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On the cost of misperceived travel time variability

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Abstract

The existence of an individual's misperception of a travel time distribution implies that using the implied reduced form of the scheduling model might fall short of capturing all costs of travel time variability. We reformulate a general scheduling model employing rank-dependent utility theory and derive two special cases as econometric specifications to study these uncaptured costs. It is found that reduced-form expected cost functions still have a mean-variance form when misperception is considered, but the value of travel time variability is higher. We estimate these two models with stated-preference data and calculate the empirical cost of misperception. We find that (i) travelers are mostly pessimistic and thus tend to choose departure times too early to achieve a minimum cost, (ii) scheduling preferences elicited using a stated-choice method can be relatively biased if probability weighting is not considered, and (iii) the extra cost of misperceiving the travel time distribution might be nontrivial when time is valued differently over the time of day and is substantial for some people.

Keywords: travel time variability, schedule delay, departure time choice, rank-dependent utility

1. Introduction

The concept of the value of travel time has been well established in the long history of economics (Becker, 1965; DeSerpa, 1971). The value accounts for a significant share of the social benefit of infrastructure investments and the social cost of traffic congestion. However, travelers are confronting increasingly uncertain travel times because pervasive congestion makes the trip duration more sensitive to non-recurrent variations (e.g., unexpected incidents). This uncertainty leads to additional scheduling costs and psychological anxiety for users, making travel time variability (unreliability) as costly as mean travel time. Thus, policy makers have been gradually shifting their focus to how travel time variability should be valued and how to provide a reliable level of service in road networks.

A behaviorally consistent and pragmatic approach for analyzing the value of travel time variability (VTTV) is a central question. In a substantial body of research, the mean-variance model (Brownstone and Small, 2005; Small et al., 2005) and the scheduling model (Small, 1982; Noland and Small, 1995) are two mainstream methods. The former is the only viable option for cost–benefit analysis relevant to travel time reliability because its results are directly associated with statistical measures of variability (e.g., the standard deviation and inter-quantile range). However, it is a black box model, where the microeconomic foundation of how travel time variability incurs a scheduling cost is hidden. In contrast, the scheduling model is micro-founded, whereby the stochastic travel time unavoidably makes a traveler arrive early or late relative to his/her preferred arrival time and thus causes disutility. Nonetheless, its formulation stands on

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the individual's perspective, making it unsuitable for appraisal purposes. A desirable solution combining the advantages of both approaches is to first estimate an individual's scheduling preferences and to then convert them to the VTTV.

This solution requires the mean-variance model to be a reduced form of the scheduling model. Noland and Small (1995) and Bates et al. (2001) show that this condition holds when assuming (i) the travel time is exponentially or uniformly distributed, (ii) there is no change in recurrent delay, (iii) there is no discrete late-arrival penalty, and (iv) travelers maximize expected utility. Fosgerau and Karlström (2010) further generalizes this result to any distribution, so long as its standardized distribution is independent of the departure time. Studies (Fosgerau and Engelson, 2011; Engelson and Fosgerau, 2011) adopt other assumptions on scheduling preferences and obtain reduced forms corresponding to variability measures that have better mathematical properties in practice. However, important discrepancies between the reduced-form scheduling model and its ad-hoc counterpart are found in some empirical studies (Börjesson et al., 2012), indicating that the former does not capture all disutility of the travel time variability.

We conjecture that the use of expected utility theory (EUT) contributes these discrepancies. In particular, the independence axiom in EUT is likely to fail when being applied to trip-timing decisions, as was the case in experiments of Allais (1953). For example, people may underweight the occurrence probability of an extremely long travel time if they believe it results from an accident and is less likely to be of concern on a daily basis. Thus, we argue that it is necessary to verify whether using EUT in a scheduling context is viable before taking advantage of its mathematical convenience. Otherwise, two types of errors are likely to occur.

The first type of error is *misspecification.* The estimation of the scheduling model relies mainly on data from stated-preference experiments, where risk is generally taken as one of the design attributes (e.g., occurrence frequency of a given travel time). If respondents do not act consistently with EUT, letting the expected value enter as a proxy for certainty equivalent is likely a misspecification (De Palma et al., 2008) and might bias the model estimates. The second type of error is *undervaluation.* Deriving a reduced-form model under EUT would ignore the cost of probability misperception; i.e., the additional cost relating to a traveler's subjectively optimized departure time cannot perfectly minimize the objective travel cost. Bates et al. (2001) mention the cost of misperception (see Figure 1) but give no analytical result. Essentially, the cost of misperception depends on how much the perceived travel time distribution deviates from the objective distribution and the convexity of the scheduling disutility function. We refer to these two errors as type i and type ii errors, respectively. A mixed effect of these errors may exist in many relevant empirical studies conducted to date.

One generalization of EUT for accommodating the above behavioral anomalies and capturing the cost of probability misperception is using rank dependence[1]; i.e., an individual processes the objective probability to decision weight non-linearly according to his/her preference for the given outcome[2]. Koster and Verhoef (2012) formulate a rank-dependent scheduling model and show that the cost of probability weighting accounts for 0%–24% of the total travel cost, using a series of values of weighting parameters. Hensher and Li (2012) estimate a rank-dependent model but implicitly assume that the marginal cost of time equals that of a scheduling delay. Wang et al. (2012) estimate weighting parameters in a scheduling context, but do not analyze the cost of probability weighting. There are studies that attempt to capture the travel time

[1] Other sources of misperception are possible but are expected to be minor under well-organized experimental conditions.

[2] From a normative point of view, this approach is preferable than cumulative prospect theory because an individual's reference point is hardly measurable and is subject to change.

perception error and its effect on travel behavior. Carrion and Levinson (2012a) and Xu et al. (2013) propose $T_s = T_o + \varepsilon|T_o$, where subjective travel time T_s is the sum of objective time T_o and a conditional perception error $\varepsilon|T_o$. Carrion and Levinson (2012a) measure $\varepsilon|T_o$ by comparing Global Positioning System trajectories and reported travel times. However, whether the reported travel time is a good proxy for the subjective travel time, which travelers use for scheduling, remains doubtful because it is subject to unpredictable accidents and perhaps a reporter's strategic behavior. Peer et al. (2013) provide further evidence on the lack of connection between the reported travel time and travel time perception. Thus, we seek to provide an alternative explanation for perception error.

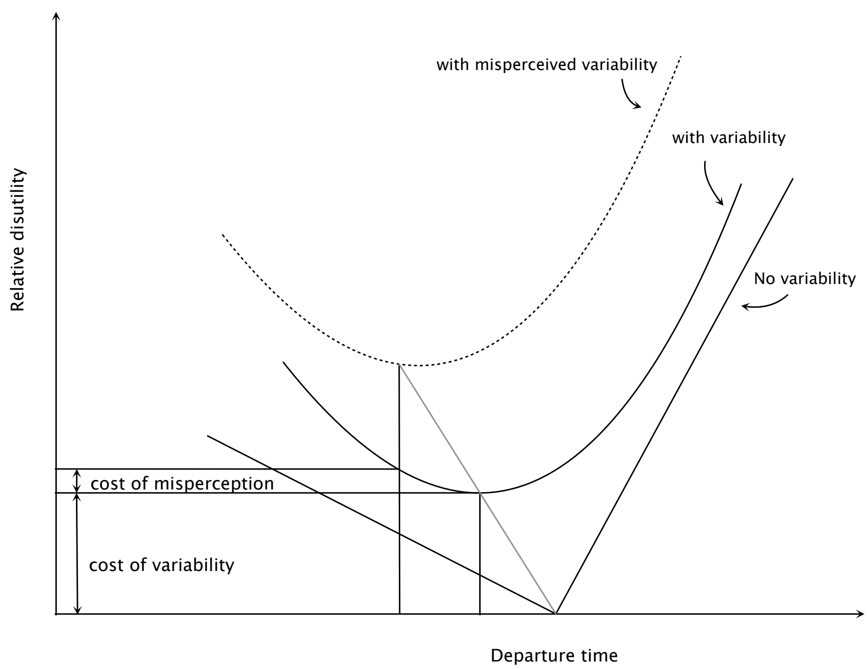

Figure 1: Additional cost due to misperception of variability (Bates et al., 2001)

To summarize, the objectives of this paper are (i) to see if a pragmatic reduced-form cost function (i.e., one linear to the mean travel time and its variability measure) still exists when probability misperception is considered, (ii) to empirically estimate a rank-dependent scheduling model and measure the aforementioned two types of errors, and (iii) to examine whether the cost of misperception is sizable enough to be considered in a cost–benefit analysis and its contribution to the discrepancy as found in Börjesson et al. (2012).

The remainder of this paper is organized as follows. In section 2, we first define the misperception from a behavioral economics point of view. We then reformulate a general scheduling model (Vickrey, 1973) with rank-dependent utility (Quiggin, 1982), analyze its properties, and derive reduced forms in two special cases. Section 3 presents details of the stated-preference (SP) experimental design and data description. Section 4 specifies the empirical model for estimating our data and the measurement of the two types of errors. Section 5 discusses the model estimation result and its implications and section 6 concludes the paper.

