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PAPER *Special Section on Digital Signal Processing*

Multiplierless Arrays for Realization of Lowpass and Highpass Linear Phase FIR Digital Filters

Saed SAMADI†, Akinori NISHIHARA††, *and* Nobuo FUJII††, *Members*

SUMMARY A class of type 1 linear phase FIR digital filters is proposed. The filter can be realized using a parallel, modular and regular array structure. It is shown that, under some simple constraints, the consisting modules of the array can be realized free of multiplier coefficients. Such two dimensional mesh arrays are specially suitable for realization with special-purpose systolic hardware for high-speed digital signal processing tasks. Compared to the array structure, proposed by the authors, for multiplierless realization of maximally flat FIR digital filters, this class needs less adders to fulfill the same magnitude response requirements. Another attractive property of the proposed array is that a number of highpass or lowpass filters with different passband widths can be realized simultaneously in a very economical way.

key words: *FIR digital filters, parallel structures, multiplierless structures, systolic arrays, digital signal processing*

1. Introduction

Array structures for FIR digital filters (DFs) based on different approximation techniques have been investigated by the authors in recent years [1]–[7]. The main incentive of the study has been finding array structures that enjoy features critical to the implementation of high-speed DFs using special-purpose parallel hardware. The approximation techniques considered, range from Bernstein approximation and Hermite interpolation to less familiar Stancu approximation operator. These array structures enjoy several merits. First of all, the structures are regular and modular with connections between neighboring modules (building blocks or processing elements) only, and thus lend themselves to implementation as VLSI systolic arrays [4]. The next important merit is that multiplierless arrays with identical modules can be obtained for a class of FIRDFs called maximally flat (MAXFLAT) DFs and a special case of flat filters [4], [6]. Also, one can obtain a number of filters with different passband or stopband widths at very economical hardware cost [2]. Finally, a study of the features of the MAXFLAT array also revealed that the structure possesses intrinsic robustness with respect to the destruction of modules [5]. Specifically, this important feature ensures graceful degradation of the filter performance when some modules are disconnected from the array because of hardware or software errors. The above encouraging fact show that our approach, that has deep connections with approximation theory and the so-called triangular structures [8] for evaluating polynomials expressed as linear combination of certain basis functions, can result in highly sophisticated structures. However, a problem remained unanswered, namely, the possibility of obtaining arrays with identical modules that realize non maximally flat filters, i.e., filters with ripples in passband or stopband. In a filter, ripples play the important role of narrowing the transition band, and thus reducing the required filter order. This can be interpreted as less hardware cost and complexity. The above-mentioned arrays can realize filters with ripples, however, the modules are no more identical and/or multiplierless, and generally require different types of multipliers in modules at different locations of the array [3]. The purpose of this paper is to propose a novel array structure that can realize lowpass and highpass filters with ripples through identical building blocks. The building blocks or modules can be realized in a multiplierless manner or need very simple multipliers. The structure is based on a modification of the existing MAXFLAT array and includes it as a special case. It results in a class of lowpass and highpass filters with narrower transition bands than MAXFLAT filters. This, in turn, leads to less amount of hardware for realizing the filter. For examples of existing multiplierless and systolic approaches to DF design, including discrete (powers-of-two) coefficient designs see [9]–[14]. The organization of the paper is as follows. In Sect. 2, we review the MAXFLAT array structure and introduce a recurrence defining what we shall call "q-filters." Properties of the frequency response of q-filters are also investigated. Section 3 presents a modular array structure for multiplierless realization of q-filters. A comparison is also made to show the superiority of q-filters, in terms of transition band-width, with respect to MAXFLAT filters. Finally, Sect. 4 concludes the paper.

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2. A New Class of Multiplierless Arrays Realizing Lowpass and Highpass FIRDFs

Array realization of MAXFLAT FIRDFs is based on the De Casteljau algorithm and can be expressed mathematically, in polynomial form, by the recurrence [1]

$$H_{i,j}(x) = (1-x)H_{i,j-1}(x) + xH_{i-1,j}(x). \tag{1}$$

The boundary conditions for the lowpass case are $H_{i,0}(x) = 1$ and $H_{0,j}(x) = 0$. Integer i varies from 1 to L (degree of flatness at the origin) while integer j varies from 1 to K (degree of flatness at $x = 1$). Transforming x to the frequency domain through

$$x = \frac{1-\cos\omega}{2} \tag{2}$$

results in a lowpass linear phase MAXFLAT filter of order $2(L+K-1)$. Interestingly, the above recurrence and transformation do not induce multiplier coefficients except simple powers of two.

