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LETTER

A Synthesis of a Class of Complex Digital Filters Based on Circuitry Transformations

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SUMMARY This letter proposes a simple synthesis of a class of complex digital filters whose transfer functions are obtained from real transfer functions by substitution of $e^{-j\theta}z$ for z. Such filters are constructed by simple circuitry transformations directly applied to real circuits.

1. Introduction

The frequency responses of a class of complex digital filters which are transformed from real transfer functions by substituting $-jz$ for z are shifted by $\pi/2$ from their original ones. Consequently, they are named orthogonal filters(1). Sim has proposed the design of orthogonal biquad digital filters(2), where the structures of the orthogonal direct form, minimum norm and optimal state space are presented. They are synthesized in terms of the signal flow descriptions of complex difference equations or state space equations derived from the orthogonal transfer functions. The derivation of these equations refers to that of their real counterparts. As the synthesis proposed in Ref. (2) consists of the same procedure as the case of real transfer functions, it seems redundant if corresponding real circuitry already exists.

This letter proposes a new synthesis of a class of complex digital filters whose transfer functions are obtained from real transfer functions by substitution of $e^{-j\theta}z$ for z. Since the frequency responses of such complex transfer functions are shifted by θ from their original ones, they form a general class of complex digital filters more than orthogonal filters, which are included at $\theta=\pi/2$. Therefore, it can be said that the proposed method is more widely applicable than Ref. (2). It is constructed by a kind of circuitry transformation directly applied to real circuits, and is divided into two steps. The first step is to map a real structure into the complex domain. The second is to replace the delay units of the resultant complex structure by the circuitry realization of $e^{-j\theta}z$. Thus the procedure is very simple and easy to apply. As the consequent complex circuits inherit the coefficient-sensitivity characteristic of their real prototypes, excellent complex structures realizing the above mentioned transfer functions can be easily obtained.

2. Synthesis by Circuitry Transformation

On the assumption that $H(z)$ is a real transfer function whose circuitry realization is known, let us consider that it is desired to realize a complex transfer function $H(e^{-j\theta}z)$, which is obtained from $H(z)$ by substitution of $e^{-j\theta}z$ for z. To this end, the following two-step procedure is proposed. In the complex domain a real transfer function is interpreted as a complex transfer function without imaginary part. When a real structure with a real transfer function $H(z)=Y/X$ shown in Fig. 1 is mapped into the complex domain in this sense, the complex structure with the same real transfer function represented by Fig. 2 is obtained. The relationship of its input and output is given by

$$Y_R+jY_I=H(z)(X_R+jX_I) \qquad (1)$$

As $H(z)$ is real,

$$H(z)=\frac{Y_R}{X_R}=\frac{Y_I}{X_I} \qquad (2)$$

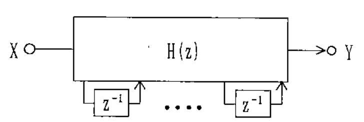

Fig. 1 Real structure realizing real $H(z)$.

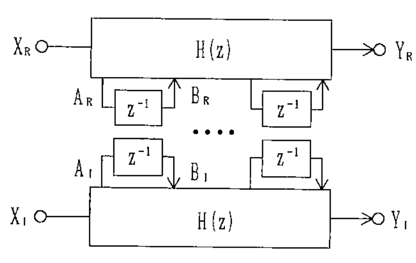

Fig. 2 Complex structure realizing real $H(z)$.

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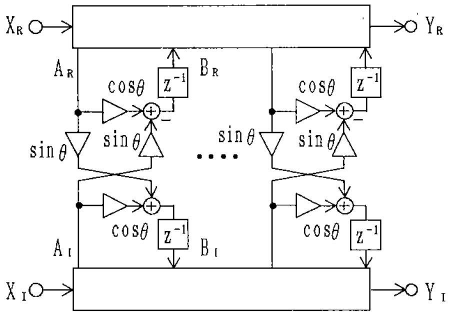

Fig. 3 Complex structure realizing complex $H(e^{-j\theta}z)$.

also holds. Such mapping is the first step.