2. Theoretical framework

2.1. Definition of misperception

Denote the set of all possible travel times by $S = \{s_1, ..., s_n\}$, and let its power set 2^S be the set of events (where an event corresponds to a subset of all possible travel times). There then exists a non-additive probability measure v (Schmeidler, 1989) on 2^S, such that $v(\emptyset) = 0$ and $v(S) = 1$, and the preferences can be reflected by a utility function u and a probability measure v independently (e.g., $\int u \cdot \mathrm{d}v$ while $\mathrm{d}v$ and u are independent). v is interpreted as subjective probability; i.e., the number a person uses to calculate the expectation of a random variable. On the other hand, we have an objective probability measure ρ. It is not correct to take $v(e) = \rho(e)$ for all $e \in 2^S$ for granted. Thus, the general definition of misperception is essentially the discrepancy between $v(e)$ and $\rho(e)$. Assuming probability-sophisticated individuals, we can find a unique non-decreasing distortion (i.e., probability weighting) function W to decompose[3] $v(\cdot)$ to $W(P(\cdot))$, where P is an additive probability measure on 2^S (see Wakker, 2010, for review). We thus argue that for a probability-sophisticated individual, misperception is a product of two effects: incorrect belief (from ρ to P) and probability weighting (from P to v). The former is about how a person believes an event's occurrence departs from reality, which can be perfected in a Bayesian manner as experience increases. The latter is relevant to how agents behaviorally transform probabilities to decision weights. For example, if in a laboratory experiment, the decision-maker behaves as if the event will occur with a 20% probability when told explicitly that the occurrence probability is 10%, probability weighting likely plays a role. However, in field data, when the decision-maker acts as if the occurrence probability is 20%, while the researcher assesses it as 10%, both effects might exist. Note that with incorrect beliefs, providing information about the travel time distribution would likely improve decision making, whereas probability weighting would not be affected by the provision of information. Despite the difference, the two situations incur extra scheduling costs similarly. However, we will focus on probability weighting because commuters should have established beliefs approximating objective probabilities, and our empirical data are taken from an SP experiment with explicit probabilities.

2.2. Rank dependence

The first-order stochastic dominance of two acts must be preserved to ensure the existence of the distortion function W. This further implies that rank dependence must be a derived property of W[4]. Rank-dependent probability weighting has been employed to address the Allais (1953) paradox. Its behavioral interpretation is that an individual does not perceive objective probabilities as they are, but rather transforms them into decision weights based on the ranked position (relative goodness or badness) of their corresponding outcomes (Quiggin, 1982). The shapes of the weighting function, as shown in Figure 2, represent an individual's risk attitude: in the case of worsening ranked outcomes, (i) a convex W reflects pessimism, because the probability of a good outcome is always underestimated while that of the bad outcome is overestimated; (ii) a concave W reflects optimism because a good outcome is overestimated; (iii) an inverse S-shaped W means that the individual focuses too much on the extreme outcomes (e.g., short and long travel times) and is insensitive to intermediate outcomes, which implies that the subjective standard deviation is large; and (iv) an S-shaped W represents a kind of regression to the perception of a single value for the travel time (possibly unequal to

[3] Such a decomposition rules out ambiguity aversion as discussed in Wakker (1990).

[4] One example that does not satisfy this condition is prospect theory (Kahneman and Tversky, 1979).

the mean travel time), which in general leads to a lower perceived standard deviation. Next, we introduce rank dependence into a general scheduling model.

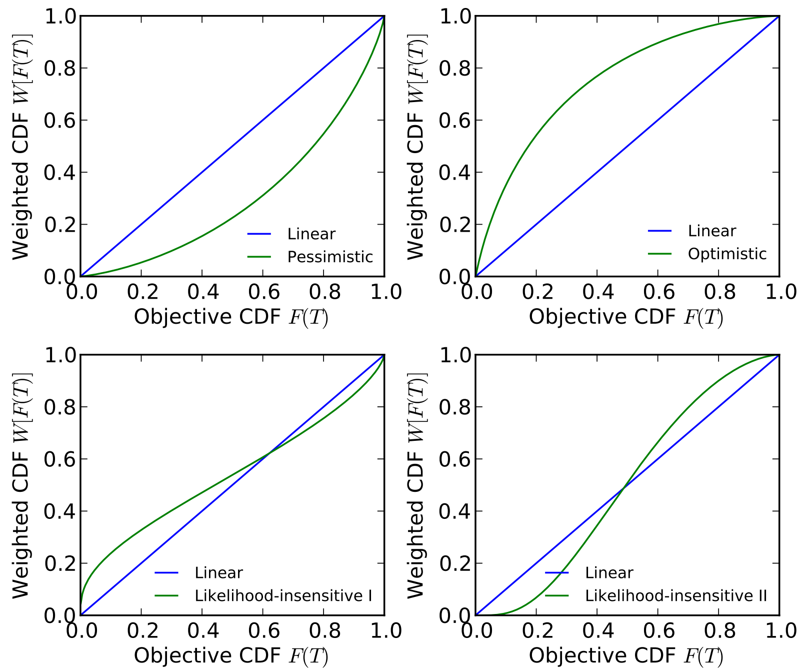

Figure 2: Shapes of the probability weighting function for worsening ranked positions

2.3. General scheduling preferences

The model builds on the general scheduling preferences as studied by Vickrey (1973) and Tseng and Verhoef (2008) and Fosgerau and Engelson (2011). Table A.9 summarizes symbols used in the remainder of this section. We consider a daily commuter who needs to travel from home to work. She/he wants to maximize the utility of conducting activities at both ends. The in-vehicle time is assumed to be completely unproductive, and the marginal utilities of time spent at home and at work relative to travel are denoted $h(t)$ and $w(t)$, respectively. $h(t)$ and $w(t)$ vary by the time of day and intersect at a^*. This intersection is interpreted as the ideal arrival time because people would be best off arriving at a^*, if travel is instantaneous. We present a graphical demonstration in Figure 3.

Assumption 1. *$h(t)$ is non-increasing, $w(t)$ is non-decreasing, $h(t) > w(t)$ for all $t < a^*$ and $h(t) < w(t)$ for all $t > a^*$.*

Assumption 1 is needed to ensure the existence of an optimal departure time and some properties shown afterwards. It is mild because it would otherwise be behaviorally implausible (e.g., there would be no need to travel if h is increasing while w is decreasing). The inequalities imply that if $h(t)$ or $w(t)$ is constant, the other should be a step function with a jump at $t = a^*$.

Assumption 2. *The travel time $T = \mu + \sigma X$ is stochastic, and its standardized random variable X is assumed to be independent of the departure time, has a bounded support $[\underline{x}, \overline{x}]$ and a distribution function G.*

In reality, there are also time-dependent parts of travel time, namely $T = \mu + \tilde{\mu}(t) + (\sigma + \tilde{\sigma}(t))X$. Therefore, this is a relatively strong assumption, stating that the mean travel time and the standard deviation change sufficiently slowly with regard to departure time, $\tilde{\mu}'(t) \approx 0$ and $\tilde{\sigma}'(t) \approx 0$. [5] The boundedness of X ensures its inverse function, G^{-1}, has a finite value. For any realization of T, the utility derived from an arbitrary time interval $[t_h, t_w]$ is a function of departure time d, given by

$$U(d;T) = \int_{t_h}^{d} h(t)\mathrm{d}t + \int_{d+T}^{t_w} w(t)\mathrm{d}t,$$

where the first term is utility derived from home and the second term is utility derived from work. Let the utility derived from instantaneous travel time, $U(a^*;0)$, be the reference utility level, and normalizing $a^* = 0$ without loss of generality, we define a cost function as

$$C(d;T) = \int_{d}^{0} h(t)\mathrm{d}t + \int_{0}^{d+T} w(t)\mathrm{d}t. \tag{1}$$

such that $C(d;T) \equiv U(0;0) - U(d;T)$.

2.4. Optimal departure times and the cost of misperception

Given Assumption 1, it immediately follows that $C(d;T)$, as a sum of two convex functions, is necessarily convex in d for each realization of T. Then, by the fact that the expectation operator and rank-dependent expectation (RDE) operator work as a weighted average, it is straightforward that $\mathrm{E}[C(d;T)]$ and $\mathrm{RDE}[C(d;T)]$ are also convex in d. Thus, there exists a unique interior solution d^* that minimizes the expected travel cost, and also d^*_w that minimizes the rank-dependent-expected travel cost. For continuous $h(t)$[6], the first-order optimality condition for minimizing the expected cost, $\mathrm{E}[C(d;T)]$, is

$$h(d^*) = \mathrm{E}[w(d^* + T)] = \int_0^1 w(d^* + F^{-1}(s))\mathrm{d}s, \tag{2}$$ [7]

where d^* is the objectively optimal departure time. However, individuals minimize the rank-dependent expected utility, $\mathrm{RDE}[C(d;T)]$, which leads to a first-order optimality condition expressed as

$$h(d^*_w) = \mathrm{RDE}[w(d^*_w + T)] = \int_0^1 w(d^*_w + F^{-1}[W^{-1}(s)])\mathrm{d}s, \tag{3}$$

where W is a probability weighting function and d^*_w is the subjectively optimal (thereby suboptimal) departure time. Thus, the misperception brings an extra cost of choosing a suboptimal departure time, given by

$$\Delta = \mathrm{E}[C(d^*_w;T)] - \mathrm{E}[C(d^*;T)] \geq 0. \tag{4}$$

Δ is necessarily not less than zero, because the traveler can do no better than realize the minimum expected cost. Equations (1) to (3) imply that the size of Δ depends on $h(t), w(t), F$, and W.