Although MAXFLAT filters find application when a monotone frequency response is desirable, in many cases the application can tolerate ripples in passband and stopband. In such cases, we should examine other alternatives to obtain filters with ripples in passband and stopband and take advantage of the ripples to narrow the transition band. Unfortunately, optimal Chebyshev or least squares designs can not be associated *directly* with multiplierless arrays in general. This is the same for designs based on Lagrange interpolation. In the latter case, though we can utilize recursive triangles to realize the DF, the resulting array would consist of modules that, generally, include multiplier coefficients. Furthermore, different modules include different multiplier coefficients. The only known multiplierless array, based on recursive triangles, with one ripple in passband or stopband is the structure proposed in [7]. It should be noted that the scope of this paper is modular design with *identical* modules and other approaches to multiplierless design exist in the literature that do not lead to such modular array realizations. Here, we propose a new recurrence that results in filters with more than one ripple in each band and thus better frequency response characteristics.

The mechanism of Recurrence (1), can be explained as follows. At each stage, the filter $H_{i,j}(x)$ takes advantage of stopband characteristics of $H_{i-1,j}(x)$ at $x = 1$ and its vicinity. This can be easily observed from (1) by letting $x = 1$. In the same way, the frequency response at $x = 0$ and its vicinity is determined by $H_{i,j-1}(x)$. This is reasonable, because $H_{i-1,j}(x)$ is flatter at $x = 1$ and $H_{i,j-1}(x)$ is flatter at $x = 0$. Now, consider the Recurrence

$$\begin{aligned} H_{i,j}(x) = {} & \frac{1-x}{q}H_{i,j-1}(x) + \frac{x}{q}H_{i-1,j}(x) \\ & + \frac{q-1}{q}H_{i-1,j-1}(x) \end{aligned} \tag{3}$$

together with boundary conditions

$$H_{0,0}(x) = 0,\ H_{i,0}(x) = 1,\ H_{0,j}(x) = 0.$$

The recurrence is shown schematically in Fig. 1. It can be proved, by induction on j, that for $i = 1$,

$$H_{1,j}(x) = \left(\frac{1-x}{q}\right)^j.$$

Figure 2 shows a plot of this polynomial for two different values of j. General properties of the polynomial are as follows.

- For an arbitrary j, $H_{1,j}(x)$ possesses j zeros at $x = 1$. Hence, as j increases, $H_{1,j}(x)$ becomes flatter around $x = 1$.
- $H_{1,j}(1-q) = 1$.

These polynomials can be construed of as lowpass filters with good stopband and poor passband characteristics over the interval $[1-q, q]$ for $q \geqq 1$. Similarly, an induction on i can be used to prove that

$$H_{i,1}(x) = 1 + \frac{(1-q)x^{i-1} - x^i}{q^i}.$$

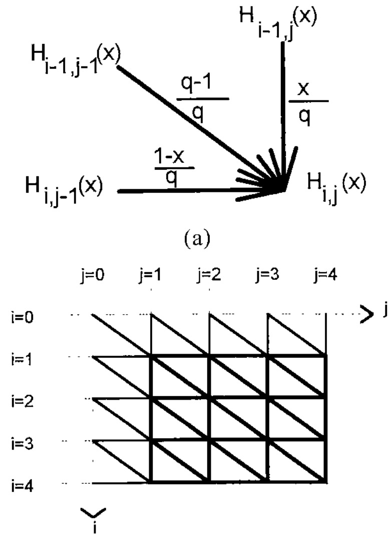


Fig. 1 Schematic representation of Recurrence (3). (a) Basic operation; (b) geometric connection.

