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論 文 要 旨

THESIS SUMMARY

専攻： Department of	数学 専攻	申請学位（専攻分野）： Academic Degree Requested	博士 Doctor of	（ 理学 ）
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要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

This thesis is three folds, which are related with two topics; nanophrase and Milnor ¥bar{μ}-invariant. First of all, we introduce these two topics and then describe a content in each chapter.

A word is a sequence of symbols, called letters, belonging to a given set, called an alphabet. Turaev developed a theory of words based on the analogy with curves on the plane, knots in the 3-sphere, virtual knots, etc. A Gauss word is a sequence of letters such that any letter appears exactly twice. It is natural to introduce combinatorial moves on Gauss words, based on Reidemeister moves on knot diagrams. The equivalence relation generated by these moves is called homotopy. We then decorate each letter with information from the diagram. This leads us to the notion of nanoword as introduced by Turaev. By introducing combinatorial moves on nanowords, the notion of homotopy can be defined for them also. From this viewpoint, homotopy of Gauss words is the simplest kind of nanoword homotopy. The theory of nanowords can be naturally generalized to the theory of nanophrases just like knot theory does to link theory.

For an ordered, oriented link in the 3-sphere, Milnor defined a family of invariants, known as Milnor ¥bar{μ}-invariants. For an n-component link L, Milnor invariant is specified by a sequence I of numbers in {1,2,⋯,n} of finite length. He defined these invariants as an extension of linking numbers. In fact, Milnor invariants with length two sequence are just linking numbers. If the sequence has no number appearing more than once, then this invariant is also link-homotopy invariant and is called Milnor link-homotopy invariant. Here, the link-homotopy is an equivalence relation generated by self-crossing changes and ambient isotopies.

We now would like to give a brief description for the following three chapters.

In chapter 2, we discuss finite type invariants for some equivalence classes of nanophrases. Vassiliev developed the theory of finite type invariants of knots, which is conjectured to classify knots. Ito defined a notion of finite type invariants for curves on surfaces, constructed a large family of finite type invariants, and showed that they become a complete invariant for stably homeomorphism classes of curves. On the other hand, Fujiwara provided a simple idea to define finite type invariants for cyclic equivalence classes of signed words by introducing a new type of crossing, called singular crossing, which plays intermediate role between an actual and virtual crossing. Here the signed word is a nanoword over α={+,-} and it is known that the set of cyclic equivalence classes of signed words bijectively corresponds to the set of stably homeomorphism classes of curves on surfaces. We extend Fujiwara's finite type invariants, to those for cyclic equivalence classes of nanophrases over a general α. To see this, we define finite type invariants for cyclic equivalence classes of nanophrases and construct the universal ones, by following Goussarov, Polyak and Viro's approach. In addition, we identify the universal finite type invariant of degree 1 essentially with the linking matrix.

In chapter 3, we develop a weaker homotopy theory of nanophrases, called M-homotopy, which is an analogue of link-homotopy. We introduce a self-crossing move on nanophrases and the associated M-homotopy allowing self-crossings. The main result establishes an M-homotopy invariant of nanophrases corresponding to virtual links as an extension of Milnor's ¥bar{μ}-invariants.

In chapter 4, we discuss a relation between Milnor's ¥bar{μ}-invariants and HOMFLYPT polynomials. There are several known results about relation between Milnor invariants of (string) links and the Alexander polynomial. On the other hand, Polyak gave a formula expressing the Milnor invariant of length 3 by Conway polynomial, and Meilhan and Yasuhara generalized it. More precisely, they showed that any Milnor invariant of length between 3 and 2k+1 can be represented as a combination of HOMFLYPT polynomial of knots obtained by certain band sum of the link components, if all Milnor invariants of length ≦k vanish. Here the HOMFLYPT polynomial is a two variable polynomial for link characterized by two relations for link diagrams. Their assumption that a link has vanishing Milnor invariants of length ≦k is essential to compute Milnor invariants of length up to 2k+1 via their formula. In fact, their formula does not hold for length 2k+2. We improve their formula to give the Milnor invariants of length 2k+2 by adding correction terms. The correction terms can be given by a combination of HOMFLYPT polynomial of knots determined by Milnor invariants of length k+1. In particular, our result gives a closed formula for any 4-components link the Milnor invariants of length 4 without any assumption.

備考 ： 論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。