[5] It is empirically demonstrated that Assumption 2 remains a good approximation by Fosgerau and Karlström (2010) and Fosgerau and Fukuda (2012), where actual traffic data are well fitted by a stable distribution $G(X)$.

[6] $w(t)$ is not required to be continuous because it can be smoothed by the expectation operator and RDE operator.

[7] $\mathrm{d}F(T)$ is substituted by $\mathrm{d}s$ for clarity, and F^{-1} is thus essentially a quantile function.

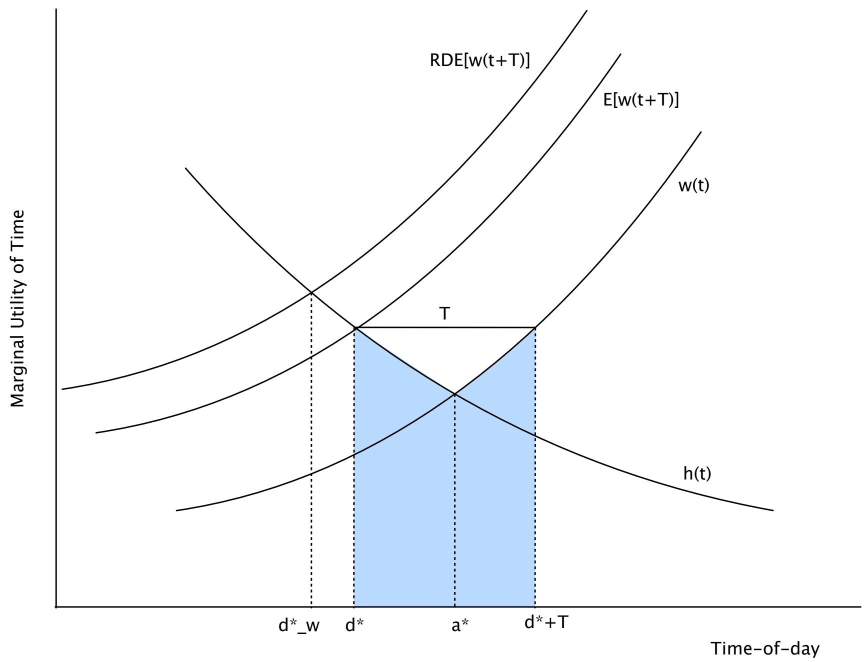

Figure 3: General scheduling preferences

2.5. Comparative statics

It remains to investigate under what condition(s) a traveler would depart earlier/later than the optimal departure time and which situation is more costly. We find that the order of stochastic dominance (Hadar and Russell, 1969) helps in answering such questions (all proofs are provided in Appendix B).

Property 1. *For any $F(T)$ that first-order stochastically dominates $W[F(T)]$ (i.e., $F(T) \geq W[F(T)]$ for every T), $d_w^* \leq d^*$ holds.*

The effects of convex and concave W as in Figure 2 can be explained by this property. It is clear that a travel time distribution dominates its pessimistically weighted counterpart (convex W), and pessimism thus indicates earlier-than-optimal departure and optimism (concave W) conversely indicates later-than-optimal departure. In particular, d_w^* is equal to d^* if $w(t)$ is a constant because, in this case, the departure time is always the intersection of $w(t)$ and $h(t)$ regardless of an individual's probability perception. The effects of S-shaped and inverse S-shaped W are ambiguous because they do not satisfy Property 1. Nonetheless, when $w(t)$ is linearly increasing, performing an integration by parts once more shows that the mean of $F(T)$ being larger than the mean of $W[F(T)]$ is sufficient for earlier-than-optimal departure. However, the effects become ambiguous again if W is a mean-preserving transformation.

Property 2. *If W is a mean-preserving transformation and $w(t)$ is convex, for any $F(T)$ that second-order stochastically dominates $W[F(T)]$ (i.e., $\int_T^\infty F(s)\mathrm{d}s \geq \int_T^\infty W[F(s)]ds$ for every T), $d_w^* \leq d^*$ holds.*

This property provides a way to check the effects of S-shaped and inverse S-shaped weighting functions on the departure time. Intuitively, if a probability weighting function preserves the means but fattens the tails of given distributions (i.e., increases variance), it is likely for there to be a larger perceived loss if $w(t)$ increases more quickly than linearly. A mean-preserving S-shaped W will never cause $W[F(T)]$ to be dominated by $F(T)$, and travelers thus always choose later-than-optimal departure. It remains to investigate in which direction subjective optimal departure time shifts are more costly.

Property 3. *If $h(t)$ and $w(t)$ are twice differentiable and* $\mathrm{E}[w''(d^*+T])>h''(d^*)$*, then marginally late departure is more costly; otherwise, marginally early departure is more costly.*

This situation occurs if $w(t)$ is strictly convex and $h(t)$ is concave. In such a case, a pessimist will be better off than a comparable optimist. As preliminaries of the econometric specification, we demonstrate two special cases of the general scheduling model as follows.

2.6. Special case 1: piece-wise constant marginal utility

This parameterization is proposed by Small (1982). We refer to it as step model because $w(t)$ is a step function; i.e., $h(t)=\alpha$ and $w(t)=\alpha-\beta+(\beta+\gamma)\cdot 1\{d+T\geq 0\}$, where $1\{\cdot\}$ is an indicator function. The travel cost then becomes

$$C(d;T)=\alpha T+\beta\max(-(d+T),0)+\gamma\max(d+T,0), \tag{5}$$

where $\max(-(d+T),0)$ is schedule delay early (SDE) and $\max(d+T,0)$ is schedule delay late (SDL). The subjective optimal departure time (see Appendix C.1 for the derivation) is then given by

$$d^*_w=-\mu-\sigma G^{-1}\left[W^{-1}(\frac{\gamma}{\beta+\gamma})\right]. \tag{6}$$

Substituting this expression into Eq. (5) and applying an expectation operator yields the expected cost for a traveler who departs at a subjectively optimal departure time:

$$\mathrm{E}[C(d^*_w;T)]=\alpha\mu+\sigma\left([(\beta+\gamma)W^{-1}\left(\frac{\gamma}{\beta+\gamma}\right)-\gamma]G^{-1}\left[W^{-1}\left(\frac{\gamma}{\beta+\gamma}\right)\right]+(\beta+\gamma)\int_{W^{-1}(\frac{\gamma}{\beta+\gamma})}^{1}G^{-1}(s)\mathrm{d}s\right), \tag{7}$$

which is linear in terms of the mean and standard deviation of the travel time. The equation shows that the value of the mean travel time (VMTT), α, is the same as in the standard scheduling model, whereas the VTTV is not. The cost of misperception is given by

$$\Delta=\sigma\left([(\beta+\gamma)W^{-1}\left(\frac{\gamma}{\beta+\gamma}\right)-\gamma]G^{-1}\left[W^{-1}(\frac{\gamma}{\beta+\gamma})\right]-(\beta+\gamma)\int_{\frac{\gamma}{\beta+\gamma}}^{W^{-1}(\frac{\gamma}{\beta+\gamma})}G^{-1}(s)\mathrm{d}s\right). \tag{8}$$

If the traveler has a correct perception (i.e., $W^{-1}(\frac{\gamma}{\beta+\gamma})=\frac{\gamma}{\beta+\gamma}$), the right-hand side of the above equation becomes zero.

2.7. Special case 2: time-dependent linear marginal utility

We refer to the second special case as the slope model because the marginal utility of time is specified as a linear function of the time of day. Equation (1) can be rewritten as

$$C(d;T)=\int_d^0(\beta_0+\beta_1 t)\mathrm{d}t+\int_0^{d+T}(\gamma_0+\gamma_1 t)\mathrm{d}t. \tag{9}$$

Without loss of generality, we normalize the intersection of $h(t)$ and $w(t)$ to be zero; i.e., $\gamma_0=\beta_0$. Applying the RDE operator and first-order condition (see Appendix C.2) yields

$$d^*_w=-\frac{\gamma_1}{\gamma_1-\beta_1}\mu_w, \tag{10}$$

where μ_w is the weighted mean travel time and $\mu_w = \int_0^1 F^{-1}[W^{-1}(s)]\mathrm{d}s$. The equation implies that the subjectively optimal departure time does not depend on travel time variability. The suboptimal travel cost due to misperception is

$$\mathrm{E}[C(d_w^*;T)] = \frac{\gamma_1^2}{2(\gamma_1-\beta_1)}\mu_w^2 + \frac{1}{2}\gamma_1\mu^2 + \frac{1}{2}\gamma_1\sigma^2 + \mu\left(\gamma_0 - \frac{\gamma_1^2}{\gamma_1-\beta_1}\mu_w\right). \tag{11}$$

The extra expected cost given a shift of d_w^* relative to d^* is symmetric because of the quadratic term in Eq. (12), making marginally early and late departures equally costly:

$$\Delta = \frac{\gamma_1^2}{2(\gamma_1-\beta_1)}(\mu-\mu_w)^2. \tag{12}$$

The equation also indicates that, if a traveler has an unbiased perception of the mean travel time, the cost of misperception is zero because the traveler coincidentally chooses the same departure time as in the standard scheduling model.

