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Realization of Multi-Port Networks Using Operational Amplifiers and Its Applications

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SUMMARY A systematic method for synthesizing multi-port networks specified by scattering matrices is proposed. First, a basic circuit for converting the port parameters is introduced. At each port the basic circuit converts the voltage and the current into the incident and the reflected wave quantities. Then each element of the scattering matrix is realized using the conventional techniques for the realization of one -input one-output transfer functions. When all the elements of the scattering matrix are either positive constant or zero, the resultant network can be simplified. With the proposed method, some known structures can be obtained very easily and two new structures for hybrid networks are derived. One of the salient feature of the proposed method is that it also provides the physical interpretation of the structures derived. Experimental results are also given.

1. Introduction

Multi-port networks, such as bidirectional amplifiers, circulators and hybrid networks, perform important roles in various transmission systems.

Transfer characteristics of multi-port networks are most properly described by scattering matrices (S -matrices) because each element of S -matrices directly expresses the transmission between the ports and S -matrices exist for all kinds of circuits, while impedance, admittance or other matrices cannot always be defined.

In general active elements are necessary for the realization of multi-port networks, and they are indispensable for the applications to amplifying networks such as multi-directional amplifiers. At present time, integrated operational amplifiers are considered to be the most versatile and economical active elements.

In this paper a simple and systematic method is proposed to synthesize multi-port networks using operational amplifiers.

One of the authors showed that a multi-port network specified by S -matrix can be synthesized by converting the S -matrix into admittance matrix (Y -matrix) [(1),(2)], and some structures for bidirectional amplifiers, circulators and multi -directional amplifiers were developed. However, that method is complicated and unfortunately very restrictive because the specified S -matrix must be converted into Y -matrix in the first step of the procedure. S -matrices can be defined for every kind of multi-port networks, while Y -matrices cannot be defined for certain sets of specifications or even essentially Y -matrices do not exist for some classes of networks such as ideal transformers, generalized immittance converters, etc. In this paper a method for realizing any kind of multi-port networks directly from S -matrices with a single procedure is introduced. With the use of this method, we can casily obtain the same results as Refs. (1) and (2), and some new hybrid networks are derived.

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57−43 [P−32]

2. Realization of S-Matrices

In the N -port network shown in Fig. 1, the voltage incicent wave a_k and the voltage reflected wave b_k at the k -th port are defined by

$$a_k = v_k + R_k i_k \tag{1a}$$

$$b_k = v_k - R_k i_k \tag{1b}$$

where R_k is the characteristic resistance. S -matrix denotes the relation between a_k 's and b_k 's as

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} \tag{2}$$

When the voltage source v_{ko} with a source resistance R_k is connected to the respective port as shown in Fig. 1 and the condition of impedance matching is satisfied at each port, the elements of the S -matrix are given by

$$S_{ii} = 0$$

$$S_{ij} = \left.\frac{V_i}{V_j}\right|_{v_{mo}=0(m\neq j)} = 2\left.\frac{V_i}{V_{jo}}\right|_{v_{mo}=0(m\neq j)} \tag{3}$$

Equation (3) shows that S_{ij} is directly related to the voltage gain from port j to port i .

For the straightforward realization of the transfer characteristics specified by S -matrix, the information of

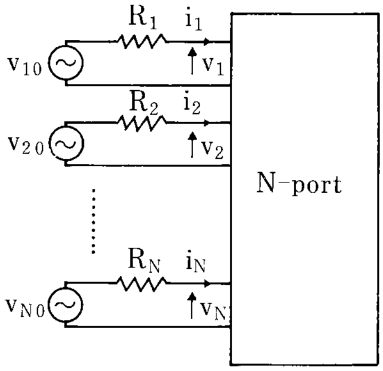

Fig. 1 N-port network.

