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Reduction and Possible Elimination of Coating Thermal Noise Using a Rigidly Controlled Cavity with a Quantum-Nondemolition Technique

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Thermal noise of a mirror is one of the most important issues in high-precision measurements such as gravitational-wave detection or cold damping experiments. It has been pointed out that thermal noise of a mirror with multilayer coatings can be reduced by mechanical separation of the layers. In this Letter, we introduce a way to further reduce thermal noise by locking the mechanically separated mirrors. The reduction is limited by the standard quantum limit of control noise, but it can be overcome with a quantum-nondemolition technique, which finally raises a possibility of complete elimination of coating thermal noise.

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Overview.—Brownian motion of coating layers on a test mass is one of the limiting noise sources for high-precision measurements such as gravitational-wave detection [1] or cold damping experiments aimed at the observation of the quantum behavior of a harmonic oscillator [2]. Mechanical loss of the coatings tends to be higher than that of the mirror substrate, and the thermal fluctuation of the coatings prevents the measurement of the mass location. Thermal noise is also the most significant source of environmental decoherence that prevents us from observing the quantum behavior of a macroscopic object [3]. The reduction of classical noise is a key for various subjects in modern physics.

Several ideas to reduce thermal noise have been proposed [4], among which the mechanical separation of coating layers proposed by Khalili [5] is the easiest and the most sensational one, especially for an interferometer with high-reflective, multilayer-coated mirrors. The mechanical separation can be realized by using two mirrors with fewer coatings locked to the antiresonance so that the reflectivity of the antiresonant cavity is as high as a single mirror with more coatings.

Figure 1 shows the configuration of a typical gravitational-wave detector with the end test mass (ETM) replaced by the antiresonant cavity. The differential mode of the two resonant arm cavities is measured at the dark port of the Michelson interferometer while the common mode and the dc component of the incident light return to the other side, reflected back by the power-recycling mirror (PRM). The reflectivity of the compound mirror in the case without optical losses is given by

$$r_c = \frac{r_{\text{IETM}} + r_{\text{EETM}}}{1 + r_{\text{IETM}}r_{\text{EETM}}}, \quad (1)$$

which is closer to unity than r_{IETM} or r_{EETM} alone. Coating thermal noise in displacement is proportional to the square root of the coating thickness. The higher the reflectivity of the mirror is, the thicker the coating should be. The single

mirror with higher reflectivity can be replaced by the cavity so that the reflectivity is almost the same but coating thermal noise of the input end-test-mass (IETM) is smaller than that of the original ETM. In Ref. [5], r_{IETM} is assumed to be reasonably high, so that thermal noise of the end end-test-mass (EETM) appears negligible.

The surface of the mirrors toward the inside of the resonant cavity is coated. The light in the resonant cavity probes the motion of the coatings on the IETM, and the light leaking through the antiresonant end-mirror cavity probes thermorefractive fluctuation of the IETM substrate [6] and coating thermal noise of the EETM [7]. The reflectivity balance can be optimized so that the total noise level from both mirrors is minimized. It is important to do the same optimization with the PRM and the input test mass (ITM)—4-mirror coupled cavity system. Any noise on the PRM does not appear on measuring the differential signal, but noise on the beam splitter (BS) appears instead. Noise on the BS is negligible if r_{ITM} is reasonably high, but

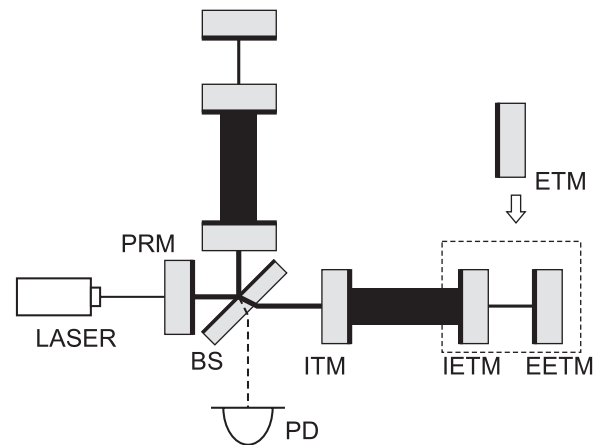


FIG. 1. Power-recycled Michelson interferometer with Fabry-Perot cavity in the arms and the end test masses are replaced by an antiresonant cavity.

it becomes nontrivial after the optimization just like noise on the EETM.

