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A Study of Priors and Algorithms for Signal Recovery by Convex Optimization Techniques

Thesis Outline

In this study, we propose novel design and use of priors and develop new algorithms for signal recovery, in the framework of convex optimization. As the diversity of the data and applications in signal processing having grown rapidly, exhaustive use of *signal-specific* a priori knowledge has been strongly demanded for satisfactory signal recovery. On the other hand, many existing priors are not *fully designed* to capture such information. In particular, most convex priors for digital images rely on their smoothness or sparsity in transformed domain, both of which only fit global structure of digital images. This implies that local details, i.e., *textures*, cannot be recovered in principle by the power of these priors. For multi-channel (e.g., color) images, *inter-channel correlation* has also not fully exploited, resulting in undesired appearance of artifacts in restored images. To resolve the above issues, we need to design priors that suitably model these image-specific features.

The said growth also suggests that we require efficient algorithms for solving convex optimization problems that are *intractable* by state-of-the-art algorithms based on the so-called proximity operator, The notion of the proximity operator is a generalization of the well-known metric projection, so that convex constraints that these algorithms can handle are *naturally limited to* the intersection of a finite number of closed convex sets onto which the metric projections are computable. On the other hand, quite a few constraints that are useful for expressing valuable candidates of desired signals actually *do not* have computationally-efficient closed-form expressions of the metric projections. Thus developing first-order algorithms that well deal with such constraints would be of great important subject.

Before proceeding to the main contents, we provide mathematical preliminaries necessary for the discussions in this study in Chapter 2, where we introduce minimal elements in convex analysis and fixed point theory and explain several algorithms. Then, Chapter 3–5 are devoted to providing novel image-specific priors mainly for high quality image restoration, by exploiting underlying physical properties of digital images that have not been fully utilized in conventional studies, see below for details.

- Chapter 3: We will develop an image restoration framework based on decomposition and component-wise use of priors. Specifically, we model an image to be estimated as the sum of three meaningful components, namely, smooth, edge, and texture components, and with use of the total variation prior and two different frame transforms, we design three convex priors each of which promotes specific properties of the corresponding component. We then formulate a nonsmooth convex optimization problem involving the priors over a Cartesian product space of the components, where restored components are characterized as its solution. Finally, the problem is reformulated by variable splitting, which leads to an efficient algorithmic solution to the problem via the so-called alternating direction method of multipliers (ADMM). Experimental results illustrate that the proposed method well restores the three meaningful components and is superior to a state-of-the-art method.

- Chapter 4: We will propose a novel convex prior suitable for repetitive textures, with its rigorous theoretical properties and applications to the extraction and restoration of textures. The characterization rests on our observation that the texture component enjoys a blockwise low-rank nature with possible overlap and shear, because texture in general is globally dissimilar but locally well-patterned. More specifically, one can observe that any local block of the texture component consists of only a few individual patterns. Based on this premise, we first introduce a new convex prior, named the block nuclear norm (BNN), leading to a suitable characterization of the texture component. We also propose a cartoon-texture decomposition model as a convex optimization problem, where the simultaneous estimation of the cartoon and texture components from a given image or degraded observation is executed by minimizing the TV prior and BNN. A noteworthy property of the model is that patterns of texture extending in different directions are extracted separately, which is of benefit to texture analysis and other applications. The model can handle various types of degradation occurring in image processing, including blur+missing pixels with several types of noise. By rewriting the problem via variable splitting, ADMM becomes applicable, resulting in an efficient algorithmic solution to the problem. Numerical examples illustrate that the proposed model is very selective to patterns of texture, which makes it produce better results than state-of-the-art decomposition models.

- Chapter 5: We are interested in removing color artifacts in color image restoration, where we will newly introduce two convex priors that exploit inter-channel dependency of color images for the said purpose. One is named the local color nuclear norm (LCNN), which is designed to promote a property inherent in natural color images - in which their local color distributions often exhibit strong linearity - and is thus expected to reduce color artifact effectively. The other is a novel multi-channel version of the TV prior, named the decorrelated vectorial total variation (D-VTV) prior, which measures the discrete gradients of the luminance component and that of the chrominance one in a separated manner, leading to a significant reduction of color artifact. The very nature of both LCNN and the D-VTV prior allows us to incorporate them into various types of color image restoration formulations, with the associated convex optimization problems solvable using proximal splitting algorithms. Experimental results illustrate their utility.

In Chapter 6 and 7, we go back to general signal recovery settings and propose two types of first-order algorithms both relying on a fixed point optimization technique called *hybrid steepest descent method* to deal with useful convex constraints onto which the metric projections are unavailable. Details are as follows.

- Chapter 6: We will propose a hierarchical convex optimization algorithm that can automatically select a desired solution among all the solutions of a given convex optimization problem (referred to as the first stage problem), by leveraging the fixed point set characterization behind a primal-dual splitting (PDS) type method. Hierarchical convex optimization, i.e., minimizing another convex function over the solution set of the first stage problem, is an ideal strategy when the first stage problem has infinitely-many solutions because of the non-strict convexity of its objective function, which could arise in various scenarios, e.g., convex feasibility problems. In this chapter, first, the fixed point set characterization behind the PDS method is incorporated into the framework of hierarchical convex optimization, which enables the framework to cover a broad class of first stage problem formulations. Then, a pair of efficient algorithmic solutions to the hierarchical convex optimization problem, as nontrivial realizations of hybrid steepest descent method, are provided with guaranteed convergence. We also present a specialized form of the proposed framework to focus on a typical scenario of inverse problems in signal recovery, and show its application to signal interpolation based on a TV minimization.

- Chapter 7: We will establish a computationally-efficient algorithm for solving convex optimization problems with sophisticated data-fidelity constraints. Imposing data-fidelity constraints directly on optimization would often facilitate the choice of the involved parameters, because the parameters are closely related to some statistical information on noise contamination. However, some of these constraints have no closed-form expressions of the metric projections, implying that proximal splitting algorithms cannot accept them. To resolve this dilemma, we activate certain quasi-nonexpansive operators to characterize sophisticated data-fidelity constraints as their fixed point sets. Thanks to this characterization, we can mobilize the so-called hybrid steepest descent method to solve convex optimization problems with such a constraint. Different from existing methods, the proposed method accepts an extensive class of convex data-fidelity constraints in a unified manner, without using any geometric approximation to the constraints. Moreover, the proposed method has the following merits: (i) it allows various designs of convex objective functions and other constraints; and (ii) it requires no computationally-expensive procedure such as operator inversion and inner loop. As applications of the proposed method, we provide image restoration under non-Gaussian noise contamination with illustrative examples.

Finally, in Chapter 8, we summarize the results obtained in this dissertation and give an outlook on future research.