The next step is a circuitry transformation in the complex domain. In Fig. 2 the input-output relation of each complex-pair of delay units is denoted by

$$B_R+jB_I=z^{-1}(A_R+jA_I) \tag{3}$$

where A_R+jA_I and B_R+jB_I are the input and the output, respectively. Substituting $e^{-j\theta}z$ for z in Eq. (2) results in

$$B_R+jB_I=e^{j\theta}z^{-1}(A_R+jA_I), \tag{4}$$

that is to say,

$$B_R=z^{-1}(A_R\cos\theta-A_I\sin\theta) \tag{5}$$

and

$$B_I=z^{-1}(A_R\sin\theta+A_I\cos\theta). \tag{6}$$

In circuitry this substitution means replacing each complex-pair of delay units in Fig. 2 by the circuitry realization of both Eqs. (5) and (6). Then Fig. 3 is obtained. Thus the objective structure realizing the transformed complex transfer function $H(e^{-j\theta}z)$ is synthesized. Its coefficient-sensitivities to multipliers excluding $\cos\theta$ and $\sin\theta$ are also shifted by θ along the frequency axis from those of its prototype. If $\cos\theta$ and $\sin\theta$ are rounded so that the squared sum of their rounded values can be approximately equal to one, the influence of their rounding causes the deviation of the shifting frequency. Therefore, Fig. 3 can inherit the sensitivity-characteristic from the prototype.

The proposed procedure is applicable to all real structures, whereas Sim[2] deals with only three biquad structures. Besides it is more straightforward since it is made up of only two circuitry transformations.

3. Example

In this section a simple example is given to demonstrate efficiency of the proposed method. The real prototype structure used here is the real minimum norm biquad structure[3] shown in Fig. 4. Its transfer function is given by

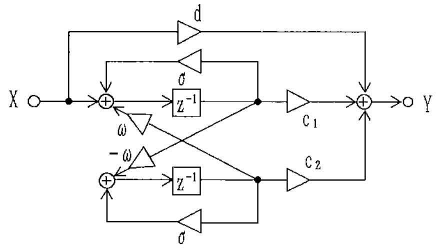

Fig. 4 Real minimum norm biquad.

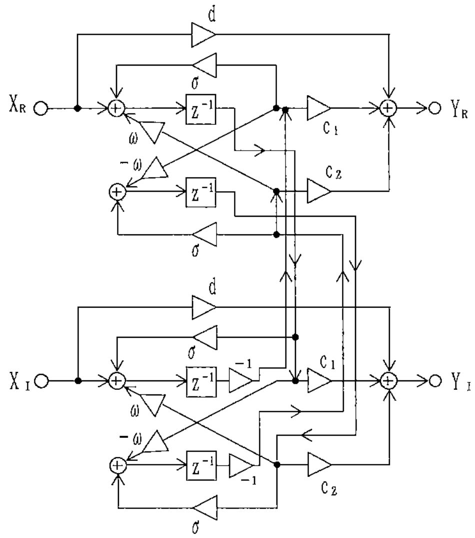

Fig. 5 Minimum norm orthogonal biquad.

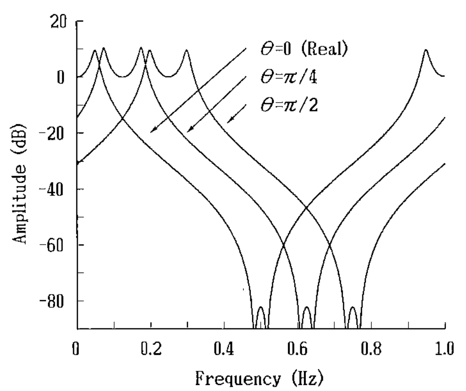

Fig. 6 Amplitude responses realized using 8-bit mantissa.

$$H(z)=d+\frac{c_1z^{-1}-(c_1\sigma+c_2\omega)z^{-2}}{1-2\sigma z^{-1}+(\sigma^2+\omega^2)z^{-2}} \tag{7}$$

First it is mapped into the complex domain to obtain a complex structure corresponding to Fig. 2. Secondly the circuitry transformation mentioned above is applied as Fig. 2 is transformed into Fig. 3. Then the complex circuit realizing the transformed transfer function $H(e^{-j\theta}z)$ is obtained. Figure 5 shows a orthogonal case of $\theta=\pi/2$. It is equivalent to Ref. (2), but the synthesis process of the proposed method is simpler than Ref. (2).

Figure 6 shows the amplitude responses at $\theta=0$ (real case), $\pi/4$ and $\pi/2$. The filters are realized using floating-point coefficients with 8-bit mantissa. The coefficients in Eq.(7) are given by

$$d=0.025,\ \sigma=0.9,\ \omega=0.3,$$

$$c_1=0.095,\ c_2=-0.293333.$$

It can be said that the coefficient-sensitivity property is inherited from the real prototype, because the amplitude responses realized using finite-wordlength coefficients are shifted by θ along the frequency axis with little distortion from the one of their real prototype.

4. Conclusion

This letter has proposed a new synthesis of complex digital filters whose transfer functions are obtained from real transfer functions by substitution of $e^{-j\theta}z$ for z. It is very simple and easy to apply, because it is constructed by simple circuitry transformations directly applied to real circuits. As the consequent complex circuits inherit the coefficient-sensitivity of their real prototypes, excellent complex structures realizing such transfer functions are easily obtained.

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