In the presence of optical losses, the reflectivity of the compound mirror with low r_{IETM} cannot be as high as that of a single mirror with an adequate number of coatings even if the EETM could be perfectly reflective. The reflectivity of the mirror coated by the SiO_2 - Ta_2O_5 doublets is roughly given by the following equation:

$$r \simeq \sqrt{1 - 2.8 \times 0.49^N}, \quad (2)$$

with N the number of Ta_2O_5 layers (SiO_2 has one layer fewer). This approximation is fine unless $N < \sim 3$; the exact amplitude reflectivity for thin coatings is 0.49 ($N = 1$), 0.72 ($N = 2$), and 0.85 ($N = 3$). Accepting a 10% increase of the optical loss compared with a single mirror with $N = 15$, we can reduce N of the IETM down to 3. Accepting a 50% increase, we can reduce it down to 1, with which, however, the noise contribution of the EETM prevents the further improvement of the noise level. The lowest noise level is achievable with $N = 2$.

Since the optimal result we obtained includes thermal noise from the EETM almost nearly as much as thermal noise from the IETM, the noise level will be even lower if the EETM motion can be completely isolated, or suppressed by the feedback control. In the current gravitational-wave detectors, the motion of the PRM, for example, is suppressed by the use of a weak control field but only at low frequencies so that sensing noise does not impose too much additional motion to the mirror [8]. Here we use a control field much stronger than the main beam (carrier light) in the end-mirror cavity so that control noise can be sufficiently small.

Let us assume that the control field is a frequency-shifted sideband to the carrier light and it resonates in the end-mirror cavity. The sideband senses the motion of the EETM, together with the thermorefractive fluctuation of the IETM, $(1 + r_{\text{IETM}})/(1 - r_{\text{IETM}})$ more than the antiresonant carrier light. The sideband power being increased, the shot-noise level at the measurement of the end-mirror cavity can be lower than its original thermal fluctuation. On the other hand, quantum radiation pressure noise is imposed on the IETM and it cannot be suppressed by the control. There exists a quantum limit of excess control noise, which indeed can be exceeded by one of the quantum-nondemolition (QND) techniques. We will see this in the following sections with the detailed calculation of quantum radiation pressure noise.

Quantum limit of control noise.—Figure 2 shows the input-output relation of the classical and vacuum fields in the end-mirror cavity. Each bold letter is a vector with the amplitude quadrature and the phase quadrature, which will be depicted by subscripts 1 and 2, respectively. The classical fields have amplitude-quadrature components only and the mirror motion appears in the phase-quadrature component. The only difference between the carrier and

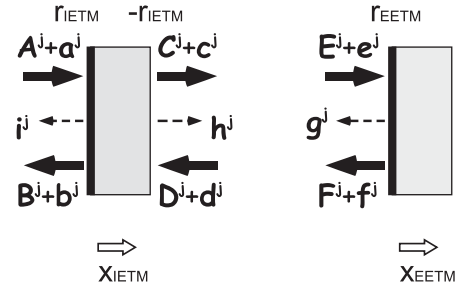


FIG. 2. Input-output relation of the end-mirror cavity. Capital letters and small letters indicate classical fields and vacuum fluctuations. Superscript j identifies the carrier (c) or the control sideband (s). Dashed lines are for loss vacuums.

sideband is the phase shift between the two mirrors. The output fields $\mathbf{B}^j$ have the information of x_{EETM} , so a certain combination of $\mathbf{B}^c$ and $\mathbf{B}^s$ has no thermal noise of the EETM. Let us ignore the optical losses of both mirrors and the transmittance of the EETM for simplicity; as our purpose is to reduce the coating layers of the IETM, the transmittance of the EETM is low enough. Feeding back the phase-quadrature information of the control sideband to the EETM so that x_{EETM} can be canceled out, we obtain a new output vector:

$$\mathbf{z} = \mathbf{b}^c + \begin{pmatrix} 0 \\ b_2^s \end{pmatrix} \frac{A_1^c(1-r)^2}{A_1^s(1+r)^2}, \quad (3)$$

the phase quadrature of which is

$$z_2 = a_2^c - \tilde{r} \mathcal{K} a_1^c + \frac{\sqrt{2\mathcal{K}}}{x_{\text{SQL}}} \tilde{r} x + \frac{A_1^c(1-r)^2}{A_1^s(1+r)^2} a_2^s - \tilde{r} \mathcal{K} \frac{A_1^s}{A_1^c} a_1^s. \quad (4)$$