The derivation of the VTTV is not as straightforward as that of the step model, because μ_w in Eq. (11) could be a function of σ. We show their relationship as follows. Because of W, X is perceived as X', which is subject to a new distribution $W[G(X')]$. Given that X' does not necessarily have zero mean and unity standard deviation, we normalize it such that $X' = \mu_\Delta + \sigma_\Delta Y$, where Y is the new normalized random variable, μ_Δ is the location shift and σ_Δ is the change of scale. It immediately follows that the perceived travel time $T_w = \mu + \sigma X' = \mu + \sigma\mu_\Delta + \sigma\sigma_\Delta Y$. This shows that T_w has mean $\mu_w = \mu + \sigma\mu_\Delta$ and standard deviation $\sigma_w = \sigma\sigma_\Delta$. Substituting these variables into Eq. (11) and taking the derivative with respect to μ. Although Y is not subject to the distribution G, the VMTT and VTTV remain the same as if Y is subject to G, because they are shape irrelevant.

We provide an example with restricted assumptions such that the effects of the probability weighting function are tractable. Given Assumption 2, the travel time has a lower bound $\mu + \sigma\underline{\mathrm{x}}$ with $\underline{x} < 0$. Let $W[F(T)] = a + (1-a)F(T)$. Such a probability weighting function essentially puts the mass density a at the lower bound of T, and lowers the density elsewhere (extremely optimistic). Straightforwardly, we have $\mu_w = a(\underline{\mathrm{x}}+\mu)+(1-a)\mu = \mu + \sigma a\underline{\mathrm{x}}$ and $\frac{\partial\mu_w}{\partial a} = \sigma\underline{\mathrm{x}} < 0$. This means that a reduces the perceived mean travel time, and travelers thereby depart later than is optimal, which is consistent with Property 1. Furthermore, Eq. (12) becomes $\frac{\gamma_1^2}{2(\gamma_1-\beta_1)}(\sigma a\underline{\mathrm{x}})^2$, indicating that the cost of misperception quadratically increases with a.

2.8. Valuation

Supposing that utility is a money metric, VMTTs and VTTVs derived from $\mathrm{E}[C(d_w^*;T)]$ in the step and slope models are summarized in Table 1. We also provide the $\mathrm{E}[C(d^*;T)]$ counterparts (see Fosgerau and Karlström, 2010; Engelson and Fosgerau, 2011) for comparison. Table 1 shows that VTTVs in both step and slope models are affected by probability weighting, whereas VMTTs are not. Put alternatively, the cost of misperception rests solely on the cost of travel time variability. This implication is consistent with the empirical results of Börjesson et al. (2012), where VMTTs derived from the scheduling model are very close to VMTTs estimated from the mean-variance model, whereas VTTVs are very different.

3. Data

Data were collected from an Internet-based SP experiment conducted in 2010 in Japan, where the respondents were asked to put themselves in a day-to-day car-commuting route-choice scenario.

Table 1: Derived VMTTs and VTTVs

Measure	Step		Slope	
	$\mathrm{E}[C(d^*;T)]$	$\mathrm{E}[C(d^*_w;T)]$	$\mathrm{E}[C(d^*;T)]$	$\mathrm{E}[C(d^*_w;T)]$
μ	α	α	$\gamma_0 - \frac{\beta_1\gamma_1}{\gamma_1-\beta_1}\mu$	$\gamma_0 - \frac{\beta_1\gamma_1}{\gamma_1-\beta_1}\mu$
σ	$(\beta+\gamma)\int_{\frac{\gamma}{\beta+\gamma}}^{1} G^{-1}(s)\mathrm{d}s$	$[(\beta+\gamma)W^{-1}(\frac{\gamma}{\beta+\gamma})-\gamma]G^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})]$ $+(\beta+\gamma)\int_{W^{-1}(\frac{\gamma}{\beta+\gamma})}^{1} G^{-1}(s)\mathrm{d}s$	-	-
σ^2	-	-	$\frac{\gamma_1}{2}$	$\frac{\gamma_1}{2} + \frac{\gamma_1^2\mu_\Delta^2}{2(\gamma_1-\beta_1)}$

The experiment is constructed as follows. We first generate 1000 random draws $X = \{x_1, ..., x_{1000}\}$ based on a stable travel time distribution (see Figure 4 and Appendix E for details) calibrated with electronic toll collection data for the Tomei Express, an inter-city toll road connecting Tokyo and its southwest suburbs. Subsequently, we specify a 5^3 fractional factorial SP experimental design, with the attribute levels given in Table 2. Design attributes are the mean travel time μ, standard deviation σ, and optimal probability of lateness $\frac{\beta}{\beta+\gamma}$. The random travel time for each choice profile is generated according to $T = \mu + \sigma X$. Following Fosgerau and Karlström (2010), we calculate the optimal departure time without probability weighting as $d^* = -\mu - \sigma\int_{\frac{\gamma}{\beta+\gamma}}^{1} G^{-1}(s)\mathrm{d}s$ for each choice profile and count the frequencies of schedule delay within given intervals (e.g., $\#SDE_{0-10} = \#\{T_k : -10 \leq d^* + T_k \leq 0, k = 1, ..., 1000\}/10$). In this way, travel time variability is converted and presented as a histogram-like choice situation, as in Table 3. Each possible schedule delay interval corresponds to an occurrence frequency that the respondent is supposed to have experienced in the past 100 days. Frequencies instead of probabilities are presented to ensure a respondent with no knowledge of probability can understand the given information.

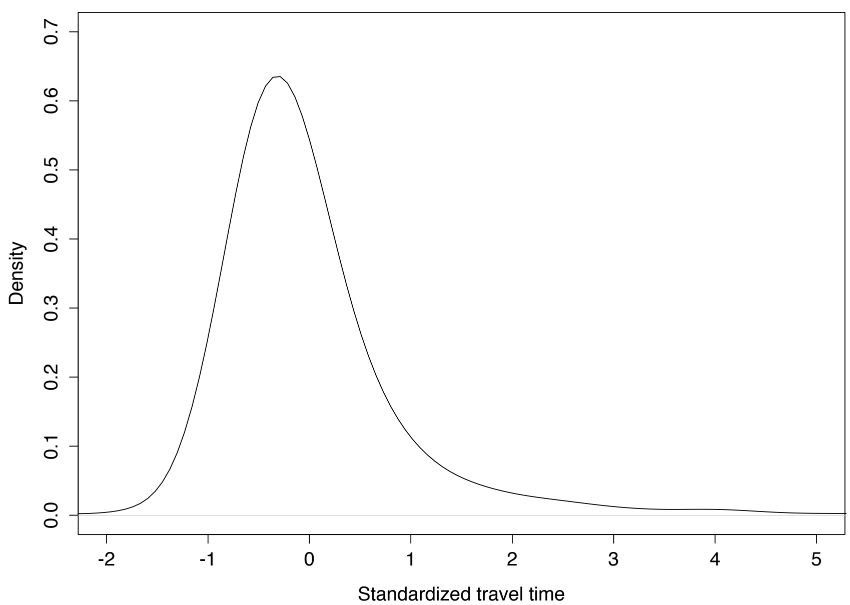

Figure 4: Stable distribution of travel time on the Tomei Express

One important point of the design needs to be clarified here. Although some scheduling-preference parameters (which construct the optimal probability of lateness) are used as design attributes and the departure time is decided by the interaction of design attributes rather than predefined levels, each alternative's departure time remains exogenous from the respondent's point of view. Additionally, the departure time for each alternative is claimed to be unchangeable (otherwise it is endogenous to respondents, making schedule delays presented in Table 3 inconsistent and hence the estimation of scheduling model unavailable). Thus, the essence, namely that the respondent is choosing a departure time given schedule constraints and stochastic travel times, still holds. This could also be viewed as a Bayesian approach where we first propose some prior belief of estimate values (i.e., β, γ), and the observations of the respondent's choice are used to update the posterior distribution.

