論文 / 著書情報
Article / Book Information

題目(和文)	代数的位相アンラップとスプライン平滑化による信号処理に関する研究
Title(English)	A Study of Algebraic Phase Unwrapping and Spline Smoothing for Signal Processing Applications
著者(和文)	北原大地
Author(English)	Daichi Kitahara
出典(和文)	学位:博士(工学), 学位授与機関:東京工業大学, 報告番号:甲第10479号, 授与年月日:2017年3月26日, 学位の種別:課程博士, 審査員:山田 功,植松 友彦,中山 実,府川 和彦,尾形 わかは,平林 晃
Citation(English)	Degree:Doctor (Engineering), Conferring organization: Tokyo Institute of Technology, Report number:甲第10479号, Conferred date:2017/3/26, Degree Type:Course doctor, Examiner:,,,,,
学位種別(和文)	博士論文
Category(English)	Doctoral Thesis
種別(和文)	要約
Type(English)	Outline

A Study of Algebraic Phase Unwrapping and Spline Smoothing for Signal Processing Applications

Daichi KITAHARA

Advisor: Prof. Isao YAMADA

Department of Communications and Computer Engineering

Tokyo Institute of Technology

February 2016

Chapter 1

Introduction

1.1 Phase Unwrapping

Phase unwrapping [1, 2] is an estimation problem of a continuous phase function from its wrapped samples. In many signal and image processing applications, the continuous phase function relates to some physical quantity, e.g., the surface profile of an object in interferometry [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13], in fringe projection [14, 15, 16, 17, 18, 19, 20], or in X-ray projection [21, 22, 23, 24], and the degree of the magnetic field inhomogeneity in magnetic resonance imaging (MRI) [25, 26, 27, 28]. However, since there exists the arbitrariness of $2\pi\eta$ ($\eta \in \mathbb{Z}$) in the phase of any nonzero complex number, we can detect only up to the principal value, *wrapped* into the interval $(-\pi, \pi]$, of the continuous phase function at every sampling point. Therefore, we must *unwrap* (noisy) samples of the principal phase function[1] (the so-called *wrapped phase*) to obtain the continuous phase function (the so-called *unwrapped phase*) relating to the physical quantity.

First of all, in order to make the following discussion clear, we classify phase unwrapping problems treated in this thesis into two types.

Problem 1 *(Phase Unwrapping Problem of Type I) Suppose that a continuous complex-valued function $F : [a, b] \to \mathbb{C}$, s.t. F is continuously differentiable almost everywhere on $[a, b]$, is given (or a continuous complex-valued function $f : \Omega \to \mathbb{C}$, s.t. f is continuously differentiable almost everywhere on Ω, and a piecewise C^1-path $\Upsilon : [a, b] \to \Omega$ are given, and define $F := f \circ \Upsilon$). Compute the unwrapped phase of F (or the unwrapped phase of f along Υ):*

$$\theta_F(t^*) = \theta_F(a) + \int_a^{t^*} \Im \left[\frac{F'(t)}{F(t)} \right] \mathrm{d}t \tag{1.1}$$

for $t^* \in [a, b]$, where $\theta_F(a) := \vartheta_0$ s.t. $F(a) = |F(a)|e^{\imath\vartheta_0}$ is given as the initial phase.[2]

Problem 2 *(Phase Unwrapping Problem of Type II) Suppose that there exists the unique unwrapped phase $\Theta_f : \Omega \to \mathbb{R}$ of an unknown continuous complex-valued function $f : \Omega \to \mathbb{C} \setminus \{0\}$, and finite noisy samples of f are observed at $\boldsymbol{x} \in \mathcal{G}$ $(\subset \Omega)$ as*

$$f(\boldsymbol{x}) + \epsilon(\boldsymbol{x}) \in \mathbb{C} \setminus \{0\}, \tag{1.2}$$

where $\mathcal{G}$ is the set of finite sampling points and $\epsilon(\boldsymbol{x}) \in \mathbb{C}$ is additive complex noise at $\boldsymbol{x}$. Estimate the unwrapped phase $\Theta_f(\boldsymbol{x})$ at every $\boldsymbol{x} \in \mathcal{G}$ from the noisy samples of f (or samples of the wrapped phase[3] *$\Theta_f^W(\boldsymbol{x}) := W(\Theta_f(\boldsymbol{x}) + \nu(\boldsymbol{x})) \in (-\pi, \pi]$, where $\nu(\boldsymbol{x}) \in (-\pi, \pi]$ is phase noise at $\boldsymbol{x}$) and the given initial phase $\Theta_f(\boldsymbol{x}_0) := \vartheta_0$ for some $\boldsymbol{x}_0 \in \mathcal{G}$.*

---

[1] The name "principal phase function" is used following [29, 30].

