frac{u^4}{4!}\phi^{(4)}(0) + \cdots) \sum_{k=0}^{\infty} \frac{(\mathbf{i}nu)^k}{k!} \\ =&1 + \frac{\mathbf{i}}{1!}\, n\, u - \frac{1}{2!}(n^2 - \phi''(0))\, u^2 - \frac{\mathbf{i}}{3!}\, \left(n^3 - 3\, n\, \phi''(0)\right) u^3 \\ &+\frac{1}{4!}(n^4 - 6\, n^2\, \phi''(0) + \phi^{(4)}(0))\, u^4 + \cdots .\end{aligned} \tag{21}$$

Expansion of the right side in a similar manner results in

$$\begin{aligned}\sum_{l=-\infty}^{\infty} \rho_{l-n} e^{ilu} =& \sum_{l=-\infty}^{\infty} \rho_{l-n} + \frac{\mathbf{i}}{1!} \sum_{l=-\infty}^{\infty} l\, \rho_{l-n}\, u \\ &-\frac{1}{2!} \sum_{l=-\infty}^{\infty} l^2\, \rho_{l-n}\, u^2 - \frac{\mathbf{i}}{3!} \sum_{l=-\infty}^{\infty} l^3\, \rho_{l-n}\, u^3 \\ &+\frac{1}{4!} \sum_{l=-\infty}^{\infty} l^4\, \rho_{l-n}\, u^4\end{aligned} \tag{22}$$

Upon equating the coefficients of like powers of u in (21) and (22), we obtain conditions of the form

$$\begin{aligned}&\sum_{l=-\infty}^{\infty} l^s \rho_{l-n} = \\ &n^s - \binom{s}{2} n^{s-2}\phi''(0) + \binom{s}{4} n^{s-4}\phi^{(4)}(0) + \cdots ,\end{aligned} \tag{23}$$

for $s = 0, 1, \ldots$.

Schoenberg concludes, in light of (14), that a smoothing formula is exact for the polynomials of degree $(2k-1)$ if and only if the Taylor expansion of $\phi(u)$ at the origin is of the form

$$\phi(u) = 1 - au^{2k} + \cdots . \tag{24}$$

This is the well-known flatness property that the linear-phase maximally flat filters exhibit around the origin.

Actuaries refer to the frequency response function as a "periodogram function." An interesting paper that shows the knowledge of the actuaries about the shape of the periodogram function is [5], where it is observed that "the highest order of polynomials reproduced by a graduator Q is equal to the order of contact of its periodogram with the horizontal at $\beta = 1$." It turns out that $\beta = 1$ corresponds to $u = 2\pi$ and, hence, to $u = 0$ because of the periodicity of $\phi(u)$.

IV. Minimum R_∞ formulas and maximally flat FIR filters

There exists an explicit expression, due to W.F. Sheppard, for the linear smoothing formulas of the length $(2n+1)$ that are exact for the polynomials up to degree $j = 2k+1$, and have a minimized R_m^2. In this paper, we reproduce this expression in the form of a transfer function by writing

$$P(z) = \sum_{l=-n}^{n} \rho_l z^l, \tag{25}$$

which is the two-sided z-transform of the coefficients ρ_l of a linear smoothing formula. Using the subscripts k, m and n to associate $P(z)$ with the particular values of these parameters, the versatile transfer function is shown to be [2]

$$\begin{aligned}&P_{k,m,n}(x) = 1 \\ &+ \frac{(-1)^k}{k!} \sum_{s=k+1}^{n} \frac{(-n)^{\overline{s}}(m+n+1)^{\overline{s}}}{s(s-k-1)!(m+k+\frac{3}{2})^{\overline{s}}} x^s\end{aligned} \tag{26}$$

where $a^{\overline{b}} = a(a+1)\cdots(a+b-1)$ and

$$x = 1 - \frac{z+z^{-1}}{2}. \tag{27}$$

Greville has proved that

$$|P_{k,m,n}(\sin^2 \frac{\omega}{2})| < 1, \qquad 0 < \omega \leq \pi. \tag{28}$$