Table 2: Attribute level setting in our SP experiment

Attributes	Level 1	Level 2	Level 3	Level 4	Level 5
Mean (min)	48	54	60	66	72
Standard Deviation (min)	8	10	12	14	16
Optimal probability of lateness ($\frac{\beta}{\beta+\gamma}$)	0.3	0.2	0.4	0.1	0.5

Table 3: Sample choice situation in our SP experiment

Route	Mean time in past 100 days	Frequency of arriving early and late in past 100 days: 20^{+} early	10-20 early	0-10 early	0-10 late	10-20 late	20-30 late	30-40 late	40^{+} late
A	60 min	0	1	72	21	4	1	1	0
B	72 min	0	3	77	14	3	1	1	0

However, ambiguity inevitably exists in this experiment. Outcomes are not specific values but only known to be bounded within intervals of probability. Identification would be a problem if we do not impose further assumptions. Thus, the travel time perceived by individuals is assumed to be distributed uniformly within these intervals, such that the average of the lower and upper bounds of a given interval can be regarded as a mass representing a specific outcome[8]. The travel time distribution is thus discretized. Despite a relatively strong distributional assumption, the discretization still makes sense because people may use histogram-shaped approximations rather than perfectly forming a travel time distribution in their mind (Tseng et al., 2009).

All the respondents are daily car commuters. After discarding samples with (i) missing data and (ii) answer times shorter than 20 min or longer than 45 min, we have 4176 observations remaining, provided by 232 respondents each of whom faced 18 choice scenarios. Descriptive statistics are summarized in Table 4.

[8] 25 min early and 45 min late were chosen arbitrarily to represent schedule delays beyond 20 min early and 40 min late because of the infinite upper bound. This arbitrary choice is not expected to affect the result greatly given the low frequencies of these extreme outcomes.

Table 4: Descriptive statistics

	Min	Median	Mean	Max
Age	20.00	45.00	43.44	60.00
Female dummy	0.00	0.00	0.39	1.00
Annual income (10^4 JPY)	50.00	600.00	587.10	1200.00
House ownership dummy	0.00	1.00	0.68	1.00
Household size dummy	1.00	3.00	2.90	8.00
Inflexible workday dummy	0.00	1.00	0.70	1.00
Commuting time (min)	1.00	20.00	26.08	110.00

4. Empirical specification

Given the dichotomous-choice data, we specify two discrete-choice models for the aforementioned special cases. The systematic utility functions[9] are given by

$$V = \begin{cases} \text{Step}: & \sum_{i=1}^{n} \pi_i(\alpha T_i + \beta SDE_i + \gamma SDL_i) \\ \text{Slope}: & \sum_{i=1}^{n} \pi_i[(\gamma_0 - \beta_0)d + \gamma_0 T_i + (\gamma_1 - \beta_1)d^2/2 + \gamma_1(T_i^2/2 + T_i d)] \end{cases} \tag{13}$$

where $\{(T_i, p_i)\}_{i=1}^{n}$ denotes n paired possible travel times with corresponding probabilities from a choice, such that $T_i > T_{i-1}$ (i.e., travel times are ranked from low to high), and $\pi_i = W(p_i + p_{i-1} + \cdots + p_1) - W(p_{i-1} + p_{i-2} + \cdots + p_1)$ is the decision weight put on each T_i. It is worth noting that $T_i > T_{i-1}$ does not necessarily imply $T_i \prec T_{i-1}$, and the implicit assumption here is thus that travelers always prefer less travel time; i.e., $\beta < \alpha$ in the step model and $\gamma_1 > \beta_1$ in the slope model. Behaviorally, it means that travelers prefer to terminate the trip when they arrive before the preferred arrival time, rather than continuing to detour. This assumption is supported by the vast majority of empirical estimates. Two popular probability weighting functions (Tversky–Kahneman (T-K) and Prelec functions) are tested:

$$W[p] = \begin{cases} \text{T-K}: & \frac{p^\theta}{(p^\theta + (1-p)^\theta)^{\frac{1}{\theta}}} \\ \text{Prelec}: & \exp(-\eta(-\ln p)^\theta) \end{cases} \tag{14}$$

The criterion for selecting a probability weighting function is whether it is flexible enough to reflect the aforementioned four types of curves. The Prelec (1998) weighting function meets this criterion because curvature (discriminality) is controlled by θ and elevation (attractiveness) is controlled by η. We use the T-K weighting function in a comparison, although it does not satisfy our criterion. The random utility for alternative j [10] is

$$U_j = V_j(\cdot) + \varepsilon_j. \tag{15}$$

The alternative-specific constant is not included because alternatives are unlabeled. Furthermore, we assume the error term ε_j is an independent and identical Gumbel distribution, so that a binary logit (BL) choice model applies. This model is estimated with a maximum log-likelihood estimator using Pythonbiogeme 2.2 (Bierlaire and Fetiarison, 2009). The bias of misspecification in the expected utility (EU)-based model and rank-dependent expected utility (RDEU)-based model can be detected readily by comparison.

[9] Note that β_0 and γ_0 are equalized to rule out the intercept terms, and thus clearly present the derived VTTV. However, this is not a good idea when we estimate the empirical model because it will constrain the crossing of h and w to be zero.

[10] Note that we economize the use of subscript j for what is supposed to be V_j, T_{ij}, and π_{ij} in Eq. (13).

5. Estimation results

5.1. *Piece-wise constant marginal utility (Step model)*

As a benchmark, we first estimate a scheduling model without probability weighting, namely $\pi_i = p_i$ in Eq. (13). According to the estimation results of the step model presented in Table 5, SDE affects the utility level weakly, as the SDE ratio is not notably different from zero. This implies that people are willing to arrive a lot earlier to save a negligible amount of travel time; i.e., they are very flexible on their workdays. This result contradicts the data, which shows 70% of subjects have inflexible workdays. Additionally, the SDL ratio is as high as 11.3. Although the model fit is not poor, such a high SDL ratio is somehow counter-intuitive and has not been reported in any study to date. These findings cast doubt on the validity of assuming that an individual is calculating objectively expected values when the experiment design is risk related. Given this, the scheduling model with probability weighting becomes a natural alternative. The model with a Prelec weighting function outperforms the others in terms of goodness of fit, whereas the T-K model is not as good, and the Akaike information criterion (AIC) implies an over-fitting issue. The SDE and SDL ratios in the Prelec model are 0.92 and 6.0, respectively. These numbers are reasonable, and we are not surprised that the SDL ratio is relatively high because we did not consider a lateness penalty.

Moreover, the two models have totally different patterns of probability weighting. θ in the T-K model is not notably different from 1, meaning that we cannot reject the null hypothesis that people are perfectly following EUT. If so, there is no need to consider probability weighting. A possible explanation is that T-K function only has one parameter, which keeps it from well fitting the true probability weighting curve. In this case, the Prelec model is more reliable, given its flexibility. The Prelec model has a skewed S-shape weighting function that is convex within [0, 0.92] and concave subsequently. In particular, probabilities lower than 0.45 are under-weighted as zero. This essentially indicates that people are very pessimistic and tend to choose an earlier departure time than they really need to. However, they also tend to undervalue the possibility of extremely bad outcomes, meaning that one who requires a very low risk of being late (less than 8%) tends to choose a departure time that makes him/her bear a slightly higher risk. It also indicates that travelers focus more on intermediate outcomes than on extremely good or bad outcomes. A possible reason for this is the effect of loss aversion; i.e., the intermediate outcomes are what delineate the boundaries of arriving early and late, and people care more about lateness. Nonetheless, we argue that the utility function has captured the effect of loss aversion by γ, and the probability weighting curve should thus be robust.

Table 5: Step model

	Standard		T-K		Prelec	
Parameter	Value	t-stats	Value	t-stats	Value	t-stats
Mean time (α)	-0.0982	-22.47	-0.0985	-22.53	-0.101	-22.46
Schedule delay early (β)	-0.0178	-1.23	-0.027	-1.65	-0.0922	-3.41
Schedule delay late (γ)	-1.11	-24.04	-1.158	-20.22	-0.607	-18.89
Theta (θ) [t-stats against 1]			1.03	1.05	1.98	5.7
Eta (η) [t-stats against 1]					10.5	3.43
No. of observation	4176		4176		4176	
Final log-likelihood	-1915.857		-1915.18		-1907.522	
Likelihood ratio ($\bar{\rho}^2$)	0.337		0.337		0.339	
Akaike information criterion (AIC)	3837.71		3838.36		3825.04	
SDE ratio	0.18		0.27		0.92	
SDL ratio	11.3		11.76		6.0	

5.2. *Time-dependent marginal utility (Slope model)*

Table 6 shows that the Prelec model again outperforms the standard and T-K models with respect to the goodness of fit for the slope-type specification, and this superiority is greater than that for the step model. The ideal arrival times (intersection of h and w) obtained as $\frac{\beta_0-\gamma_0}{\gamma_1-\beta_1}$ from these three models are negative and range from -9 to -31. This meets the setting in the slope model that the ideal arrival time should be earlier than the preferred arrival time, given non-zero T. In all models, we find that the marginal utility at home $h(t)$ decreases much more slowly than the marginal utility at work $w(t)$ increases. This implies that travelers prefer to depart earlier rather than arrive later to compensate for a change in travel time, which could be due to the tight workday of the population[11]. Put alternatively, as $h(t)$ becomes flatter, the preferred arrival time approaches the ideal arrival time, and travelers assign a larger increment of the travel time to their head start.