[2] In this thesis, $\vartheta_0 \in \mathbb{R}$ denotes the given initial phase, and $\vartheta_c \in \mathbb{R}$ (s.t. $c \in \mathbb{C} \setminus \{0\}$) denotes the phase of a nonzero complex number c (see Section 2.2).

[3] See (2.4) and (2.5) in Section 2.2 on the definition of the wrapping operator $W : \mathbb{R} \to (-\pi, \pi]$.

Problem 1 is especially important when f is a *complex polynomial*. This is because, in this case, the unwrapped phase has a close relation to the stability of a certain digital filter [31, 32, 33, 34]. Moreover, the unwrapped phase is needed to compute the *complex cepstrum* [35, 36] which plays a key role in *spectral factorization* [37, 38, 39]. In Problem 1, since F is known, we can compute the value of F at any point on the interval $[a, b]$. Therefore, the integral in (1.1) are computed by using numerical integration techniques proposed, e.g., in [40, 41, 42]. However, there is no guarantee that such numerical integration techniques give the exact unwrapped phase even if f is a complex polynomial.

The exact closed-form solution of Problem 1 for a complex polynomial can be given by *algebraic phase unwrapping* [43, 44, 45]. Algebraic phase unwrapping along the unit circle was first established in [43] by extending the discovery of direct relation between a real polynomial and its unwrapped phase along the unit circle [46]. As its continuations, algebraic phase unwrapping along the imaginary axis [44] and along the real axis [45] have been developed. These methods [43, 44, 45] do not require any numerical root finding or numerical integration technique, and are essentially based on the computations of certain *Sturm sequences*[4] generated by polynomial division type algorithms. However, in the computations of the Sturm sequences, we encounter numerical instabilities due to *coefficient growth* [49], which also happens in the computation of the *polynomial remainder sequence*, to derive the *greatest common divisor* of a pair of polynomials, by the Euclidean algorithm. As a result, direct implementations of algebraic phase unwrapping sometimes fail to compute the exact unwrapped phase, especially for complex polynomials of high degree, which restricts the practical applicability of algebraic phase unwrapping.

Problem 2 is more challenging because f is not given differently from Problem 1, and we can use only finite noisy samples of f. In particular, if $\Omega \subset \mathbb{R}^2$, Problem 2 is called a *two-dimensional (2D) phase unwrapping problem* [1, 2], which is required to be solved in many applications such as *terrain height estimation* and *landslide identification* by interferometric synthetic aperture radar (InSAR) [3, 4, 5, 6, 7, 8, 9], *seafloor depth estimation* by interferometric synthetic aperture sonar [10, 11, 12, 13], *three-dimensional (3D) measurement* by fringe projection [14, 15, 16, 17, 18, 19, 20] or X-ray [21, 22, 23, 24], and *water/fat separation* in MRI [25, 26, 27, 28]. All commonly used 2D phase unwrapping methods [5, 27, 28, 50, 51, 52, 53, 54, 55, 56, 57, 58] assume that $\Theta_f(\widetilde{x}, \widetilde{y}) - \Theta_f(x, y) \in (-\pi, \pi]$ for most pairs of neighboring sampling points $((x, y), (\widetilde{x}, \widetilde{y})) \in \mathcal{G} \times \mathcal{G}$, and hence these methods try to construct $\Theta(x, y)$ at every $(x, y) \in \mathcal{G}$, as an estimate of $\Theta_f(x, y)$, s.t. $\Theta(\widetilde{x}, \widetilde{y}) - \Theta(x, y) \approx W(\Theta_f^W(\widetilde{x}, \widetilde{y}) - \Theta_f^W(x, y))$ for as many as possible pairs of neighboring sampling points $((x, y), (\widetilde{x}, \widetilde{y})) \in \mathcal{G} \times \mathcal{G}$. Despite tremendous efforts made so far, a widely used 2D phase unwrapping method, for various applications, has not yet been established. This is because many existing methods are based on NP-hard optimization problems [53] or do not check the consistency $W(\Theta(x, y)) \approx \Theta_f^W(x, y)$ at $(x, y) \in \mathcal{G}$.