Here $r = r_{\text{IETM}}$, x is the motion of the IETM without quantum radiation pressure noise, and

$$\tilde{r} = \frac{4r}{(1+r)^2}, \quad x_{\text{SQL}} = \sqrt{\frac{2\hbar}{m\Omega^2}}, \quad \mathcal{K} = \frac{4I_0\omega_0}{m\Omega^2c^2}, \quad (5)$$

with I_0 as the carrier power on the left side of the IETM, ω_0 as the laser angular frequency, $\hbar$ as the Planck constant, c as the light speed, m as the mass of IETM, and Ω as the measurement angular frequency. While thermal noise of the EETM is suppressed by the control, shot noise moves the EETM and the motion is sensed by the leaking carrier light. Radiation pressure noise of the control sideband moves the IETM and the motion is directly sensed by the carrier light. Taking the square sum of each vacuum component of the right-hand side in Eq. (4), comparing it with the displacement x , and choosing the proper A_1^c/A_1^s , we obtain the quantum-noise level, with excess control noise being minimized at one given measurement frequency, as

$$x_{\text{QN}}^{\min} = \frac{x_{\text{SQL}}}{\sqrt{2\mathcal{K}\tilde{r}}} \sqrt{1 + \tilde{r}^2 \mathcal{K}^2 + 2\tilde{r}\mathcal{K} \frac{(1-r)^2}{(1+r)^2}}. \quad (6)$$

The last term of Eq. (6) is the contribution of excess control noise, which can be rewritten as

$$x_{\text{ctrl}}^{\min} = x_{\text{SQL}} \frac{1-r}{2\sqrt{r}}. \quad (7)$$

Figure 3 shows the comparison of excess thermal noise, which is, namely, the sum of thermorefractive noise in the IETM and coating thermal noise from the EETM, and excess control noise that we have just derived. At 100 Hz, the control-noise level is smaller than the other. The optimal number of layers is $N = 1$, and the total noise level is 1.8×10^{-21} m/ $\sqrt{\text{Hz}}$, which is about 13% better than the lowest level without the control ($N = 2$).

Note that, even without excess control noise, the minimum value by changing $\mathcal{K}$ in Eq. (6) does not reach the standard quantum limit x_{SQL} . This is due to the reduction of the effective mass. Both the carrier and sideband fields sense the position of the IETM, and the effective mass of the end-mirror cavity is $\tilde{r}$ times smaller than the single ETM.

Exceeding the quantum limit.—There exists a way to remove control noise. Using one of the QND techniques, a so-called *backaction evasion* technique [9], radiation pressure noise can be canceled out so that control noise can be infinitely small with infinitely high laser power. Instead of feeding back the phase-quadrature information b_2^s , we can feedback $b_\zeta^s = b_1^s \sin \zeta + b_2^s \cos \zeta$ to the EETM. Equation (3) becomes

$$\mathbf{z}^{\text{VR}} = \mathbf{b}^c + \begin{pmatrix} 0 \\ b_\zeta^s \end{pmatrix} \frac{A_1^c(1-r)^2}{A_1^s(1+r)^2} \frac{1}{\cos \zeta}, \quad (8)$$

the phase quadrature of which is

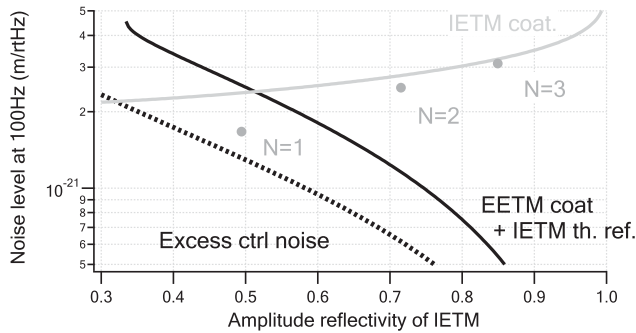


FIG. 3. Comparison of excess thermal noise and excess control noise. Gray circles show the exact value of coating thermal noise from the IETM with $N \leq 3$. Locking the end-mirror cavity, we can replace thermal noise of the cavity to control noise that is smaller. Thermal noise from the IETM coating appears equally regardless of the control.