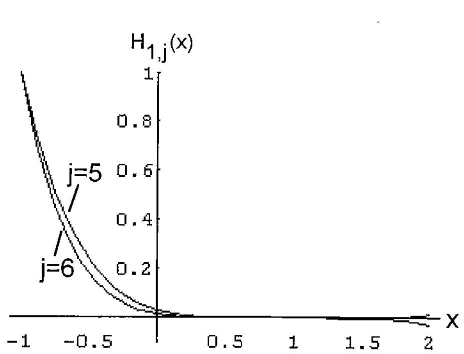


Fig. 2 Plots of $H_{1,5}(x)$ and $H_{1,6}(x)$ for $q = 2$ over $[1-q, q]$.

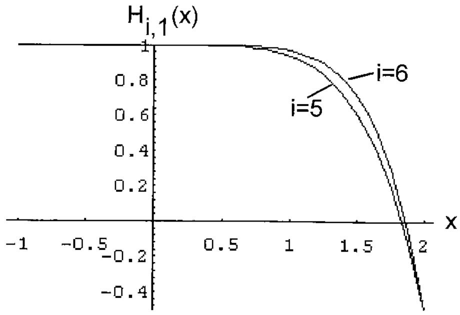


Fig. 3 Plots of $H_{5,1}(x)$ and $H_{6,1}(x)$ for $q=2$ over $[1-q,q]$.

Figure 3 shows a plot of this polynomial for two different values of i. General properties of the polynomial are as follows.

- For an arbitrary i, $1-H_{i,1}(x)$ possesses one zero at $x=1-q$, and $i-1$ zeros at $x=0$.
- $H_{i,1}(1)=1-q^{1-i}$.
- $H_{i,1}(q)=-1+q^{-1}$.

These polynomials can be viewed as lowpass filters with good passband and poor stopband characteristics over the interval $[1-q,q]$ for $q \geqq 1$.

2.1 General Stopband Properties

The above observations on the outputs of the first row and the first column of the recurrence show that we get lowpass filters with better stopband characteristics as we proceed horizontally, and better passband characteristics as we proceed vertically. Next, let us examine general outputs of the recurrence. To do this, we find a relationship yielding $H_{i,j}(x)$, in terms of $H_{i-1,k}(x)$ for $k=1$ to j. This helps us deduce stopband properties of $H_{i,j}(x)$ utilizing the properties of the outputs of the previous row, i.e., $i-1$th row. Using Fig. 4, it can be shown that

$$H_{i,j}(x)=\left(\frac{1-x}{q}\right)^{j-1}\frac{q-x}{q}+\frac{x}{q}H_{i-1,j}(x)+\frac{(q-x)(x+q-1)}{q^2}\cdot\sum_{s=0}^{j-2}\left(\frac{1-x}{q}\right)^{s}H_{i-1,j-1-s}(x). \tag{4}$$

There are two types of functions in the above equation; the attenuating functions and the past outputs. Specifically, the first term in (4) is the $j-1$th horizontal boundary output of the recurrence, attenuated by a linear decreasing function over the interval $[1,q]$ i.e., the interval including the stopband of the filter. In the second term, $H_{i-1,j}(x)$ is also attenuated by a linear decreasing function over the same interval. The last group of terms is the convolution of the outputs of the $i-1$th row and the first row attenuated by a function of second degree on, again, the same interval.

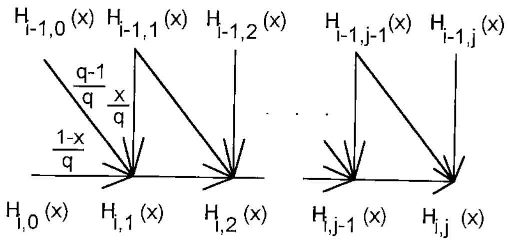


Fig. 4 Relation between two consecutive rows of Recurrence (3).