One of the main goals of this thesis is a significant expansion of the application range of algebraic phase unwrapping [43, 44, 45]. For this purpose, we resolve the numerical instabilities of algebraic phase unwrapping by avoiding the computations of the inductive polynomial division. To be concrete, we generate new Sturm sequences by modifying the polynomial division type algorithms, and the redefined Sturm sequences can be expressed with the use of the *subresultant* [49, 59, 60, 61], which is a polynomial defined as the

[4] On the standard Sturm sequence, which is used to compute the number of zeros of a real polynomial on a given real interval, see, e.g., [47, Section 6.3.III] and [48, Section 38].

determinant of a certain matrix. Therefore, in order to escape from the numerical instabilities due to the coefficient growth, we propose to replace inductive computations of the redefined Strum sequences with direct numerical computations of the determinants of the matrices. By the proposed replacements, algebraic phase unwrapping can stably compute the exact unwrapped phase even for complex polynomials of high degree.

Moreover, for solving 2D phase unwrapping problems, we propose a completely novel approach using algebraic phase unwrapping. The key of the proposed algebraic approach is *spline smoothing* introduced in the next section.

1.2 Spline Smoothing

Spline is a function which is piecewise-defined by polynomials, and which possesses certain-times continuous differentiability at the places where the polynomial pieces connect, i.e., spline is a ρ-times ($\rho \in \mathbb{Z}_+$) continuously differentiable piecewise polynomial. Spline functions have been widely used for interpolation and smoothing of data in many signal and image processing applications [62], e.g., super-resolution [63, 64], computer aided design [65, 66], and regression analysis [67, 68]. The most commonly used spline functions are *cubic splines*, i.e., univariate spline functions which are expressed, on subintervals, as polynomials of degree 3 at most. This is because cubic splines are the unique solutions of the following one-dimensional (1D) variational problems [69, 70, 71, 72].

Problem 3 (Variational Problem on 1D Interpolation) *Find $f^* \in C^2(\mathbb{R})$ minimizing*

$$\int_{\mathbb{R}} |f''(x)|^2 \, \mathrm{d}x$$

subject to

$$f(x_i) = \zeta_i \ (i = 0, 1, \ldots, n).$$

Problem 4 (Variational Problem on 1D Smoothing) *Find $f^* \in C^2(\mathbb{R})$ minimizing*

$$\sum_{i=0}^{n} |\zeta_i - f(x_i)|^2 + \lambda \int_{\mathbb{R}} |f''(x)|^2 \, \mathrm{d}x,$$

where the smoothing parameter $\lambda > 0$ controls the trade-off between the data fidelity and the smoothness.

Problem 3 is called (1D) *spline interpolation* and it is especially effective when noise-free data are available [63, 64, 65, 66]. Problem 4 is called (1D) *spline smoothing* and it is often used for design of continuous functions from noisy samples [67, 68].

However, even if an unknown continuous function to be estimated is smooth,[5] spline interpolation and spline smoothing not always work well. For example, design of *non-negative* continuous functions such as probability density function [73, 74] and power spectral density [75, 76] is also required in many applications, e.g., pattern recognition [77, 78], quantization [79], filtering [80], data analysis [81], speech enhancement [82], speech recognition [83] and sound source separation [84]. In such situations, spline inter-

[5] In this thesis, the word "smooth" means the L_2 norm of the second order (partial) derivative is small.

polation and spline smoothing have been hardly applicable because the nonnegativity of f^* is not guaranteed in general, even if $\zeta_i \geq 0$ ($i = 0, 1, \ldots, n$), as shown in [85].