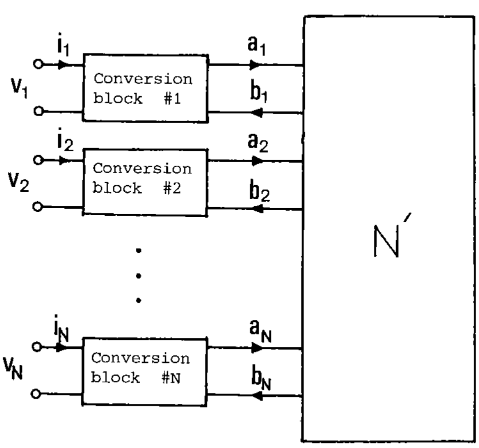

Fig. 2 Extraction of conversion blocks.

the port voltages and currents are converted into wave quantities as shown in Fig. 2. The network N' in Fig. 2 consists of some subnetworks whose transfer functions are equated to the corresponding elements of the S-matrix. This structure is especially useful for the realization of sparse S-matrices. Each conversion block produces the output voltage a_k using b_k as an input. Now Eq. (1) can be rewritten as follows;

$$a_k = 2v_k - b_k \tag{4a}$$

$$i_k = \frac{v_k - b_k}{R_k} \tag{4b}$$

The circuit satisfying Eq. (4) can be easily designed as shown in Fig. 3 with the use of one operational amplifier. This circuit is essentially the same as that proposed by Brackett[3] except that the terminal names for a_k and b_k are reversed because of the different usage.

The input polarity of the operational amplifier is determined such that the overall circuit is stable. The general solution for this problem has been given by Fujii[4]. When the diagonal element S_{kk} is equal to zero, the situation becomes much simpler, i.e., the upper input terminal is chosen to be the inverting input in the same way as the conversion circuit exists alone, since there is no transmission

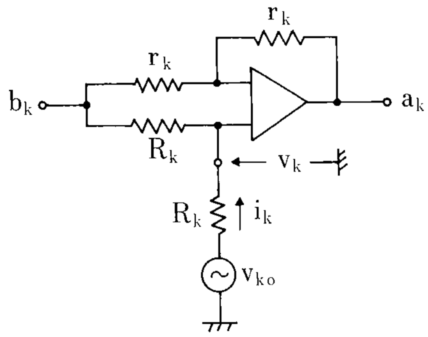

Fig. 3 Realization of the basic conversion block.

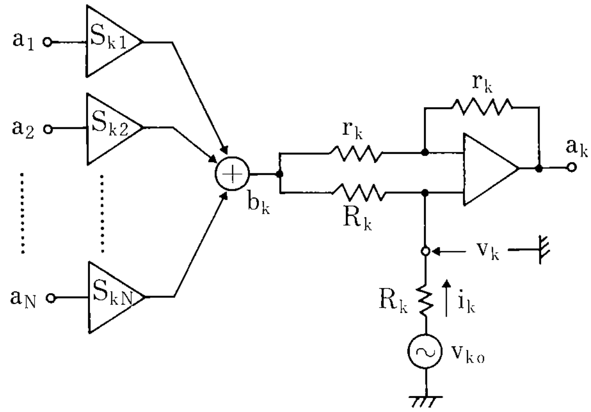

Fig. 4 Block diagram realizing the k-th row of the S-matrix.

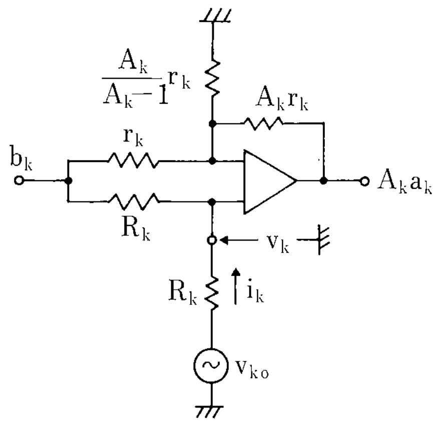

Fig. 5 Gain enhanced conversion circuit.

from a_k through N' to b_k in this case.