The estimated Prelec weighting curve (see Figure 5) has a pattern similar to that of its counterpart in the step model, reflecting a pessimistic attitude of almost the same magnitude. The decision weights put on extremely bad outcomes are not very different from being linear, meaning that travelers can perceive probabilities of these outcomes fairly well. The T-K weighting function has a completely convex curve, also indicating pessimism. These two estimated weighting functions are not mean-preserving transformations unless the travel time distribution has a very fat tail. We thus argue that they will result in an earlier-than-optimal departure. Because loss aversion is not controlled in the slope model, it is possible that the weighting parameters are contaminated. We rule out such a possibility by estimating a slope model with a kink in its $w(t)$ to accommodate the loss of late arrival (see Appendix D). No notable difference is found in the probability weighting curve.

In contrast to some literature, we find the step model generally has a better fit than the slope model. Possible reasons are that our sample contains a high fraction of people who have inflexible workdays. In such a case, we are not surprised that the step model (in which the preferred arrival time is fixed and the jump of w reflects the official work start time better than a continuous transition) has better descriptive power.

Table 6: Slope model

	Standard		T-K		Prelec	
Parameter	Value	t-stats	Value	t-stats	Value	t-stats
$\gamma_0 - \beta_0$	-0.52	-13.1	-0.496	-12.17	-0.308	-6.85
γ_0	-0.542	-27.6	-0.413	-15.57	-0.357	-26.52
$\gamma_1 - \beta_1$	-0.0324	-18.06	-0.0158	-3.18	-0.033	-11.3
γ_1	-0.0313	-16.3	-0.0129	-2.43	-0.0321	-9.84
θ [t-stats against 1]			2.71	5.52	2.29	11.94
η [t-stats against 1]					15.7	6.05
No. of observations	4176		4176		4176	
Final log-likelihood	-1957.869		-1944.492		-1914.151	
Likelihood ratio ($\bar{\rho}^2$)	0.322		0.327		0.337	
Akaike information criterion (AIC)	3923.74		3898.98		3840.30	

5.3. *Type i and type ii errors*

Because monetary cost is not included in our SP experiment, the magnitude of travel cost is measured as the equivalent mean travel time. Assuming the scheduling preferences estimated from both models with

[11] We try segmenting the population by flexibility of workday and find a steeper $w(t)$ for those who have inflexible workdays.

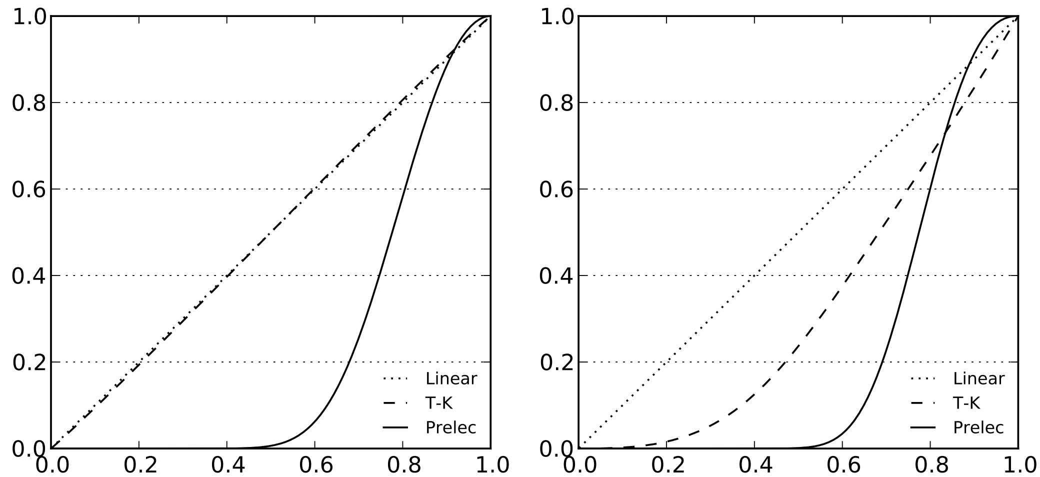

Figure 5: Objective probability (ranked travel time) versus decision weight, step model on the left and slope model on the right

the Prelec weighting function are the true preferences associated with their certainty equivalents, we can decompose the mixture of type i and type ii errors into two terms: (i) the difference between reliability ratios (RRs) calculated from estimates using models with and without probability weighting functions and (ii) the difference between RRs derived with and without probability weighting, using the estimates from the same model. Because VMTTs were previously shown to be irrelevant to probability weighting in both models, they will serve as the baseline; thus, RRs here actually reflect the levels of expected costs.

Although all estimates are supposed to be asymptotically normally distributed in our econometric setting, the distribution of the RR is unknown because it is converted from the formula given in Table 1. We thus calculate the confidence intervals (CIs) of the RR and the two types of errors by multivariate normal simulation (Armstrong et al., 2001). We generate 3000 multivariate normal random draws that are subject to the estimated variance–covariance matrix, and convert them to the RR. Subsequently, we use the Wilcoxon rank-sum test to see if the two types of errors are of statistical significance.

Let Θ denote the set of parameter estimates. In Table 7, column $\mathrm{EU}(d^*;\Theta_{EU})$ contains RRs derived from EU-based estimates without considering the misperception cost, $\mathrm{EU}(d^*;\Theta_{RDEU})$ contains RRs derived from RDEU-based estimates without considering the misperception cost, and $\mathrm{EU}(d^*_w;\Theta_{RDEU})$ contains RRs derived from RDEU-based estimates considering the misperception cost. Type i and type ii errors are then given by $\delta_1 = \frac{\mathrm{EU}(d^*;\Theta_{RDEU})-\mathrm{EU}(d^*;\Theta_{RDEU})}{\mathrm{EU}(d^*;\Theta_{RDEU})}$ and $\delta_2 = \frac{\mathrm{EU}(d^*_w;\Theta_{RDEU})-\mathrm{EU}(d^*;\Theta_{RDEU})}{\mathrm{EU}(d^*_w;\Theta_{RDEU})}$ respectively. The statistical test shows that both types of errors are significant; i.e., not random variation.

The type i error is considerable, being -23.8% in the step model and 7.95% in the slope model. It is still an open question how travel time variability should be incorporated into SP design, because estimated values of reliability are strongly affected by the design(see the discussion in Carrion and Levinson, 2012b). Probabilities have often been involved in surveys, explicitly or implicitly, in past studies. Our findings suggest that it is necessary to consider the individual's non-linear probability weighting when modeling this kind of data; otherwise, estimates can be relatively biased.

By contrast, the effect of ignoring the cost of probability misperception in deriving the reduced form (i.e., type ii error) is as low as -0.9% in the step model, provided weighting parameters are estimated correctly in our empirical application. This is good news because it suggests that we can continue to take advantage of the analytical convenience of linearity without losing much accuracy. However, the error is 7.58% in the slope model and may be non-trivial. This result is not surprising because the marginal utility of time changes over the time of day in the slope model, making a unit deviation from the optimal departure time more costly. The conclusion depends largely on whether time-varying marginal utility is the case in reality.

The use of multivariate normal simulation has, to some extent, accounted for the unobserved heterogeneity in scheduling preferences, provided that it is asymptotically normal distributed. However, if this is not the case, then the results herein might not be robust enough. It is still possible that the cost of misperception is large for some individuals, even though the population-averaged probability weighting function implies the opposite.

Table 7: Type i and type ii errors

Valuation	$\mathrm{EU}(d^*; \Theta_{EU})$	$\mathrm{EU}(d^*; \Theta_{RDEU})$	$\mathrm{EU}(d_w^*; \Theta_{RDEU})$	δ_1	z-stats	δ_2	z-stats
σ/μ	1.79	2.39	2.42	-23.8%	-39.09	-0.9%	-2.76
95% CI	[0, 2.79]	[1.77, 2.96]	[1.77, 3.02]	[-100%, -30%]		[-2.37%, 0.1%]	
σ^2/μ	0.173	0.162	0.176	7.95%	25.26	-7.58%	-24.42
95% CI	[0.150, 0.197]	[0.129, 0.202]	[0.138, 0.220]	[-17.8%, 39.6%]		[-9.59%, -5.85%]	

5.4. Effects of unobserved heterogeneity

To control the unobserved heterogeneity, we estimate panel latent class logit models for both step and slope specifications. The existence of a large amount of heterogeneity is confirmed by a significant improvement in the model fit compared with the case for the BL model. Again, the slope specification is not so good as step specification in terms of the model fit regardless of the number of classes. The reason might be, the same as it was previously, that the sample contains a large portion of workers with inflexible workdays. Therefore, we only present the result for the step model hereinafter, taking it as the correct specification.