Furthermore, $P_{k,m,n}(\sin^2 \frac{\omega}{2})$ has $(n-k-1)$ extrema in the open interval $(0,\pi)$. To obtain an intuitive understanding of the general shape of $P_{k,m,n}(\sin^2 \frac{\omega}{2})$, we have plotted the characteristic

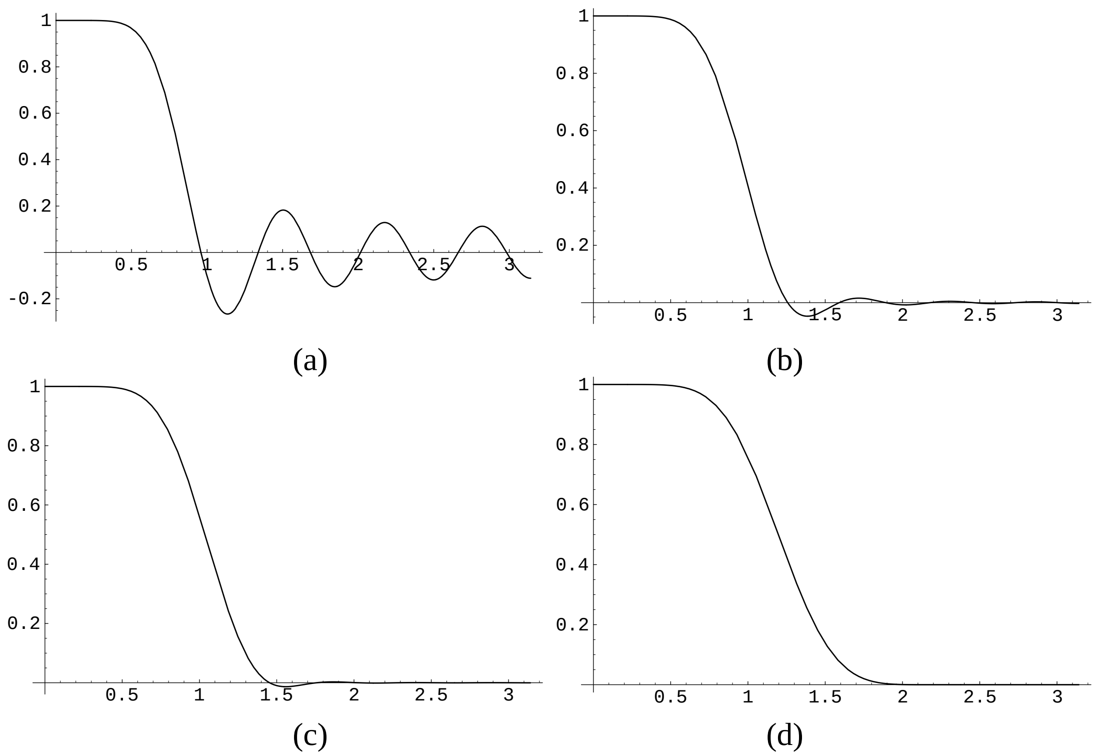

Fig. 2. Plots of $P_{3,m,10}(\sin^2 \frac{\omega}{2})$. (a) m=0. (b) $m = 5$. (c) $m = 10$. (d) $m = 50$. The horizontal axis depicts the normalized frequency ω.

function for selected values of k, m and n in Fig. 2. The frequency variable in the plots is consistent with the modern notion of the normalized frequency ω for $P_{k,m,n}(\sin^2 \frac{\omega}{2})$. The satisfaction of the tangency conditions can be visually confirmed by the flat shape of the curves around the origin. The more interesting property is the impact of the value of m determining the order in R_m^2. It can be seen that m has a remarkable influence on the shape of the curves in the vicinity of $\omega = \pi$. We can visually verify the following fact analyzed mathematically by Greville in 1966 [2]. As the value of m tends to infinity, the shape of $P_{k,m,n}(\sin^2 \frac{\omega}{2})$ tends to that of the magnitude response of a maximally flat lowpass FIR filter. A more precise expression of this fact is the following. Let ξ denote the smallest positive zero of $P_{k,m,n}(x)$ in $(0,1)$, and y (a non-positive number) denote the minimum value of $P_{k,m,n}(x)$ in $(0,1)$. Note that y is both the numerically largest negative value assumed by $P_{k,m,n}(x)$ in $(0,1)$ and the largest deviation from zero in $(\xi, 1)$. Then, for fixed k and n, both ξ and y increase monotonically with increasing m, i. e.,

$$\xi^{(m-1)} < \xi^{(m)}, \qquad y^{(m-1)} < y^{(m)},$$

where $\xi^{(m)}$ and $y^{(m)}$ are the values of ξ and y associated with a particular choice of m. In the limiting case of a minimum R_∞ smoothing formula, $\xi = 1$ and $y = 0$ is achieved.