We already know that the sequence $H_{1,j}(x)$ produces polynomials with smoother stopbands as j increases. To utilize this knowledge as the first step of an induction on i, we would like to see how the outputs of the ith row behave when the outputs of the $i-1$th row are a sequence of polynomials with improving stopband characteristics as j increases. $H_{i,j+1}(x)$ can be written as

$$H_{i,j+1}(x)=\left(\frac{1-x}{q}\right)^{j}\frac{q-x}{q}+\frac{x}{q}H_{i-1,j+1}(x)+\frac{(q-x)(x+q-1)}{q^2}\left(\frac{1-x}{q}\right)^{j-1}H_{i-1,1}(x)+\frac{(q-x)(x+q-1)}{q^2}\sum_{s=0}^{j-2}\left(\frac{1-x}{q}\right)^{s}H_{i-1,j-s}(x) \tag{5}$$

or, after some manipulations,

$$H_{i,j+1}(x)=\left(\frac{1-x}{q}\right)^{j}\frac{q-x}{q}\cdot\left(1-q^{-i}x^{i-2}(x+q-1)^2\right)+\frac{x}{q}H_{i-1,j+1}(x)+\frac{(q-x)(x+q-1)}{q^2}\sum_{s=0}^{j-2}\left(\frac{1-x}{q}\right)^{s}H_{i-1,j-s}(x) \tag{6}$$

Comparing $H_{i,j}(x)$ with $H_{i,j+1}(x)$ term to term, we recognize that the attenuating functions are the same except the first term which is further attenuated by a polynomial of degree i. On the other hand, past outputs that are used to compute $H_{i,j+1}(x)$ possess better stopbands (assumption of the induction) and hence a lower upper bound can be obtained on maximum deviation of $H_{i,j+1}(x)$ from zero over the interval $[1,q]$. Thus we can expect better stopband characteristics for the ith row as j increases, if this is also the case for the $i-1$th row. Another result from (4) is

$$H_{i,j}(q)=H_{i-1,j}(q)=\cdots=H_{1,j}(q)=\left(\frac{1-q}{q}\right)^{j}$$

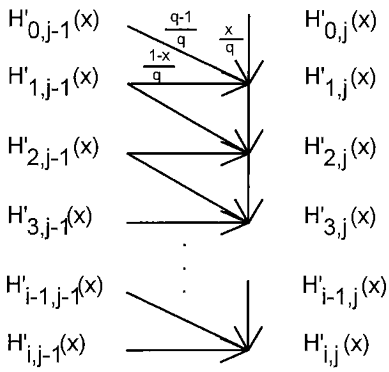


Fig. 5 Relationship between two consecutive columns of Recurrence (3).

2.2 General Passband Properties

To obtain information on the effect of i on the flatness of passband, we may examine the relationship between the outputs of the jth column and the outputs of the $j-1$th column. Our assumption is that the outputs of the $j-1$th column possess better passband characteristics as i increases. We already know that this is the case for the first column. It is more convenient to study functions of the from $H'_{i,j}(x) = 1 - H_{i,j}(x)$ instead, and again concentrate on deviation from zero. Hence, the recurrence to be considered here has boundary inputs of the form

$$H'_{0,0}(x) = 1,\ H'_{0,j}(x) = 1,\ H'_{i,0}(x) = 0$$

From Fig. 5, we can write

$$H'_{i,j}(x) = \left(\frac{x}{q}\right)^{i-1} \frac{x+q-1}{q} + \frac{1-x}{q} H'_{i,j-1}(x) + \frac{(q-x)(x+q-1)}{q^2} \sum_{s=0}^{i-2} \left(\frac{x}{q}\right)^s H'_{i-1-s,j-1}(x). \tag{7}$$

New types of first degree attenuating functions are appeared here which are effective on interval $[1-q, 0]$ for $q \geq 1$. Computing $H'_{i+1,j}(x)$ we get

$$\begin{aligned} H'_{i+1,j}(x) &= \left(\frac{x}{q}\right)^{i-1} \frac{x+q-1}{q} \left(1 - \frac{q-x}{q}\left(\frac{1-x}{q}\right)^{j-1}\right) \\ &+ \frac{1-x}{q} H'_{i+1,j-1}(x) \\ &+ \frac{(q-x)(x+q-1)}{q^2} \sum_{s=0}^{i-2} \left(\frac{x}{q}\right)^s H'_{i-s,j-1}(x). \end{aligned} \tag{8}$$

Comparing $H'_{i,j}(x)$ and $H'_{i+1,j}(x)$ term to term, we can easily verify that the attenuating functions are the same except the first term which is further attenuated by a polynomial of jth degree. Past outputs used to compute $H'_{i+1,j}(x)$ possess better passbands (assumption of the induction) and hence a lower upper bound can be obtained on maximum deviation of $H'_{i+1,j}(x)$ from zero on the interval $[1-q, 0]$. Thus we can expect better passband characteristics for $H'_{i,j}(x)$ as i increases, if this is also the case for the $j-1$th column.