$$z_2^{\text{VR}} = a_2^c - \tilde{r}\mathcal{K}a_1^c + \frac{\sqrt{2\mathcal{K}}}{x_{\text{SQL}}} \tilde{r}x + \frac{A_1^c(1-r)^2}{A_1^s(1+r)^2} (a_2^s + a_1^s \tan \zeta) - \tilde{r}\mathcal{K} \frac{A_1^s}{A_1^c} a_1^s. \quad (9)$$

Then, choosing the readout quadrature to satisfy

$$\tan \zeta = \tilde{r}\mathcal{K} \left[\frac{A_1^s(1+r)}{A_1^c(1-r)} \right]^2, \quad (10)$$

and increasing the sideband power A_1^s , control noise can be eliminated at one given measurement frequency. Radiation pressure noise of the IETM from a_1^s is canceled by driving the EETM with the same amount of vacuum fluctuation. It is only coating noise of a single layer on the IETM that appears at the measurement of this compound mirror's position.

In fact, even the single layer of coating is not necessary. The amplitude reflectivity of an uncoated silica substrate in the vacuum is not zero but $r = 0.184$. Using this reflective substrate as the IETM, we finally realize the position measurement without coating thermal noise and excess control noise.

Note that the minimization of control noise can be done at one given frequency since $\mathcal{K}$ is a frequency-dependent coefficient. Kimble *et al.* has proposed a so-called *filter cavity* to realize the frequency-dependent tuning of the readout quadrature ζ [10]. Implementation of such a filter should be considered in our case as well.

Figure 4 shows an example noise spectrum of an end-mirror cavity. Each mirror is 40 kg with 20 cm thickness, the laser power inside the arm cavity is 800 kW, mechanical losses are 4.0×10^{-5} for silica and 2.4×10^{-4} for tantalum coatings, and the numbers of layers are $N = 1$ on the IETM and $N = 17$ on the EETM. The control sideband power in the case with the conventional readout is defined so that control noise is minimized at 100 Hz. With the above parameters the required sideband power is

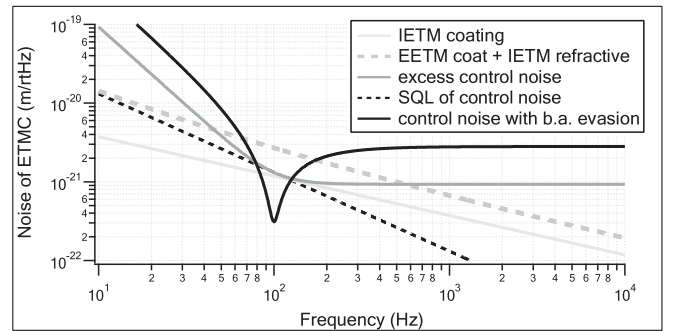


FIG. 4. Thermal noise and control noise of an end-mirror cavity with a single layer ($N = 1$) on the IETM. Without the rigid control, coating thermal noise of the EETM coating and thermorefractive noise of the IETM are added. With the rigid control, instead of those two kinds of noise, excess control noise is imposed.

57% of the carrier power. The sideband power in the case with the backaction evasion is set to be 3 times higher than that. The total noise level at 100 Hz is 2.3×10^{-21} m/ $\sqrt{\text{Hz}}$ without the rigid control, 1.8×10^{-21} m/ $\sqrt{\text{Hz}}$ with the control using the conventional readout, and 1.2×10^{-21} m/ $\sqrt{\text{Hz}}$ with the control using the backaction evasion.