Greville also shows that the minimum R_∞ smoothing formula is given by

$$P_{k,\infty,n} = (1-x)^{n-k} \sum_{s=0}^{k} \binom{n-k+s-1}{s} x^s. \quad (29)$$

This is identical to the formula of O. Herrmann [6]. He further observes that “this characteristic function is unusual in that in $[0, \pi]$ its only minimum is zero” at $\omega = \pi$ “and its only maximum is unity” at $\omega = 0$ “so that it is everywhere positive except at $\omega = \pi$ and decreases monotonically in the interval.”

V. Recent developments and concluding remarks

Most of the recent research on the maximally flat FIR filters have been reported in the literature dealing with circuits and systems or digital signal processing. We review some recent developments, mainly from the 1990's onward, without making a claim of comprehensiveness. The central theme is the closed form formulas and discrete-time structures; we have not made any attempt to cover the analytical methods that require an optimization step.

An influential insight into design of maximally-flat (MAXFLAT) FIR filters was provided in [7] through the Bernstein polynomials. The Bernstein-form of the transfer function was used in [8] to develop an exact multiplier-free structure for the maximally flat lowpass filters. For fixed values of the parameters, the structure can hierarchically realize all of the related filters of lower orders. The structure was further generalized in [9] to the structures for maximally flat bandstop and bandpass filters with an arbitrary center of flatness. Odd-order maximally flat FIR filters were reported in [10]. A new type of Bernstein approximation was introduced in [11]. The design problem of Mth band maximally flat filters was first considered in [12], [13] and completely solved in [14]. A generalization of maximally flat filters with arbitrary group delay at the origin (due to Baher), was proposed in [15] and studied further in [16]. In [17], it was shown that the maximally flat filters of Baher are a universal class of lowpass filters that contain the fractional-sample delay filters as well. Discrete-time cellular structures, including multiplier-less ones, were developed in [17], [18], [19].

The customary criteria used in the evaluation of the performance of digital filters deal with qualities such as sharpness of the transition and the minimization of the deviation from an ideal response. However, as is clear from the historical background provided in this paper, the maximally flat lowpass filters of Herrmann have a profound signal processing significance that overshadows the traditional performance criteria. They are the only minimum R_∞ solutions that pass polynomial signals. Application of this property, as shown for example in [20], may prove insightful in dealing with problems involving maximally flat digital filters.

Digital filters with smoothing properties that are exact for other types of signals are not well studied. The researchers in the field seeking new directions of inquiry are well advised to study the generalizations of the concepts covered in this paper as made by actuarial mathematicians. An interesting paper in that direction is [21] where a new criterion for judging the properties of moving averages is given and formulas are derived under a general assumption for the noise. As noted in [21], there is no need to limit the exactness condition to the polynomial signals. Extensions to the second dimension and beyond are also not thoroughly studied.

In recent years, various wavelet bases have been constructed and enthusiastically studied by engineers and mathematicians. Research on the construction of compactly supported wavelets using digital filters satisfying regularity conditions is directly related to the study of flat and maximally flat digital filters. Even though the digital filtering equivalents of the results introduced in this paper are not new to the wavelet designers, it is expected that the historical background introduced in this paper would inspire new research directions. We conclude this paper by the following excerpt from [22]. "There is a saying in the Orient, Onko-chishin, which originated with Confucius in the 6th century B.C. It means, by exploring the old, one can come to understand the new, or more simply, take a lesson from the past."

REFERENCES

[1] W.F. Sheppard, "Reduction of error by means of negligible differences," in *Proc. Fifth International Congress of Mathematicians*, Cambridge, vol. 2, pp. 348–384, 1913.