Another property of $H_{i,j}(x)$, which can be derived from (7), is

$$\begin{aligned} H_{i,j}(1-q) &= H_{i,j-1}(1-q) = \cdots \\ &= H_{i,1}(1-q) = 1 \end{aligned}$$

In conclusion, the outputs of the recurrence can be improved in passband and stopband by increasing i and j respectively. To obtain a highpass characteristic, we use the property that the coefficients of Recurrence (3) sum up to one, and replace boundary inputs 1 with 0 and vice versa. For this recurrence, the roles of i and j are reversed.

2.3 The Role of Parameter q

Parameter q reveals itself in the above relationships in two ways. It affects the attenuating functions and the past outputs involved in (4) and (7). Clearly, $q = 1$ yields the MAXFLAT recurrence introduced earlier, and as q gets larger the regions associated with the passband and the stopband get wider. These regions are $[1-q, 0]$ for the passband and $[1, q]$ for the stopband. In fact, $[1-q, 0]$ includes the whole passband and perhaps the transition band and part of the stopband. $[1, q]$ includes the whole stopband and perhaps the transition band and part of the passband. The interval $(0, 1)$ may include the transition band, part of the passband or part of the stopband of the filter, noting that starting from non-negative boundary conditions, the recurrence is guaranteed to produce non-negative outputs over $[0, 1]$.

Another important effect of q is that large values of q give rise to attenuating functions with greater maximum, over corresponding intervals, for the second degree function and one of the first degree functions. A larger maximum for such functions means less attenuation of the previous outputs and thus more deviation from desired values in the passband and stopband.

The effect of q on the realization of the recurrence as an array structure for FIRDFs is another factor that should be considered. This will be discussed in the following subsection and we suffice it to say that $1 \leq q \leq 2$ offers suitable values for this parameter. Because of the role of q in determining the amplitude response and the structure of the filter we call the class of filters, resulting from Recurrence (3), q-filters.

3. Realization of the Filter

We first need to map the union of the three above-mentioned intervals, i.e., $[1-q, q]$, to $[0,1]$ and then apply (2) to map x-domain to the frequency domain. To get the z-transform representation of the filter, we just replace $\cos\omega$ with $(z+z^{-1})/2$ and obtain

$$\begin{aligned} H_{i,j}(z) &= \frac{2+(2q-1)(z+z^{-1})}{4q} H_{i,j-1}(z) \\ &+ \frac{2-(2q-1)(z+z^{-1})}{4q} H_{i-1,j}(z) \\ &+ \frac{q-1}{q} H_{i-1,j-1}(z). \end{aligned} \tag{9}$$

Execution of (9) from $i=j=1$ to $i=L$, $j=K$, can be mapped to an array structure for a zero-phase FIRDF consisting of identical modules, as shown in Fig. 6 (a). To obtain a causal structure, the vertical boundary inputs should be replaced by delayed versions of the input and delay elements should be inserted between diagonal connections of the module. Then, the recurrence should be written as

$$\begin{aligned} H_{i,j}(z) &= \frac{2z^{-1}+(2q-1)(1+z^{-2})}{4q} H_{i,j-1}(z) \\ &+ \frac{2z^{-1}-(2q-1)(1+z^{-2})}{4q} H_{i-1,j}(z) \\ &+ \frac{q-1}{q} z^{-1} H_{i-1,j-1}(z). \end{aligned} \tag{10}$$

The above recurrence can be realized by various structures for a given q. Figure 6 (b) shows a realization suitable for systolic implementation of the array for $q=2$. This value for q is of interest for two reasons. Firstly, the properties of the amplitude response, i.e., a narrower transition band than the MAXFLAT case, and, secondly, the realization of the modules that can be done in a multiplierless manner as shown in Fig. 6 (b). Other natural powers of two can also be considered because of the multiplierless nature of the modules they yield. However, the problem is that the ripples get unacceptably large for $q>2$. Implementation of Fig. 6 (b) as an independent module in a parallel systolic structure requires six adders and two delay elements.