The other main goal of this thesis is development of some novel spline smoothing techniques, including a combination with algebraic phase unwrapping. For this purpose, we propose *nonnegative spline interpolation/smoothing* as optimization problems to design nonnegative continuous functions from nonnegative samples. To guarantee the nonnegativity of spline functions, we derive some sufficient conditions by generalizing the idea presented, for quartic splines, in [86]. Under the proposed sufficient condition, nonnegative spline interpolation/smoothing are expressed as convex quadratic programming problems, and numerical experiments show that we can construct satisfactorily smooth spline functions within the the proposed sufficient condition. Moreover, for 2D phase unwrapping, we propose *other spline smoothing formulations*, by relaxing the interpolation condition of spline interpolation, and combine them with algebraic phase unwrapping as follows.

In the spirit of *functional data analysis* [70, 71, 72], we consider the situation where the unknown continuous complex-valued function $f : \Omega \to \mathbb{C} \setminus \{0\}$ is approximated from its noisy samples in (1.2) by $f^* := f^*_{(0)} + \imath f^*_{(1)}$ through a certain smoothing technique, where $f^*_{(k)} : \Omega \to \mathbb{R}$ ($k = 0, 1$) are continuous functions having no common zero on Ω. In this situation, if the unwrapped phase of f^* is uniquely defined independently of the integration path, it is natural to estimate the unknown unwrapped phase Θ_f by

$$\Theta_{f^*}(x, y) = \Theta^{[\Upsilon]}_{f^*}(x, y) = \theta_{F^*}(b) = \theta_{F^*}(a) + \int_a^b \Im \left[\frac{F^{*\prime}(t)}{F^*(t)} \right] \mathrm{d}t, \tag{1.3}$$

where $\Upsilon : [a, b] \to \Omega$ is any piecewise C^1-path s.t. $\Upsilon(a) := (x_0, y_0)$ and $\Upsilon(b) := (x, y)$, $F^* := f^* \circ \Upsilon$, and $\Theta_{f^*}(x_0, y_0) = \theta_{F^*}(a) := \vartheta_0$ ($= \Theta_f(x_0, y_0)$) s.t. $F^*(a) = |F^*(a)| e^{\imath \vartheta_0}$ is given as the initial phase. In particular, motivated by the great success in the use of spline smoothing [62, 69, 87, 88, 89, 90, 91, 92, 93, 94] in functional data analysis, in this thesis, we construct $f^*_{(k)} \in C^2(\Omega)$ ($k = 0, 1$) as *bivariate spline functions* through the proposed spline smoothing formulation. In such a case, by choosing a suitable piecewise C^1-path Υ and dividing the interval $[a, b]$ into finite subintervals, we can exactly compute the integral in (1.3) by using repeatedly *algebraic phase unwrapping* along the real axis [45] because f^* is a piecewise complex polynomial. Furthermore, if f^* interpolates the observed value $f(x, y) + \epsilon(x, y) \neq 0$ at some sampling point $(x, y) \in \mathcal{G}$, then the desired consistency $W(\Theta_{f^*}(x, y)) = \Theta^W_f(x, y)$ is guaranteed at (x, y) without solving the NP-hard optimization problems. Numerical experiments for InSAR terrain height estimation demonstrate the effectiveness of the proposed 2D phase unwrapping scheme, and hence, the application range of algebraic phase unwrapping can be expanded, by the combination with the proposed spline smoothing formulation, to practical signal processing applications.

As related works, in this thesis, we also treat the *single-frame fringe projection profilometry* [95, 96, 97, 98, 99] and *probability density estimation* [73, 74, 100]. For the single-frame fringe projection profilometry, in order to estimate the signs of the wrapped phase on 2D lattice points before 2D phase unwrapping, we propose a branch cut type algorithm which is inspired by Goldstein's combinatorial approach to 2D phase unwrapping [5]. For probability density estimation, by modifying the idea of the proposed nonnegative spline smoothing, we estimate an unknown smooth probability density function (PDF) from its histogram as a nonnegative spline function. Numerical experiments show the effectiveness of the proposed branch cut type algorithm and the proposed PDF estimation.

1.3 Organization

In Chapter 2, as preliminaries, first we give the notation used in this thesis, and define the unwrapped phase of a complex-valued function mathematically. Second, we introduce algebraic phase unwrapping along the real axis [45] (Fact 1) and along the unit circle [43] (Fact 2) which can exactly solve phase unwrapping problems of Type I for univariate complex polynomials (Problems 5 and 6). Third, we indicate spline functions treated in this thesis, including their optimality (Facts 3 and 4), and formulate 2D spline interpolation and smoothing (Problems 13 and 14) as certain 2D extensions of Problems 3 and 4.