[2] T. N. E. Greville,"On Stability of Linear Smoothing Formulas," *SIAM Journal on Numerical Analysis*, vol. 3, pp. 157–170, 1966.

[3] H.H. Wolfenden, "On the development of formulae for graduation by linear compounding, with special reference to the work of Erastus L. DeForest," *Trans. Actuar. Soc. Amer.*, vol. 26, pp. 81–121, 1925.

[4] I.J. Schoenberg, "Contributions to the problem of approximation of equidistant data by analytic functions. Part A: On the problem of smoothing or graduation. A first class of analytic approximation formulae." *Quarterly of Appl. Math.*, vol. 4, pp. 45–99, 1946.

[5] G.H. Andrews and C.J. Nesbitt, "Periodograms of graduation operators," *Soc. Actuar. Trans.*, vol. 17, pp. 1–27, 1965.

[6] O. Herrmann, "On the approximation problem in nonrecursive digital filter design," *IEEE Trans. Circuits Syst.*, vol. 18, pp. 411–413, 1971.

[7] L.R. Rajagopal and S.C. Dutta Roy, "Design of maximally-flat FIR filters using the Bernstein polynomial," *IEEE Trans. Circuits Syst.*, vol. 34, pp. 1587–1590, 1987.

[8] S. Samadi, T. Cooklev, A. Nishihara and N. Fujii, "Multiplierless structure for maximally flat linear phase FIR filters," *Electronics Lett.*, pp. 184–185, vol. 29, 1993.

[9] S. Samadi, A. Nishihara and N. Fujii, “Modular realization of bandstop and bandpass FIR digital filters,” in Proc. *ISCAS,* vol. 2, pp. 73–76, 1994.
[10] N. Aikawa, N. Yabiku and M. Sato, “Design method of maximally flat FIR filter in consideration of even and odd order,” in Proc. *ISCAS,* vol. 1, pp. 276–279, 1991.
[11] T. Cooklev and A. Nishihara, “Maximally flat FIR filters,” in Proc. *ISCAS,* vol. 1, pp. 96–99, 1993.
[12] P.N. Heller, “Lagrange M-th band filters and the construction of smooth M-band wavelets,” In *Proc. IEEE-SP International Symposium on Time-Frequency and Time-Scale Analysis* pp. 108–111, 1994.
[13] S.-C. Pei and P.-H. Wang, “Closed-form design of maximally flat R-regular Mth-band FIR filters,” in Proc. *ISCAS* vol. 1, pp. 539–542, 2000.
[14] S. Samadi, M.O. Ahmad and M.N.S. Swamy, “Complete characterization of systems for simultaneous Lagrangian upsampling and fractional-sample delaying,” to appear in *IEEE Trans. Circuits Syst. I.*
[15] I. Selesnick and C.S. Burrus, “Maximally flat lowpass FIR filters with reduced delay,” *IEEE Trans. Circuits Syst. II,* vol. 45, pp. 53–68, 1998.
[16] J.B. Little, “Solving the Selesnick-Burrus filter design equations using computational algebra and algebraic geometry,” *Adv. Appl. Math.,* vol. 31, pp. 463–500, 2003.
[17] S. Samadi, A. Nishihara and H. Iwakura, “Universal maximally flat lowpass FIR systems,” *IEEE Trans. Signal Processing,* vol. 48, pp. 1956–1964, 2000.
[18] S. Samadi, A. Nishihara and H. Iwakura, “Filter-generating systems,” *IEEE Trans. Circuits and Systems II,* vol. 47, pp. 214–221, 2000.
[19] S. Samadi and A. Nishihara, “Concise representation and cellular structure for universal maximally flat FIR filters,” *ISCAS,* vol. 4 , pp. 201–204, 2003.
[20] M. U. A. Bromba and H. Ziegler, “Explicit formula for filter function of maximally flat nonrecursive digital filters,” *Electron. Lett.,* vol. 16, no. 24, pp. 905–906, 1980.
[21] Ø. Borgan, “On the theory of moving average graduation,” *Scand. Actuarial Journal,* pp. 83–105, 1979.
[22] K. Habara, “ISDN: a look at the future through the past,” *IEEE Communications Magazine,* vol. 26, no. 11, pp. 25–32, 1988.