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論文 / 著書情報
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種別(和文)	論文要旨
Type(English)	Summary

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（博士課程）
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論 文 要 旨

THESIS SUMMARY

専攻： Department of	数理・計算科学　　専攻	申請学位（専攻分野）： Academic Degree Requested	博士 Doctor of　（　理学　）
学生氏名： Student's Name	正井　秀俊	指導教員（主）： Academic Advisor(main)	小島　定吉
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要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

A hyperbolic 3-manifold is a Riemannian manifold of constant sectional curvature -1. We mainly consider hyperbolic 3-manifolds of finite volume. By the Mostow-Prasad rigidity theorem, two such manifolds are isometric if and only if they are homeomorphic. Hence we may study hyperbolic geometry via topology and vice versa. Further, thanks to the Mostow-Prasad rigidity, computers can be very useful tools for the study of hyperbolic 3-manifolds. In this thesis, we study hyperbolic 3-manifolds both theoretically and computationally.

In the theoretical part, we study hyperbolic volumes, and fibered commensurability. We first discuss a volume preserving move which relates hyperbolic graphs and links. By using this move, we give a method to compute the best possible upper bounds for volumes of hyperbolic link complements in terms of the twist numbers. We compute approximate values of the best possible upper bounds for the case where twist numbers are less than 10 by using the computer programs called Orb and plantri. Next, we discuss the number $N(v)$ of hyperbolic 3-manifolds of volume v. By a work of Jørgensen and Thurston, the number $N(v)$ is known to be finite for all $v \in \mathbb{R}$. We first investigated censuses on SnapPea and computed (approximate) volumes. This experiments suggest that i) $\sup_{w \leq v} N(w)$ grows exponentially as v goes to infinity, and ii) there are infinitely many v with $N(v) = 1$. We show that these are actually the case. As the final work about hyperbolic volumes, we compare two formulae by Bridgeman-Kahn and Calegari that compute the volume of a given compact hyperbolic n-manifold M with totally geodesic boundary in terms of *orthospectrum*. The orthospectrum of M is the set of length of orthogeodesics, geodesics perpendicular to the boundary ∂M at both ends. The two formulae are derived by means of different decompositions of the unit tangent bundle. Since both formulae hold for every such hyperbolic manifold, it is natural to ask if their formulae coincide. For $n = 2$, Calegari showed that they are identical. We verify that their formulae coincide for any $n \geq 2$. Both Bridgeman-Kahn and Calegari showed that their formulae can be expressed by certain integrals which are complicated. We derived a simple expression of their formulae for the case where $n = 3$; for a piece associated to an orthospectrum of length l, it will be $\frac{2\pi(l+1)}{e^{2l}-1}$. We then consider fibered commensurability of fibrations on a hyperbolic 3-manifold. The notion of fibered commensurability is introduced by Calegari-Sun-Wang. We first give a necessary condition for manifolds to have non-symmetric but commensurable fibrations. Then, we also show that for any $n \in \mathbb{N}$, there exists a hyperbolic 3-manifolds with at least n mutually non-symmetric but commensurable fibrations.

We now turn to a computational part of this thesis. We first developed the package *hikmot* that enables us to prove the hyperbolicity of a given triangulated 3-manifold. Since, to prove the hyperbolicity, we deal with equations with complex variables, so-called gluing equations, the notion of *interval arithmetic* plays a key role. Note that computers can only deal with approximate values. Instead of dealing with approximate values, in interval arithmetic, we compute intervals that contain rigorous values. Thus we can prove inequalities. To solve a gluing equation, we use Newton's method. There are several theorems that give sufficient conditions, which are inequalities, for the convergence of Newton's method. Putting those theorems and interval arithmetic together, we can prove that there is a solution for a gluing equation and hence the hyperbolicity of a manifold. By using hikmot we prove hyperbolicity of all manifolds in several censuses on SnapPy. Further, we use hikmot together with a python code based on the codes by Martelli-Petronio-Roukema to classify all the exceptional surgeries along alternating knots. Thurston has proved that for a given hyperbolic knot, all but finitely many surgeries give hyperbolic manifolds. We call those finitely many surgeries exceptional. The codes by Martelli-Petronio-Roukema enumerate exceptional surgeries by using floating point arithmetic. We modified their codes so that each computation is done by interval arithmetic. Thanks to several known works, to classify the exceptional surgeries along alternating knots, it suffices to prove that about 30000 links with a condition on the slopes for certain components do not admit exceptional surgeries. By applying our codes to those links, we see that none of such links admit any unexpected exceptional surgeries. The computational cost is quite expensive and not durable for standard personal computers. Hence we used TSUBAME, the supercomputer of Tokyo Tech. Thus with a big aid of computer, we have a classification of the exceptional surgeries along the alternating knots.

備考 ： 論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。

Note : Thesis Summary should be submitted in either a copy of 2000 Japanese Characters and 300 Words (English) or 1copy of 800 Words (English).