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Frequency-Domain Simulator of Digital Networks from the Structural Description

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SUMMARY A user-friendly simulator PANDA (Program for the Analysis of Networks with Digital Arithmetic) is introduced, which can analyze the frequency responses, coefficient sensitivities and roundoff noise of given linear shift-invariant digital networks from their structural descriptions.

1. Introduction

Linear shift-invariant digital networks play important roles in various applications, and it is often necessary to evaluate their frequency responses (magnitude, phase and group-delay) and finite wordlength effects. There are many excellent structures of digital networks such as wave digital filters[1], which are less sensitive to their coefficients and have low parasitic oscillation. Instead of their marvelous characteristics, they often have very complicated structures, and it is tiresome work to evaluate their frequency responses.

In this letter, a simulator PANDA (Program for the Analysis of Networks with Digital Arithmetic)[2] is introduced, which analyze 1-dimensional linear shift-invariant networks in the frequency domain. PANDA has user friendly input description; only connections of network nodes are described in arbitrary order. Therefore PANDA is especially useful when the given networks have complicated structures.

2. Analysis in Frequency-Domain

2.1 Frequency Response

When a given linear digital network is represented in a Signal-Flow-Graph (SFG) notation, its incident matrix $A(z)$ can be easily obtained. For a network of N nodes, the node signal vector $\boldsymbol{X}(z)=\{x_k(z),\ k=1, 2, \cdots, N\}$ satisfies

$$\boldsymbol{A}(z)\boldsymbol{X}(z)+\boldsymbol{U}(z)=0, \qquad (1)$$

where $\boldsymbol{U}(z)=\{u_k(z),\ k=1, 2, \cdots, N\}$ is the input vector. By solving Eq. (1) for $\boldsymbol{X}(z)$, any node value $x_k(z)$ is expressed in terms of the system input signal $u_i(z)$. The choice $u_i(z)=1$ leads to the transfer function $H(z)$ from the input to a certain node of the given network. Then its frequency response is given by

$$H(e^{j\omega})=H(z)|_{z=e^{j\omega T}}. \qquad (2)$$

In PANDA, Eq. (1) is numerically solved for each frequency ω by substituting $z=e^{j\omega T}$, where LU-decomposition method is employed with the sparse matrix techniques. Then magnitude $|H(e^{j\omega})|$ and phase arg $(H(e^{j\omega}))$ are evaluated, and furthermore group-delay is calculated by the numerical differentiation of phase.

2.2 Sensitivity Analysis

The variations of the frequency response due to coefficient quantization is measured by the sensitivity. The derivative of a given transfer function $H(z)$ with respect to a multiplier of coefficient m, which is connected from node p to q, is given by[3],[4]

$$\frac{\partial H}{\partial m}=H_{ip}H_{qo}, \qquad (3)$$

where H_{ip} and H_{qo} are the transfer functions from the system input to node p, and from node q to the system output, respectively, which are easily calculated by the method indicated in the last subsection. Using $H(e^{j\omega})=|H|e^{j\theta}$, the left side of Eq. (3) is calculated as

$$\frac{\partial H}{\partial m}=\frac{H}{|H|}\frac{\partial|H|}{\partial m}+jH\frac{\partial\theta}{\partial m}. \qquad (4)$$

By taking real and imaginary part of Eq. (4), the sensitivity of the magnitude and phase are obtained respectively. There are several definitions of sensitivities, and in PANDA, absolute, relative and semi-relative sensitivities are provided, which differ by a factor of m and/or $|H|$. Among these definitions, appropriate sensitivity has to be chosen in dependence on the measure of the frequency response deviation (absolute or relative variation), and on the number representation in the system (fixed-point or floating-point representation).

The total effect of the coefficient quantization is measured by some way of summation about the sensitivities for all multipliers in the network. In PANDA, they are defined as follows:

- sensitivity in the worst case

$$S^H_{\text{all}}=\sum_{\text{all mult.}k}|S^H_{m_k}|$$

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• Sensitivity in the worst case assuming the track of identical coefficients

$$S^H_{\text{all}}=\sum_{\text{Distinct mult.}k}\left|\sum_{(m_k=m_j)} S^H_{m_j}\right|$$

• sensitivity in the statistic sense

$$S^H_{\text{all}}=\sum_{\text{all mult.}k}|S^H_{m_k}|^2$$

In the second definition, identical coefficients are assumed to remain the same after quantization.

2.3 Roundoff Noise Analysis

After each multiplication, the product has longer wordlength than its inputs, and has to be rounded afterward. This roundoff operation can be modeled by an equivalent white noise source of power $\sigma_e^2=\Delta^2/12$(Δ : quantization step size). The noise power at the output introduced by the multiplier k is given as

$$\sigma_k^2=\frac{\sigma_e^2}{2\pi j}\oint H_{ko}(z)H_{ko}(z^{-1})\frac{dz}{z}$$

$$=2\sigma_e^2\int_0^{\pi}|H_{ko}(e^{j\omega})|^2 d\omega \qquad (5)$$

where $H_{ko}(z)$ is the transfer function from multiplier k to the system output node o. The value of $H_{ko}(e^{j\omega})$ is available at every frequency point ω, as indicated in Sect. 2. 1. The above equation is evaluated by numerical integration. Noise sources are assumed to be mutually uncorrelated, and then the total average power of output noise is the sum of roundoff noise power from each multiplier as

$$\sigma^2=\sum_{\text{all mult.}k}\sigma_k^2 \qquad (6)$$

The signal levels at all nodes in a network are obtained by the analysis of Sect. 2. 1, which can be used to scale the node values to prevent overflow. Signal-to-noise ratio is then evaluated.