Although the Bayesian information criterion suggests the optimal number of classes to be four, we find that signs of the parameters become increasingly inconsistent with theory as the number of classes increases. For example, the marginal utility of SDE, β, turns out to be positive in some classes. A positive β implies a negative value of the optimal probability of being late, which is obviously not plausible, making the derived VTTV unavailable. On the basis of extensive tests on alternative specifications and taking the interpretation of coefficients into consideration, we select a model with two latent classes as our final model.

The estimation result is presented in Table 8. All coefficients are class-specific except β, which is restricted to zero in class 2 to avoid an otherwise positive estimate. It is worth noting that class 2 accounts for approximately 53.9% of the population, which implies that about half of respondents take SDE as an indication of a good travel condition. A possible explanation is that such people derive mental comfort or other sorts of utility by arriving early. We also note that β in class 2 is not significantly (t-stat 1.44) different from zero if not being restricted to zero, which entails another explanation; i.e., as detailed representation of SDEs and SDLs might be burdensome to some people, they adopt simplified decision-making strategies, such as only focusing on the mean travel time and SDL. If so, the estimates of class 2 do not represent the true preferences, and people in class 2 might therefore derive disutility from SDE in reality. The difference in SDLs between class 1 and class 2 (3.75 versus 11.2) indicates that class 1 might have a higher value of time or fewer schedule constraints.

Addressing the weighting parameters, we find that η of class 2 is relatively large, implying a strong probability of the under-weighting behavior for both short and long travel times. This means that people in class 2 tend to perceive a single value for the travel time distribution, and care much about whether this perceived value will result in him/her arriving on time. However, given that $\beta = 0$, the derived VTTV of class 2 is zero. By contrast, class 1 has weaker probability under-weighting behavior, as its weighting parameters are closer to 1 and not significant. This does not necessarily mean people almost perfectly perceive probability, but might be a signal of unobserved heterogeneity (otherwise η would have been close to 1). Overall, in agreement with the results for the BL model, these weighting parameters reveal that all classes have a pessimistic attitude to some extent.

Because β is ether restricted to zero or insignificantly different from zero, we can only calculate the reliability ratio for class 1. We find that, for class 1, RR is 7.7% higher when the cost of misperception is incorporated. Even when this ratio is reduced with the inclusion of class 2 (with RR equal to zero), it is still notably higher (3.55%) than that for the BL model. Given that the latent class model is preferable in terms of the model fit, it is reasonable to conclude that there is a considerable cost due to misperception for some people.

Table 8: Panel latent class step model

	Class 1		Class 2	
Parameter	Value	t-stats	Value	t-stats
Mean time (α)	-0.141	-9.56	-0.092	-7.32
Schedule delay early (β)	-0.088	-2.11	0	-
Schedule delay late (γ)	-0.529	-3.17	-1.03	-6.42
Theta (θ) [t-stats against 1]	1.01	0.07	2.19	5.92
Eta (η) [t-stats against 1]	2.0	0.83	11.0	3.35
Sample fraction	0.461	4.95	0.539	-
SDE ratio	0.62		0	
SDL ratio	3.75		11.2	
RR (no misperception)	1.55		0	
RR	1.67		0	
No. of Observations	4176			
Final log-likelihood	-1745.651			
Likelihood ratio ($\bar{\rho}^2$)	0.393			
Bayesian information criterion (BIC)	3574.673			

5.5. Comparison with the existing literature

It is interesting to compare our estimation result with that of Wang et al. (2012) because the two studies use similar model specifications. Before we do so, several distinctions should be clarified. Firstly, the data used in the cited study have a limited variation of probability (i.e., $p = \{0.025, 0.05, 0.1, 0.2\}$), which is not the case in our study. Secondly, the cited study has a control variable for the shape of G in the utility function, which is not necessary in our case because G is the same in all scenarios. Thirdly, the survey in the cited study was for two separate experiments: one estimating the scheduling model and the other estimating the reduced-form model. That approach was superior to that of our study because it allows a direct comparison to be made between the VTTV derived from the scheduling model and the VTTV derived from the reduced-form model using the same sample. Fourthly, the cited study focused on the estimate values of reduced-form model (i.e., $\mathrm{RDE}[C(d_w^*; T)]$) while our study focuses on the expected travel cost with a suboptimal departure time (i.e., $\mathrm{E}[C(d_w^*; T)]$).

Despite their differences, both studies confirmed that, when the optimal probability of arriving at time $\frac{\beta}{\beta+\gamma}$ is high, travelers tend to be optimistic and choose a departure time later than optimal. However, the comparison for the low $\frac{\beta}{\beta+\gamma}$ case is unavailable owing to the lack of variation of probability in the cited study.

Another common finding is that, even when the probability weighting is considered, the aforementioned difference between the derived VTTV and estimated VTTV from the reduced-form model is not eliminated.

6. Concluding remarks

In this paper, we first cast doubt on valuing travel time variability on the basis of WTPs converted from scheduling preferences, because it might be subject to error if travelers misperceive the travel time distribution to some degree. We argued that the effects of such violations are two-fold: the estimated scheduling preferences may be biased and the generalized travel cost may be underestimated. We reformulated a general scheduling model under rank-dependent utility to address part of the perceptual errors and analyzed the models properties. It was found that the shape of the weighting function and the way the marginal utility of time changes determine how the traveler's suboptimal departure time deviates from the optimal departure time, which allows us to determine the cost of misperception.

With the data collected in a stated-preference experiment, we then estimated two special cases of the proposed model. The estimation results indicated that (i) travelers are mostly pessimistic, (ii) the scheduling preference estimates are biased (around 20%) without considering probability weighting in a risky choice situation, and (iii) the cost of probability misperception may be as little as 1% or as large as 8%, depending on how the marginal utility of time varies by the time of day and the unobserved heterogeneity.

From a practitioner's point of view, if data are obtained from SP experiments involving probabilities, it is a good idea to use a non-EU scheduling model (with a flexible probability weighting function) to estimate the data. This would, to some extent, avoid the bias from misspecification (i.e., mistakenly taking the expected value as a certainty equivalent). On the other hand, it remains a good approach for future practice to estimate a scheduling model based on SP experiments without presenting probabilities and to calibrate the derived VTTVs by applying the formulas in Table 1 (and by using weighting parameters from the literature). Moreover, it would be good to have individual-level estimates of weighting parameters given that using the population-average estimates might undervalue the aggregate cost of misperception.

On the other hand, the probability misperception, though it exists, is not likely the cause of a discrepancy between the estimated VTTV and derived VTTV as large as that found in Börjesson et al. (2012). However, additional empirical studies are needed to confirm our argument and to investigate the effects of other sources of misperception in future work.

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Appendix A. List of notations

Table A.9: List of notations

Variable	Definition
T	Random travel time, $T = \mu + \sigma X$ with mean μ, std. σ and standardized random variable X
d	Departure time from home
h	Marginal utility of time spent at home
w	Marginal utility of time spent at workplace
a^*	Ideal arrival time, the intersection of h and w
W	Probability weighting function
F	Cumulative distribution function of T
f	Density function of T
G	Cumulative distribution function of X
π	Decision weight
E	Expectation operator
RDE	Rank-dependent expectation operator
Δ	Cost of misperception

Appendix B. Proofs of properties

Appendix B.1. Property 1

Proof. Because $h(t)$ is non-increasing, $d^*_w \leq d^* \Leftrightarrow h(d^*_w) \geq h(d^*) \Leftrightarrow \mathrm{RDE}[w(d+T)] \geq \mathrm{E}[w(d+T)]$ for any d. We show $\mathrm{RDE}[w(d+T)] \geq \mathrm{E}[w(d+T)]$ for any d as follows.

$$\mathrm{RDE}[w(d+T)] - E[w(d+T)] = \int_{T=0}^{T=\infty} w(d+T)\,[\mathrm{d}W[F(T)] - \mathrm{d}F(T)] \tag{B.1}$$

Using integration by parts, the right-hand side of Equation (B.1) becomes

$$w(d+T)\,[W[F(T)] - F(T)]\,|_0^\infty - \int_0^\infty w'(d+T)[W[F(T)] - F(T)]\mathrm{d}T \tag{B.2}$$

Because $W[F(0)]-F(0) = 0$ and $W[F(T)]-F(T) = 0$ for large T, the first term is zero. By $w'(d+T) \geq 0$ and $F(T)-W[F(T)] \geq 0$ for any realization of T, the second term is nonnegative. Consequently $\mathrm{RDE}[w(d+T)] \geq \mathrm{E}[w(d+T)]$ for any d and thereby $d^*_w \leq d^*$. □

Appendix B.2. Property 2

Proof. Equation (B.2) can be rewritten as

$$-\int_{T=0}^{T=\infty} w'(d+T)\mathrm{d}(\int_0^T [W[F(s)] - F(s)]\,\mathrm{d}s) \tag{B.3}$$

Applying integration by parts again, Equation (B.3) becomes

$$-w'(d+T)\int_0^T [W[F(s)] - F(s)]\,\mathrm{d}s|_0^\infty + \int_0^\infty \left(w''(d+T)\int_0^T [W[F(s)] - F(s)]\mathrm{d}s\right)\mathrm{d}T \tag{B.4}$$