For this realization, the maximum computation time is the time needed to execute three addition operations. This structure has two important and novel properties. First of all, it consists of identical multiplier-less modules interconnected in a regular way. And it is the first such structure that yields filters with ripples in passband and stopband. Another advantage of the structure is that it can be converted easily to a MAXFLAT filter by putting switches at appropriate positions.

Other choices of q may also yield structures with reasonable amplitude response and multipliers that can be realized by addition of powers of two. One such selection is $q=1/(1-2^{-b})$ where b is a positive integer. Different values of b yield q varying over $(1,2]$. The actual value of b can be selected by taking round-off noise and amplitude response into account. A large

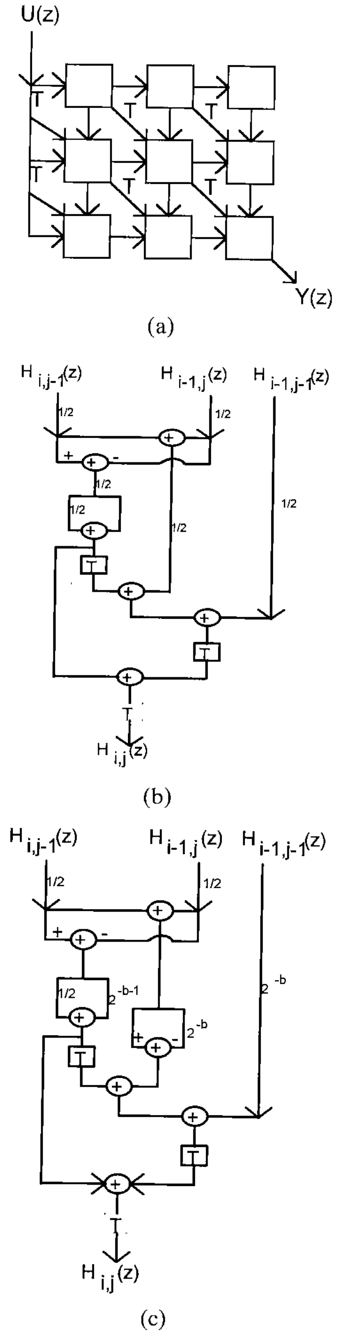
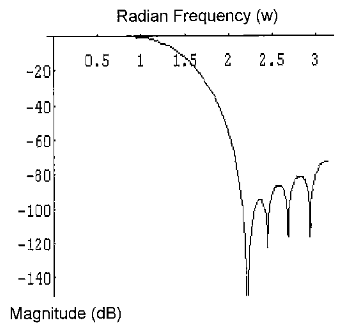


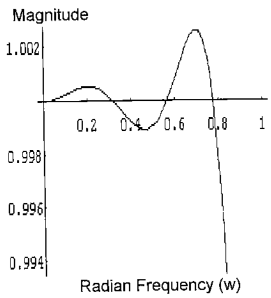
Fig. 6 (a) Array realization of a causal lowpass q-filter for $L=K=3$; $U(z)$ and $Y(z)$ denote the z-transforms of the input and output signals, respectively, and are related as $Y(z)=H_{3,3}(z)U(z)$ (note that modules of the first row should receive zero signals as vertical and diagonal inputs); (b) a causal realization of the modules of (a) for $q=2$; (c) a causal realization of the modules of (a) for $q=1/(1-2^{-b})$.

b amounts to an amplitude response close to that of a MAXFLAT filter with the same L and K. A realization of the modules for such values of q is given in Fig. 6 (c). The structure requires seven additions for this case. The improvement of frequency response characteristics over the MAXFLAT filter is shown for a q-

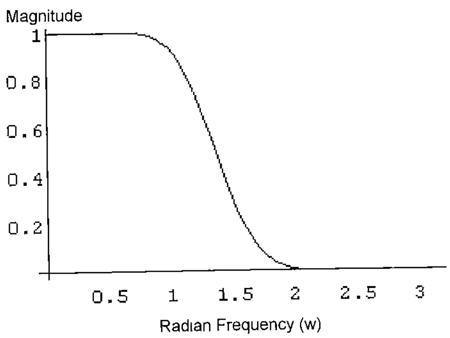
filter with $L = 4,\ K = 6,\ b = 2$ in Fig. 7. The filter has comparable amplitude response characteristics to a MAXFLAT filter with $L = 6$ and $K = 9$. Specifically, defining passband and stopband edge frequencies of a



(a)



(b)



(c)

Fig. 7 (a) Magnitude response of a q-filter with $L = 4, K = 6$ and $q = 4/3$ in dB; (b) detailed passband characteristics of (a); (c) magnitude response of a MAXFLAT filter with approximately the same frequency characteristics as (a) ($L = 6, K = 9$ and $q = 1$).