The realization of the k-th row of the specified S-matrix is illustrated in Fig. 4. Since S_{ki}'s in Fig. 4 are transfer functions with one input and one output, they can be easily realized by the aid of the conventional techniques.

When S_{ki}'s are real, they are realized as simple amplifiers or attenuators. The amplification can be performed by the slightly modified conversion circuit as shown in Fig. 5. In this circuit, the incident voltage wave is amplified by the gain of A_k. Further modification enables the circuit to perform the weighted summation at the same time, i.e.,

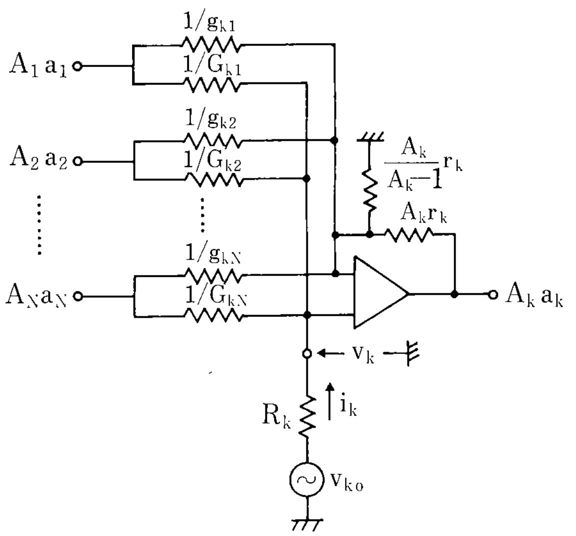

Fig. 6 Realization of the k-th row of the S-matrix with positive elements.

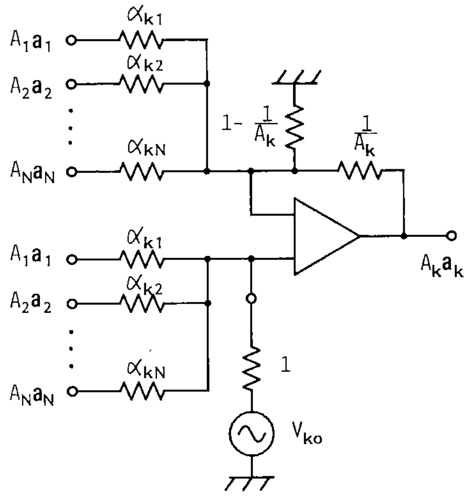

Fig. 7 Modification of Fig. 6.

some input terminals are made by splitting the resistors without changing their total admittance values. Figure 6 shows the resultant circuit realizing the k-th row of the S-matrix. In this figure, the following equations are satisfied.

$$\sum_i G_{ki} = \frac{1}{R_k} \quad , \tag{5a}$$

$$\sum_i g_{ki} = \frac{1}{r_k} \quad , \tag{5b}$$

and

$$R_k G_{ki} = r_k g_{ki} \ . \tag{5c}$$

Then b_k is calculated as

$$b_k = R_k \sum_i G_{ki} A_i a_i \ . \tag{6}$$

Therefore the values of G_{ki}'s and A_i's are chosen such that

$$R_k G_{ki} A_i = S_{ki} \tag{7}$$

is satisfied for every k and i.

When S_{ki} is less than unity, the corresponding attenuation is performed by making G_{ki} small in the same circuit.

The operation of this circuit is illustrated as follows. From Eqs. (5), α_{ki} is defined as

$$\alpha_{ki} = R_k G_{ki} = r_k g_{ki} \ , \tag{8}$$

and

$$\sum_i \alpha_{ki} = 1 \ . \tag{9}$$

When all the resistors connected to the lower input terminal of the operational amplifier are normalized by R_k and the other resistors are normalized by r_k, the circuit in Fig. 6 can be redrawn as shown in Fig. 7, where the resistors are represented in the dimension of conductance. In this figure, the matching condition is satisfied at the port and the signal of the source v_{k0} is amplified to produce the output $A_k a_k$, while this output of the operational amplifier is independent of the inputs $A_i a_i$'s. Therefore this circuit performs the directional coupling.