3. PANDA and Its Simulation Example

Here, input description of PANDA is explained, and

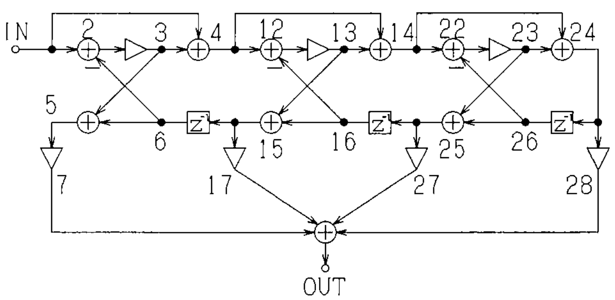

Fig. 1 Third-order gray-markel lattice filter.

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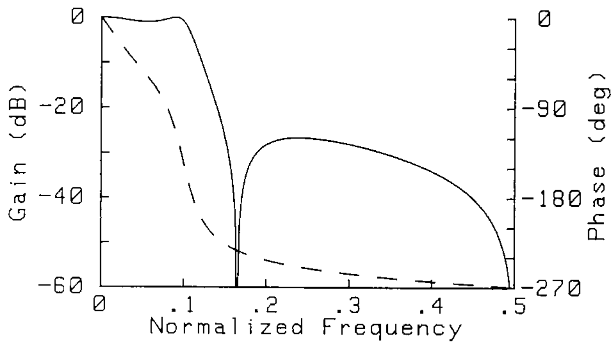
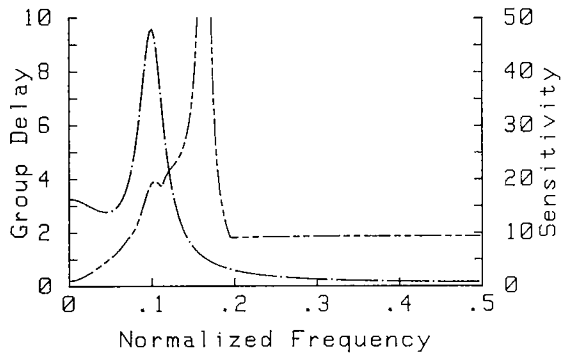
TITLE 3rd Order Gray-Markel
1=IN
2= 1 - 6
3= 2 * (-0.543528517)
4= 3 + 1
5= 6 + 3
6= DLY(15)
7= 5 * (0.0563191111)
11= 4
12= 11 - 16
13= 12 * (0.8888331116)
14= 13 + 11
15= 16 + 13
16= DLY(25)
17= 15 * (0.2606599722)
21= 14
22= 21 - 26
23= 22 * (-0.8715286686)
24= 23 + 21
25= 26 + 23
26= DLY(24)
27= 25 * (0.1095842127)
28= 24 * (0.573436699)
99= 7+17+27+28
OUT=99
FS=1
FREQ 0 0.5 POINT 200
GAIN(DB) PHASE(DEG) GDELAY
SENS (GAIN,RELATIVE) ALL(WORST)
PLOT GAIN(-40,0) PHASE GDELAY ALL
TERM
END
```

(a)



(b)



(c)

——Magnitude, ·········Phase,
- - - - Group delay, and -------Sensitivity

Fig. 2 Example of analysis by PANDA.
(a) Input description for Fig.1.
(b),(c) Frequency responses of Fig.1.

a simple example is shown. To describe a given network, put distinct number to every node, and then, write the relation of nodes for individual elements, such as adders, multipliers and delays. The description order is arbitrary and there is no need to consider the actual computational orders between elements. Furthermore, users may employ the language DIMPL (Digital networks IMPlementation Language)[5] to describe the network more effectively using subcircuit expressions. In this case the same description is used for the DIMPL complier to obtain assembly codes for DSPs (Digital Signal Processors) automatically.

For example, a lattice filter[6] as depicted in Fig.1, is described as shown in Fig.2(a). Figures 2(b) and(c) show the analysis results, which are, in this case, the frequency response of magnitude, phase, group delay and sensitivity.

The simulator PANDA written in FORTRAN77 and PASCAL (Turbo Pascal), can run on engineering workstations and personal computers, respectively. The time required for the analysis is short enough and is approximately proportional to the number of nodes/elements involved in the network. For the above example (the number of nodes and elements are 24 and 21, respectively), it takes about 9 seconds on NWS1750 (SONY) and about 100 seconds on PC9801VX21 (NEC).

4. Conclusion

This letter introduces a convenient simulator PANDA, which can easily analyze frequency response, sensitivity, and roundoff noise of a given network from the structural description. Extension of PANDA for the analysis of multi-rate systems and multi-dimensional systems are now investigated.

Acknowledgement

The first version of PANDA was written by Mr. Akira Haga when he was a student at Tokyo Institute of Technology. The authors wish to thank him for his pioneering work.

References

(1) A. Fettweis : "Wave digital filters : Theory and practice", Proc. IEEE, **74**, 2, pp.270-327(Feb. 1986).

(2) A. Haga : "Computer-aided analysis of digital networks", B. E. dissertation, Department of Electrical and Electronic Engineering, Tokyo Institute of Technology (Feb. 1984).

(3) A. V. Oppenheim and R. W. Schafer : "Digital Signal Processing", Prentice-Hall (1975).

(4) R. E. Crochiere and A. V. Oppenheim : "Analysis of linear digital networks", Proc. IEEE, **63**, 4, pp. 581-595(April 1975).

(5) N. Sugino, S. Ohbi and A. Nishihara : "Computational ordering of digital networks under pipeline constraints and its application to DSP compilers", Trans. IEICE, **E72**, 12, pp.1299-1306 (Dec. 1989).

(6) A. H. Gray and J. D. Markel : "Digital lattice and ladder filter synthesis", IEEE Trans. Audio Electroacoust., **AU-21**, pp.491-500(Dec. 1973).