In Chapter 3, in order to firmly establish the application of algebraic phase unwrapping to spline functions, we mention the conditions, on the starting point of the integral in (1.1), required in [43, 45] and the numerical instabilities in the direct computer implementations of algebraic phase unwrapping. At first, we modify the original polynomial division type algorithms (Algorithms 1 and 2) in [43, 45] and generate new Sturm sequences. The modified algorithms (Algorithms 3 and 4) enable us to compute the exact unwrapped phase, as refined algebraic phase unwrapping (Theorems 1 and 3), without the conditions required in [43, 45]. Moreover, we show that the redefined Sturm sequences have close relations to the *subresultant* [49, 59, 60, 61] and the newly defined subresultant, the *self-reciprocal subresultant*, which are polynomials defined as the determinants of certain matrices. Then, after explaining typical numerical instabilities observed in the computer implementations of algebraic phase unwrapping, we propose to replace the inductive computations of the redefined Strum sequences with the direct numerical computations of the subresultants and the self-reciprocal subresultants, i.e., the computations of the determinants of the matrices (Theorems 5 and 6). Numerical experiments show that algebraic phase unwrapping can be significantly stabilized by the proposed replacements.

In Chapter 4, we propose a novel smoothing technique, *nonnnegative spline smoothing*, to estimate a nonnegative continuous function from its noisy nonnegative samples. The proposed nonnegative spline smoothing, including nonnegative spline interpolation, (Problems 15, 16, 19, and 20) are formulated as natural extensions of the standard spline interpolation and smoothing in Chapter 2. Since it is very difficult to give useful criteria which are necessary and sufficient conditions for guaranteeing the nonnegativity of spline functions for any point in their domains, we generalize the sufficient conditions derived in [86] for univariate spline functions and bivariate spline functions on rectangular grid (Theorems 7 and 8), and newly derive some sufficient conditions for bivariate spline functions on triangular grid (Theorem 9), as linear inequalities on the coefficients of the spine functions. In consequence, under the proposed sufficient condition, nonnnegative spline interpolation and smoothing are expressed as convex quadratic programming problems (Problems 17 and 18). Numerical experiments demonstrate that we can construct nonnegative and satisfactorily smooth spline functions within the proposed sufficient condition.

In Chapter 5, for solving phase unwrapping problems of Type II s.t. $\Omega \subset \mathbb{R}^2$, i.e., *2D phase unwrapping problems* [1, 2], we propose an algebraic approach based on algebraic phase unwrapping and spline smoothing. After clarifying the condition for guaranteeing the path independence of the 2D unwrapped phase (Theorem 10), we estimate an unknown continuous phase function as the unwrapped phase Θ_{f^*} of a twice continuously differentiable complex-valued function $f^* := f^*_{(0)} + \imath f^*_{(1)}$ s.t. $f^*_{(k)} : \Omega \to \mathbb{R}$ $(k = 0, 1)$. $f^*_{(k)}$ $(k = 0, 1)$ are constructed by 2D spline interpolation (Problem 21) or the *generalized*

2D spline smoothing formulations (Problems 22 ans 23), and Θ_{f^*} is exactly obtained by algebraic phase unwrapping along the real axis [45] if $f^*_{(k)}$ $(k = 0, 1)$ have no common zero on Ω. To avoid the occurrence of common zeros due to phase noise in the observed wrapped phase, we also propose a denoising step, as preprocessing, which selectively smooths unreliable samples by using convex optimization (Problem 24). The smoothness of Θ_{f^*} is guaranteed globally over the domain without losing the desired consistency with any reliable wrapped sample. Numerical experiments for InSAR terrain height estimation exemplify the effectiveness of the proposed 2D phase unwrapping scheme.