On the one hand, in matched networks, the following equations must be met.

$$S_{ii} = 0 \tag{10a}$$

$$b_i = S_{ij} a_j \tag{10b}$$

It is obvious that arbitrarily specified S_{ij}'s can be easily realized with the use of directional couplers. Thus the circuit shown in Fig. 6 is found to be quite suitable for the realization of S-matrices.

3. Effects of Finite Amplifier Gain-Bandwidth Product

It is well known that the non-ideality of operational amplifiers affect the performance of the networks employing them. The main defect of actual IC operational amplifiers is the finite gain-bandwidth product and it is analyzed in the basic circuit of Fig. 3.

Let A be the open-loop gain of the operational amplifier, then it can be shown that the relationship of Eq. (1) is modified as

$$a_k = \frac{A}{2+A}(v_k + R_k i_k) \tag{11a}$$

$$b_k = v_k - R_k i_k \tag{11b}$$

In case of internally compensated operational amplifiers, A is characterized by the one pole rolloff model as

$$A = \frac{GB}{s+\omega_c} \cong \frac{GB}{s} \tag{12}$$

where GB represents the gain-bandwidth product, and the cutoff angular frequency ω_c is usually negligible compared with the signal frequency.

Substituting Eq. (12) into Eq. (11a) yields

$$a_k = \frac{\dfrac{GB}{2}}{s + \dfrac{GB}{2}} (v_k + R_k i_k) \tag{13}$$

Equation (11b) indicates that the finite GB has no effects on the reflected wave, while Eq. (13) shows that the first-order lowpass filter having its cutoff frequency at $GB/2$ is inserted in the path of the incident wave. This effect is usually negligible because $GB/2$ is large enough.

From another viewpoint, the signal a_k can be regarded as the output of the circuit having two inputs, i.e. b_k and v_{k0}. Then it can be shown that it is independent of b_k even if GB is finite, i.e., a_k is always separated from b_k. Therefore any sets of S_{ij} 's will never produce a loop, and the overall network is always stable provided that S_{ij} is stable.

4. Realization of Networks Characterized by Specific S-Matrices

4.1 One-Port

One-port networks are the simplest case, in that S -matrix has only one element as

$$b = Sa \tag{14}$$

where a, b and S are scalar values. We assume S is a real constant. When $-1<S<1$, the circuit can be realized using only one resistor. $S=1$ and $S=-1$ represent open and short circuit respectively, therefore no components are necessary for their realization. $S>1$ and $S<-1$ correspond to negative resistance circuits. It is expected that the signs of S correspond to the two groups of actual negative resistance circuits, i.e. short-circuit-stable and open-circuit-stable ones. In fact, for example, a short-circuit-stable negative resistance must be driven by a source with an internal resistance less than the absolute value of the negative resistance in order to assure the system stability. This implies S is greater than 1. Similarly when an open-circuit-stable negative resistance is working stably, S is less than -1.

A short-circuit-stable negative resistance circuit, S of which is greater than 1, can be easily realized by simply connecting the two terminals on both sides of the circuit in Fig. 5. The gain A is equated to the value of S. The circuit is shown in Fig. 8(a), and the one simplified by combining the parallel resistors is shown in Fig. 8(b), which is the well-known NIC (Negative Immittance Converter) terminated by a resistor. Input resistance R_{in} is calculated as

$$R_{in} = -\frac{S+1}{S-1} R \tag{15}$$

The polarity of the operational amplifier is determined so that the circuit is stable for $R<|R_{in}|$.

In case that S is less than -1, the procedure requires an extra operational amplifier for inverting the output signal of the circuit in Fig. 5. However, simple reversing the sign of S in Fig. 8 completes the required circuit. Though one of the resistors in the intermediate circuit becomes negative, the final circuit does not contain negative elements.