Table 1 Comparison of computational and memory complexity of an array of size $L \times K$ for different values of q. Parentheses show the number of delays for pipelined realization.

	Number of Adders	Number of Delays
MAXFLAT Array ($q = 1$)	$4LK$	$2LK + L - 1$ ($3LK + 2L - 2$)
q-filter ($q = 2$)	$6LK$	$3LK - K$ ($5LK - 2K$)
q-filter ($q = \frac{1}{1-2^{-b}}$)	$7LK$	$3LK - K$ ($5LK - 2K$)

MAXFLAT filter as frequencies where the amplitude response attains 0.95 and 0.05 respectively, the transition band width of the MAXFLAT filter of Fig. 7 (c) is equal to 0.849 while we get 0.787 for the q-filter of Fig. 7 (a) using the same definition of edge frequencies. Using the structure proposed in [2], this MAXFLAT filter would require $6 \times 9 \times 4$ adders while the novel structure requires only $4 \times 6 \times 7$ adders.

Table 1 presents a complexity comparison between a q-filter of size $L \times K$ and a MAXFLAT array of the same size. Note that the delay element shown by a dotted block in Figs. 6 (b) and (c) should be used for the purpose of pipelining the operation of the modules. When this delay element is operational, the delay elements on the vertical boundary inputs and diagonal connections of Fig. 6 (a) should be doubled ($T \rightarrow 2T$) to maintain the order of computations.

4. Conclusion and Discussions

A novel array structure for type 1, linear phase FIRDFs was proposed, resulting in modular realizations consisting of identical modules or processing elements. The structure includes MAXFLAT filters as special cases and can be realized free of multiplier coefficients through a careful choice of its parameter. Furthermore, compared to MAXFLAT filters, the amplitude response of the proposed filter is more favorable and same specifications can be satisfied with lower orders and thus less hardware. For this filter, there are three design parameters that should be adjusted carefully; namely, L and K, determining array size and performance in passband and stopband, i.e., passband edge and transition band-width, and q determining the magnitude of ripples. However, there is no rule-of-thumb to guess the values of these parameters for given specifications. A suggestible approach is to start from a MAXFLAT approximation to the specifications and then modify it by selecting appropriate values of q, to narrow transition band width or equivalently reduce the order of the filter. Fig. 8 and Table 2 show the effect of varying L and K while keeping their sum constant. Combinations with smaller K result in larger stopband ripples compared to others. Allowing a wider transition band width, the ripple size can be reduced by adopting a smaller q.

The array structure is unique, compared to the MAXFLAT array, in having diagonal connections between modules. It seems that these diagonal connections play an important role in creating ripples for a structure consisting of identical modules. Although the discussion has been focused on the lowpass case so far, one can obtain a highpass filter by changing the boundary conditions as $H_{0,0}(x) = 1, H_{i,0}(x) = 0, H_{0,j}(x) = 1$. Another point to mention is that unlike MAXFLAT filters, a q-filter with $L = K$ is not a half-band filter. A way to get around this is to alter boundary input $H_{0,0}(x) = 0$ as $H_{0,0}(x) = \frac{1}{2}$ and leave the remaining