In Chapter 6, as a related work, we treat the *single-frame fringe projection profilometry* [95, 96, 97, 98, 99] for 3D measurement of moving objects. In this application, differently from the standard fringe projection profilometry [14, 15, 16, 17, 18, 19, 20] where we can use at least three fringe images and detect the wrapped phase values on 2D lattice points, we can detect only up to the absolute values of the wrapped phase because we can use only a single fringe image. Therefore, before 2D phase unwrapping, we have to estimate the signs of the wrapped phase on 2D lattice points. For this purpose, we newly formulate a certain energy minimization problem (Problem 25) so that we use a minimizer of the energy as an estimate of the signs of the wrapped phase. To approximately solve this binary combinatorial optimization problem, we propose a branch cut type algorithm which is inspired by Goldstein's combinatorial approach to 2D phase unwrapping [5]. Numerical experiments demonstrate that the proposed method provides a remarkable improvement over an existing path-following method [101] especially around the edges of objects.

In Chapter 7, we estimate smooth PDFs with the use of nonnegative spline functions. In this situation, nonnegative spline interpolation and smoothing proposed in Chapter 4 are hardly applicable because the data points to be interpolated (or to be approximated) are not available, and we can only achieve finite observed values, of a random variable, which are generated from an unknown PDF. By slightly changing the idea of nonnegative spline smoothing, we regard a histogram constructed from the observed values as known noisy data, and hence, the proposed (1D and 2D) PDF estimation (Problems 27 and 29) are formulated by replacing the data fidelity terms in nonnegative spline smoothing with the square error between the ratio of the observed values belonging to each bin of the histogram and the integrated value of a spline function over each bin. Numerical experiments, in the situation where unknown smooth PDFs are Gaussian mixtures, show the effectiveness of the proposed PDF estimation by comparison with *kernel density estimation* [73, 74] which has been widely used as a nonparametric PDF estimator.

Finally, in Chapter 8, we conclude this thesis.

Publication List

Journal Paper

1. Daichi Kitahara and Isao Yamada, "Algebraic phase unwrapping along the real axis: extensions and stabilizations," *Multidimensional Systems and Signal Processing*, vol. 26, no. 1, pp. 3–45, Jan. 2015.

2. Daichi Kitahara and Isao Yamada, "Algebraic phase unwrapping based on two-dimensional spline smoothing over triangles," *IEEE Transactions on Signal Processing*, vol. 64, no. 8, pp. 2103–2118, Apr. 2016.

International Conference

1. Daichi Kitahara and Isao Yamada, "Algebraic phase unwrapping for functional data analytic estimations—extensions and stabilizations," in *Proceedings of 2013 IEEE International Conference on Acoustics, Speech and Signal Processing*, 2013, pp. 5835–5839.

2. Daichi Kitahara and Isao Yamada, "Algebraic phase unwrapping over collection of triangles based on two-dimensional spline smoothing," in *Proceedings of 2014 IEEE International Conference on Acoustics, Speech and Signal Processing*, 2014, pp. 4996–5000.

3. Daichi Kitahara and Isao Yamada, "Probability density function estimation by positive quartic C^2-spline functions," in *Proceedings of 2015 IEEE International Conference on Acoustics, Speech and Signal Processing*, 2015, pp. 3556–3560.

4. Daichi Kitahara, Masao Yamagishi, and Isao Yamada, "A virtual resampling technique for algebraic two-dimensional phase unwrapping," in *Proceedings of 2015 IEEE International Conference on Acoustics, Speech and Signal Processing*, 2015, pp. 3871–3875.

5. Daichi Kitahara and Isao Yamada, "Two-dimensional positive spline smoothing and its application to probability density estimation," in *Proceedings of 2016 IEEE International Conference on Acoustics, Speech and Signal Processing*, 2016, pp. 4219–4223.

6. Kenji Kakimoto, Daichi Kitahara, Masao Yamagishi, and Isao Yamada, "Stabilization of adaptive eigenvector extraction by continuation in nested orthogonal complement structure," in *Proceedings of 2016 IEEE International Conference on Acoustics, Speech and Signal Processing*, 2016, pp. 4189–4193.

7. Masao Yamagishi, Daichi Kitahara, and Isao Yamada, "A fast dual iterative algorithm for convexly constrained spline smoothing," in *Proceedings of 2016 IEEE InternationalConference on Acoustics, Speech and Signal Processing*, 2016, pp. 4538–4542.

Domestic Conference

1. Daichi Kitahara and Isao Yamada, "A stable sign estimation for algebraic phase unwrapping," in *Proceedings of 26th IEICE SIP Symposium*, 4 pages, 2011.