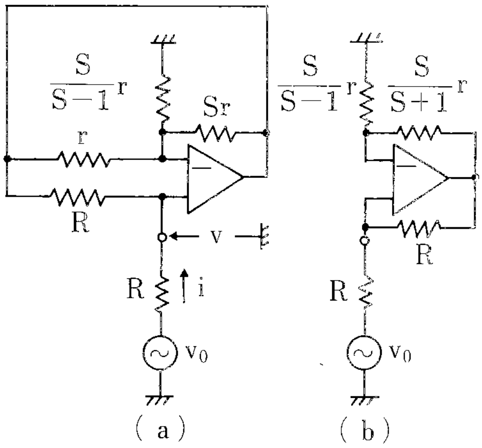

Fig. 8 Short-circuit-stable negative resistance.

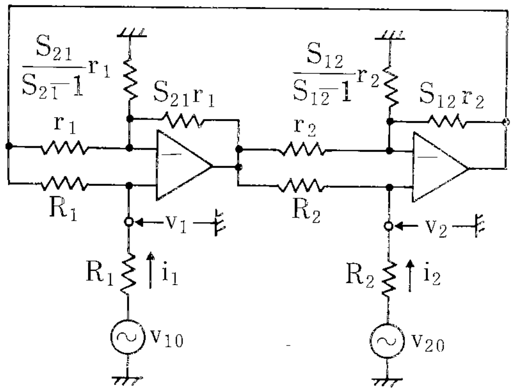

Fig. 9 Matched bidirectional amplifier.

4.2 Two-Port

In case of multi-port networks, matched circuits are practically important. Since there is no reflection at every port of a matched network, the diagonal elements of the S -matrix are zero.

The S-matrix of a matched two-port circuit is given by

$$(S) = \begin{bmatrix} 0 & S_{12} \\ S_{21} & 0 \end{bmatrix} \tag{16}$$

When both S_{12} and S_{21} are real and greater than 1, the circuit is referred to as a bidirectional amplifier and the circuit can be easily realized as shown in Fig. 9 by connecting two of the circuit shown in Fig. 5.

When S_{12} and S_{21} are not constant but have frequency characteristics, the circuit shown in Fig. 10 is obtained directly from the structure of Fig. 4. In Fig. 10, S_{12} and

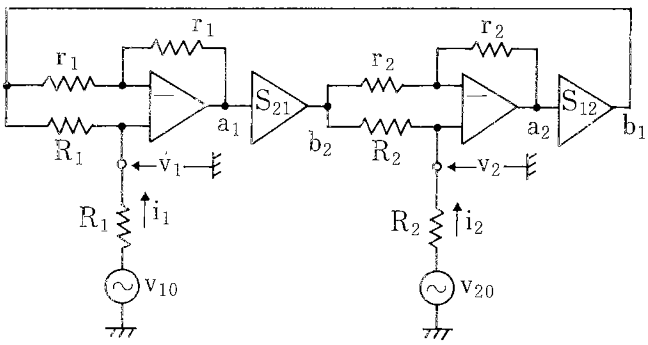

Fig. 10 Generalized bidirectional amplifier.

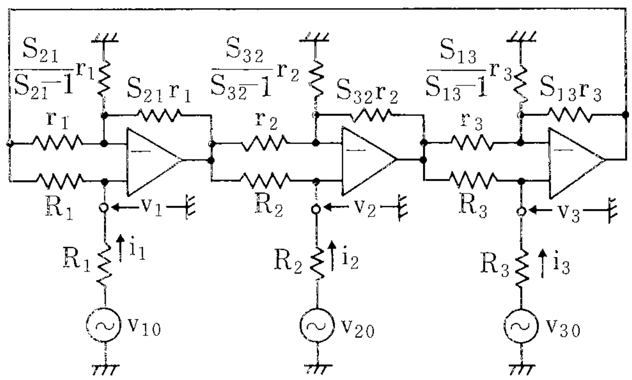

Fig. 11 Matched three-port circulator.