Summary and discussions.—Reduction of coating thermal noise is the goal of this study. We started from the previous work by Khalili to realize the mechanical separation of the coating layers using an antiresonant end-mirror cavity. First we pointed out that one can take more advantage of the separation using a 4-mirror coupled cavity with the optimally balanced numbers of coating layers, and demonstrated the optimization. Second, as the main work of this Letter, we suggested locking the end-mirror cavity by control using a sideband field. Thermal noise from one of the mirrors in the cavity is replaced by shot noise imposed by the control and radiation pressure noise of the sideband. Since excess control noise turned out to be smaller than thermal noise, the total noise level was further improved. At last, we introduced a way to remove control noise by the backaction evasion technique, with which we can choose the reflectivity of the IETM even lower than that of the single-layer coating, namely, no coatings, thus no coating thermal noise.

There are two issues to be discussed here. First, the cancellation of thermal noise strongly relies on the assumption that thermal fluctuations sensed by the carrier light and the sideband are the same. In practice, the mode and the beam centering cannot be perfectly the same between the two fields, so this will be one of the issues. The rigid control on both coupled cavities PRM-ITM and IETM-EETM would be realized, for example, using three sets of control sidebands; one does not resonate in any cavity, one resonates in the power-recycling cavity, and the other resonates in all three cavities (the power-recycling, the arm, and the end-mirror cavity) and taking the beats.

The second issue is about other kinds of coating thermal noise. Thermal expansion due to the Brownian motion or the fluctuation of the refraction index causes the fluctuation of the complex reflectivity of a mirror. These are called thermoelastic noise [11] and thermorefractive noise [12], respectively, which are not included in the calculation of this Letter. Brownian thermal noise tends to be larger than thermoelastic or thermorefractive noise,

but it decays faster at higher frequencies, so the final optimization should be done taking the frequency dependence into account.

The cancellation of thermal noise will be a key issue for various experiments in the years to come. This Letter brings up the idea of the thermal-noise cancellation using the rigid control, which would be applicable to various kinds of high-precision measurement. Experimental demonstrations should be given in the near future.

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- [1] A. Abramovici *et al.*, Science **256**, 325 (1992); C. Bradaschia *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **289**, 518 (1990); B. Willke *et al.*, Classical Quantum Gravity **19**, 1377 (2002); M. Ando *et al.*, Phys. Rev. Lett. **86**, 3950 (2001).
 - [2] S. Gigan *et al.*, Nature (London) **444**, 67 (2006); O. Arcizet *et al.*, Nature (London) **444**, 71 (2006); D. Kleckner and D. Bouwmeester, Nature (London) **444**, 75 (2006); J. Harris *et al.*, Rev. Sci. Instrum. **78**, 013107 (2007); A. Schliesser *et al.*, Phys. Rev. Lett. **97**, 243905 (2006); T. Corbitt *et al.*, Phys. Rev. Lett. **99**, 160801 (2007).
 - [3] H. Müller-Ebhardt *et al.*, Phys. Rev. Lett. **100**, 013601 (2008).
 - [4] M. Evans *et al.*, Phys. Rev. D **78**, 102003 (2008); V. Galdi *et al.*, Phys. Rev. D **73**, 127101 (2006); M. Ohashi *et al.*, Classical Quantum Gravity **20**, S599 (2003); H. Kimble *et al.*, Phys. Rev. Lett. **101**, 260602 (2008); V. Braginsky *et al.*, Meas. Sci. Technol. **10**, 598 (1999).
 - [5] F. Khalili, Phys. Lett. A **334**, 67 (2005).
 - [6] V. Braginsky and S. Vyatchanin, Phys. Lett. A **324**, 345 (2004).
 - [7] G. Harry *et al.*, Classical Quantum Gravity **19**, 897 (2002).
 - [8] K. Somiya *et al.*, LIGO Report No. LIGO-DCC T060272, 2006; K. Somiya, LIGO Report No. LIGO-DCC T0900154, 2009.
 - [9] S. P. Vyatchanin and A. B. Matsko, JETP **77**, 218 (1993); S. P. Vyatchanin and E. A. Zubova, Phys. Lett. A **201**, 269 (1995).
 - [10] H. J. Kimble *et al.*, Phys. Rev. D **65**, 022002 (2001).
 - [11] M. Fejer *et al.*, Phys. Rev. D **70**, 082003 (2004).
 - [12] V. Braginsky and S. Vyatchanin, Phys. Lett. A **312**, 244 (2003).