By $\int_0^0 [(W[F(s)] - F(s)]\mathrm{d}s = 0$, the first term becomes $-w'(t+\infty)\int_0^\infty [(W[F(s)] - F(s)]\mathrm{d}s$, which is nonnegative because $w'(\cdot) \geq 0$ and $\int_0^\infty [(W[F(s)] - F(s)] \leq 0$ by the definition of second-order stochastic dominance. For the second term of Equation (B.4), because $w''(d+T) \geq 0$ and $\int_0^T [W[F(s)] - F(s)]\mathrm{d}s = -\int_T^\infty [W[F(s)] - F(s)]\mathrm{d}s \geq 0$, it is also nonnegative. Therefore Equation (B.4) is nonnegative and $d_w^* \leq d^*$ immediately follows. □

Appendix B.3. Property 3

Proof. If $\frac{\partial^2 \mathrm{E}[C(d;T)]}{\partial d^2}$ is a constant, then the cost is symmetric about d^*. Therefore, making a marginally late departure more costly than a marginally early departure will suffice to let $\frac{\partial^2 \mathrm{E}[C(d;T)]}{\partial d^2}$ be an increasing function, at least in the neighborhood of d^*. This is implied by $\frac{\partial^3 \mathrm{E}[C(d;T)]}{\partial d^3}|_{d=d^*} = \mathrm{E}[w''(d^*+T)] - h''(d^*) > 0$. □

Appendix C. Derivation of models

Appendix C.1. Derivation of RDEU-step model

The rank-dependent expected cost is given by

$$\mathrm{RDE}[C(d;T)] = \alpha\mu_w + \beta\int_0^{-d} -(d+T)\frac{\partial W(F)}{\partial F}f(T)\mathrm{d}T + \gamma\int_{-d}^{\infty}(d+T)\frac{\partial W(F)}{\partial F}f(T)\mathrm{d}T \tag{C.1}$$

where $\mu_w = \int_0^1 F^{-1}[W^{-1}(s)]\mathrm{d}s$. The first-order condition is

$$\frac{\partial \mathrm{RDE}[C(d;T)]}{\partial d} = \gamma - (\beta+\gamma)W[F(-d)] = 0 \tag{C.2}$$

Solving it yields the subjectively optimal departure time

$$d_w^* = -F[W^{-1}(\frac{\gamma}{\beta+\gamma})] = -\mu - \sigma G^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})] \tag{C.3}$$

Substituting d_w^* back into $C(d;T)$ and applying an expectation operator over T, we have

$$\begin{aligned}
\mathrm{E}[C(d_w^*;T)] = \alpha\mu &+ \beta\int_0^{F^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})]}(F^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})] - T)f(T)\mathrm{d}T \\
&+ \gamma\int_{F^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})]}^{\infty}(T - (F^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})]))f(T)\mathrm{d}T \\
= (\alpha-\beta)\mu &+ [\beta W^{-1}(\frac{\gamma}{\beta+\gamma}) - \gamma + \gamma W^{-1}(\frac{\gamma}{\beta+\gamma}))]F^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})] \\
&+ (\beta+\gamma)\int_{F^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})]}^{\infty} Tf(T)\mathrm{d}T \\
= \alpha\mu &+ \sigma\left([(\beta+\gamma)W^{-1}(\frac{\gamma}{\beta+\gamma}) - \gamma]G^{-1}[W^{-1}(\frac{\gamma}{\beta+\gamma})] + (\beta+\gamma)\int_{W^{-1}(\frac{\gamma}{\beta+\gamma})}^{1} G^{-1}(s)ds\right)
\end{aligned} \tag{C.4}$$

Appendix C.2. Derivation of RDEU-slope model

The rank-dependent expected cost is given by

$$\mathrm{RDE}[C(d;T)] = \gamma_0\mu_w + (\gamma_1 - \beta_1)d^2/2 + \frac{\gamma_1}{2}\int_0^\infty T^2\frac{\partial W(F)}{\partial F}f(T)\mathrm{d}T + \gamma_1 d\mu_w \tag{C.5}$$

The first-order condition is

$$\frac{\partial \mathrm{RDE}[C(d;T)]}{\partial d} = -\beta_1 d + \gamma_1(d+\mu_w) = 0 \tag{C.6}$$

Solving it yields the subjectively optimal departure time

$$d_w^* = -\frac{\gamma_1 \mu_w}{\gamma_1 - \beta_1} \tag{C.7}$$

Substituting d_w^* back into $C(d;T)$ and applying an expectation operator over T, we have

$$\mathrm{E}[C(d_w^*;T)] = \frac{\gamma_1^2 \mu_w^2}{2(\gamma_1-\beta_1)} + \frac{1}{2}\gamma_1\mu^2 + \frac{1}{2}\gamma_1\sigma^2 + \mu(\gamma_0 - \frac{\gamma_1^2\mu_w}{\gamma_1-\beta_1}) \tag{C.8}$$

Appendix D. Estimation result of RDEU-slope model with a kink

We construct a RDEU-slope model with a kink by allowing a constant increment on the slope of $w(t)$ when arrival time $d+T > d+\mu_w$ via a new parameter γ_2. Its cost function is given by

$$C(d;T) = \int_d^0 (\beta_0 + \beta_1 t)\mathrm{d}t + \int_0^{d+T} (\gamma_0 + \gamma_1 t)\mathrm{d}t + \int_0^{d+T} \gamma_2 \max(0, t-d-\mu_w)\mathrm{d}t \tag{D.1}$$

Table D.10 presents the estimation results of such a model. The statistical significance of γ_2 confirms the existence of the loss aversion, which is also why this model performs best among those we tested. However, the estimated probability weighting curve (Prelec2) shown in Figure D.6 is S-shaped, indicating that accounting for loss aversion does not fail the conclusion we have drawn on the pessimistic attitude in scheduling behavior.

Table D.10: Parameter estimates of RDEU-slope-kink model

Parameter	Value	Robust std.	t-stats	p-value
$\gamma_0 - \beta_0$	-0.422	0.0403	-10.47	0.00
γ_0	-0.457	0.0162	-28.16	0.00
$\gamma_1 - \beta_1$	-0.024	0.0019	-13.02	0.00
γ_1	-0.023	0.0020	-11.17	0.00
γ_2	-0.475	0.0624	-7.61	0.00
η [t-stats against 1]	5.22	1.81	2.33	0.00
θ [t-stats against 1]	1.85	0.226	3.76	0.00
No. of Observations		4176		
Final log-likelihood		-1891.815		
Likelihood ratio ($\bar{\rho}^2$)		0.344		
Akaike information criterion (AIC)		3797.63		

Appendix E. Estimation of stable distribution for standardized travel times

The electronic toll collection (ETC) data for estimating the distribution of standardized travel times were provided by Central Nippon Expressway Company for a single section of the Tomei Expressway (Atsugi IC to Yokohama-Machida IC, inbound) with a length of 15.3 km. The ETC system recorded the entry time and the exit time of each toll road user. We used data from weekdays between 6 a.m. and 10 p.m. during the period July to September 2008. The sample size (i.e., the number of vehicles) was 231769.

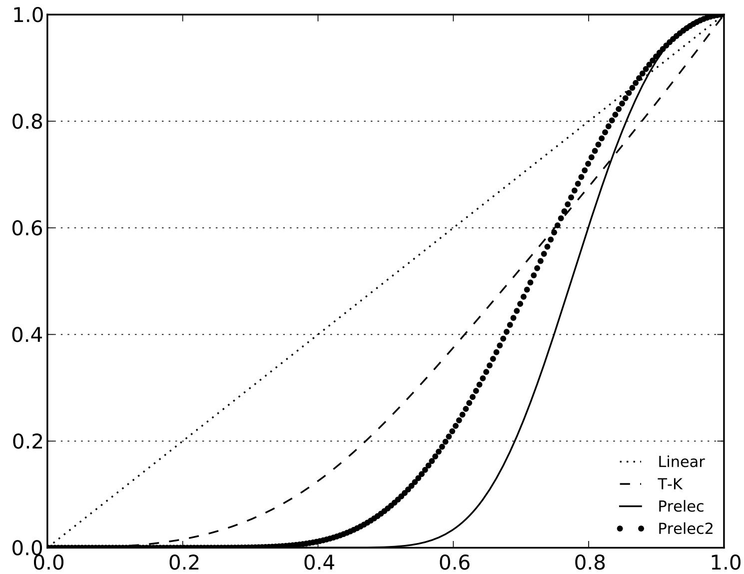

Figure D.6: Probability weighting curve from slope-kink model

First, travel time was transformed into a standardized form using the corresponding mean and standard deviation. Then, we fit the data with a stable distribution (e.g. Zolotarev, 1986) using a maximum likelihood method (see Nolan, 1997). Four parameters were obtained to describe a stable distribution: a stability parameter = 1.250, a skewness parameter = 0.786, a scale parameter = 0.293, and a location parameter = 0.318. Discussions on the properties of these parameters can be found in Fosgerau and Fukuda (2012). The corresponding distribution is illustrated in Figure 4.