S_{21} represent the filter circuits satisfying given specifications. Though this circuit is exactly the same as the results of Ref. (5), the design procedure is much simpler here.

4.3 Three-Port

Among matched three-port circuits having positive S -parameters, circulators and hybrid networks are useful in transmission systems.

4.3.1 Circulator

The S -matrix of a matched three-port circulator is given by

$$(S) = \begin{bmatrix} 0 & 0 & S_{13} \\ S_{21} & 0 & 0 \\ 0 & S_{32} & 0 \end{bmatrix} \tag{17}$$

When all S_{ij} 's are greater than 1, the circuit can be realized in the similar way to the case of the bidirectional amplifier. The resultant structure is illustrated in Fig. 11.

When one or some of the S -parameters, say S_{km} is less than 1, then the circuit is modified at the k-th port to reduce the magnitude of the reflected wave using the technique suggested in chapter 2.

4.3.2 Hybrid Network

Hybrid networks are widely used at the junction points of bidirectional and unidirectional transmission systems such as telephone terminals. A hybrid coil shown in Fig. 12 is a well-known example of hybrid networks. In Fig. 12, the unidirectional systems are shown by T (transmitter) and R (receiver). Z_b is the impedance of the balancing network, which is designed to be equal to Z_a , the input impedance of the bidirectional line. Under the condition of balance, the four-port hybrid coil behaves as a three-port and signal of T is not transmitted to R. Since coils prevent integration of the systems, active realization of hybrid networks is desirable.

The S-matrix of the balanced hybrid coil is expressed by

$$(S) = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & 0 \\ S_{31} & 0 & 0 \end{bmatrix} \tag{18}$$

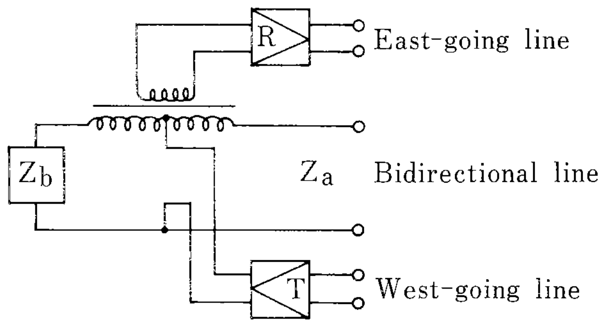

Fig. 12 Hybrid coil.

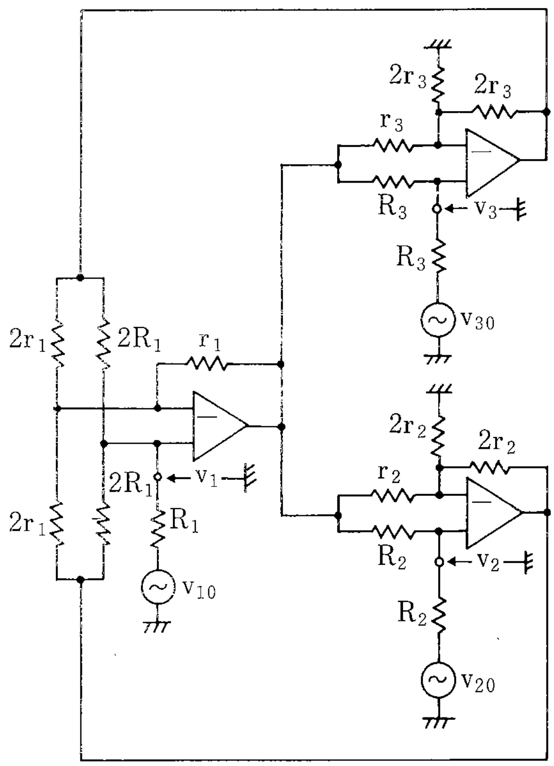

Fig. 13 Hybrid network.