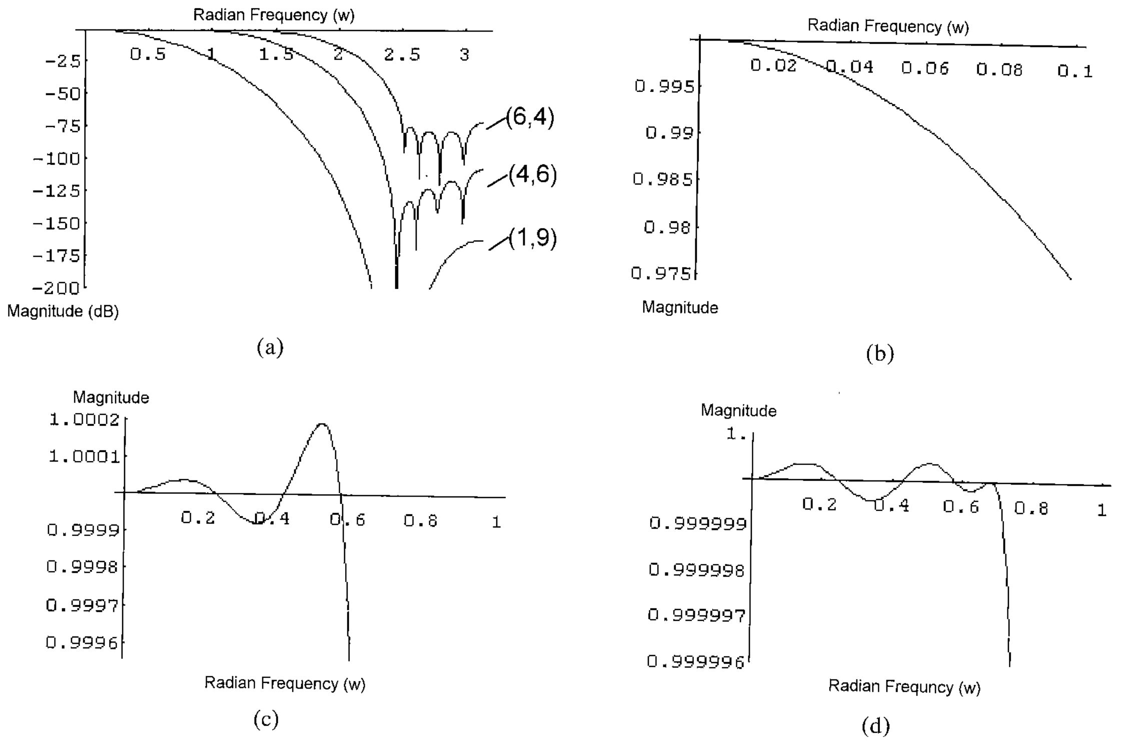


Fig. 8 (a) Magnitude responses of three q-filters of order $18(=2(L+K-1))$ using $q=\frac{8}{7}$; from left to right (L,K) varies as: $(1,9)$, $(4,6)$, $(6,4)$; (b) detailed passband characteristics of (a) for $(1,9)$; (c) detailed passband characteristics of (a) for $(4,6)$; (d) detailed passband characteristics of (a) for $(6,4)$.

Table 2 Peak passband ripple and minimum stopband attenuation for 10 possible combinations of L and K adding up to 10.

(L,K)	Peak Passband Ripple (Absolute Value)	Minimum Stopband Attenuation (dB)
(1, 9)	No Ripples in Passband	162.56
(2, 8)	6.1 E-3	144.49
(3, 7)	1.3 E-3	126.43
(4, 6)	1.9 E-4	108.37
(5, 5)	1.2 E-5	90.31
(6, 4)	4.5 E-7	72.25
(7, 3)	3.5 E-8	54.19
(8, 2)	3.5 E-9	36.12
(9, 1)	3.2 E-10	18.06

boundary inputs unchanged. This configuration would result in a half-band filter for $L=K$, but unlike the initial configuration, the amplitude response would no longer assume unity for DC inputs.

Finally, using the same technique as [2], we can obtain all N possible q-filters of a fixed given degree $2N$ (with a fixed value for q), using a total number of $\frac{N(N+1)}{2}$ modules. In this case, the average cost of each filter is $\frac{N+1}{2}$ modules. This is comparable with a direct form realization considering the number of adders.

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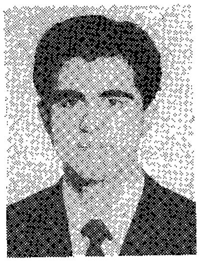

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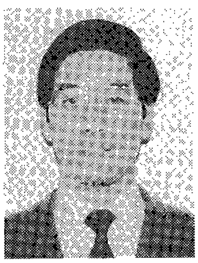

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