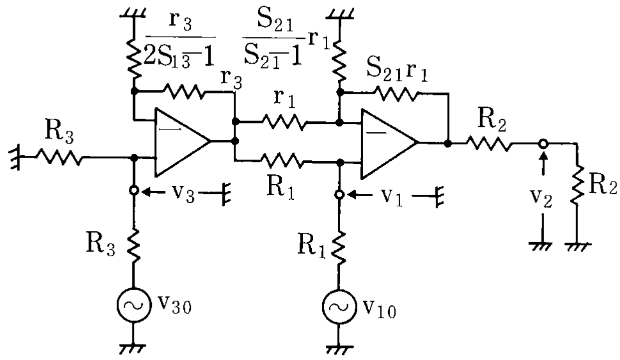

Fig. 14 Simplified hybrid network.

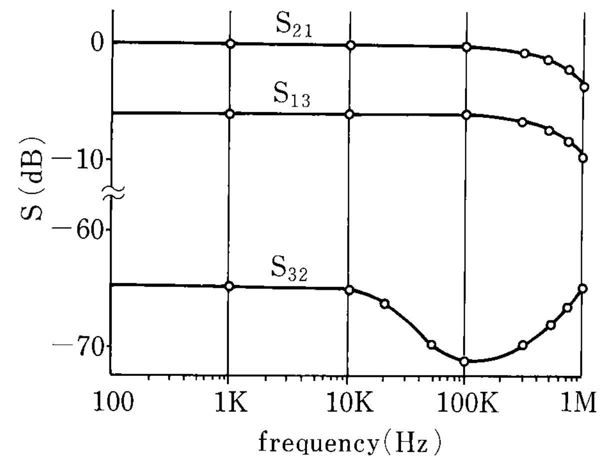

Fig. 15 Measured frequency characteristics of the hybrid network.

Usually the values of S_{ij}'s may be chosen to unity. When the line impedance is resistive, the hybrid network is realized as shown in Fig. 13 by connecting the basic circuits of Figs. 5 and 6. This circuit is superior to that of Ref. (6) in the number of resistors and the spread in their velues. In addition, the design procedure is much simpler.

Since the circuit in Fig. 13 is designed by simulating a hybrid coil, all of three ports have bidirectional characteristics, which are not necessary for the ports to be connected to unidirectional systems. Removing the unnecessary elements, a simplified S-matrix is introduced as

$$(S) = \begin{bmatrix} 0 & 0 & S_{13} \\ S_{21} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{19}$$

Comparing Eqs. (18) and (19), a simple circuit reduced by removing unnecessary parts can be obtained as shown in Fig. 14. In this circuit, the left operational amplifier works as a mere buffer amplifier, and the essential part as a hybrid network is the right half section which is just the same as the circuit in Fig. 5. In other words, the circuits in Fig. 5 and Fig. 3 are found to have the property of hybrid networks.

As an example, selecting $S_{21}=1$ and $S_{13}=0.5$, the circuit of Fig. 14 is constructed using μA741 operational amplifiers and resistors with 0.1% tolerance. The measured frequency characteristics are shown in Fig. 15. In this figure S_{32} represents the crosstalk which is designed to be zero ($-\infty$ dB). All the characteristics are favorable in the frequency range of 0—1 MHz.

5. Conclusion

A simple and systematic procedure for realizing multi-port networks specified by S-matrices is proposed. Converting the voltage and the current into the incident and reflected wave quantities at each port, the S-matrix is directly realized. The method is especially suited for the realization of sparse S-matrices. The application of the proposed technique yields some known circuits and new hybrid networks. The results of this paper also give a physical interpretation of the circuits derived.

All the circuits in this paper are designed under the ideal matching condition. The problems of mismatching are left to be studied.

Acknowledgement

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