2. Daichi Kitahara and Isao Yamada, "A stabilization of algebraic phase unwrapping by subresultant," in *Proceedings of 27th IEICE SIP Symposium*, 5 pages, 2012.

3. Daichi Kitahara and Isao Yamada, "Algebraic phase unwrapping over collection of triangles and its application," in *Proceedings of 28th IEICE SIP Symposium*, 4 pages, 2013.

4. Daichi Kitahara and Isao Yamada, "One-dimensional probability density function estimation by positive quartic C^2-spline interpolation and smoothing," in *Proceedings of 29th IEICE SIP Symposium*, pp. 462–467, 2014.

5. Daichi Kitahara, Masao Yamagishi, and Isao Yamada, "A preprocessing using convex optimization for algebraic two-dimensional phase unwrapping," in *Proceedings of 29th IEICE SIP Symposium*, pp. 369–374, 2014.

6. Masao Yamagishi, Daichi Kitahara, and Isao Yamada, "A fast Gauss-Seidel-like splitting algorithm for convexly constrained spline smoothing," in *Proceedings of 29th IEICE SIP Symposium*, pp. 64–67, 2014.

7. Daichi Kitahara and Isao Yamada, "A smoothness-aware phase unwrapping by convex optimization technique," in *Proceedings of 40th SICE Symposium on Remote Sensing*, pp. 1–2, 2015.

8. Daichi Kitahara and Isao Yamada, "Positive quartic and biquartic C^2-spline and their applications to two-dimensional probability density estimation," in *Proceedings of 30th IEICE SIP Symposium*, pp. 344–349, 2015.

9. Daichi Kitahara and Isao Yamada, "Two-dimensional phase unwrapping based on selective smoothing and inconsistency correction," in *Proceedings of 30th IEICE SIP Symposium*, pp. 361–366, 2015.

10. Kenji Kakimoto, Daichi Kitahara, Masao Yamagishi, and Isao Yamada, "On a projection step by generalized orthogonal complement matrices for adaptive subspace tracking," in *Proceedings of 30th IEICE SIP Symposium*, pp. 328–331, 2015.

11. Daichi Kitahara and Isao Yamada, "Algebraic phase unwrapping based on two-dimensional spline smoothing," in *Proceedings of 8th SICE Symposium on Computational Intelligence*, pp. 11–18, 2015.

12. Daichi Kitahara and Isao Yamada, "A branch cut type sign estimator for 3D measurement from a single fringe pattern," in *Proceedings of 9th SICE Symposium on Computational Intelligence*, pp. 73–76, 2015.

13. Daichi Kitahara and Isao Yamada, "A branch cut type sign estimator for single-frame fringe projection prolometry," in *Proceedings of 31th IEICE SIP Symposium*, pp. 157–161, 2016.

14. Kazuya Mori, Daichi Kitahara, Masao Yamagishi, and Isao Yamada, “A robust global positioning system with triple phase differences and Huber loss function,” (in Japanese), in *Proceedings of 31th IEICE SIP Symposium*, pp. 427–432, 2016.

15. Isao Yamada and Daichi Kitahara, “Continuation of phase—Algebraic phase unwrapping and its applications,” (in Japanese), in *Proceedings of IEICE Society Conference 2016*, 2 pages, 2016.

16. Daichi Kitahara and Isao Yamada, “A stabilization of algebraic phase unwrapping along the unit circle with self-reciprocal subresultant,” in *Proceedings of IEICE Society Conference 2017*, 1 page, 2017.

Other Publication

1. Daichi Kitahara and Isao Yamada, “Mixed trigonometric interpolation techniques for fast and stable algebraic phase unwrapping,” in *IEICE Technical Report*, pp. 303–307, Mar. 2012.

2. Daichi Kitahara and Isao Yamada, “A robust algebraic phase unwrapping based on spline approximation,” in *IEICE Technical Report*, pp. 1–6, Jul. 2012.

Patent

1. 山田 功，北原 大地．信号処理装置、信号処理方法およびプログラム．特許第 6041325 号，出願日 2013/2/22，登録日 2016/11/18．

Award

1. 電子情報通信学会 基礎境界ソサイエティ 信号処理研究専門委員会 SIP 若手奨励賞 (27th IEICE SIP Symposium, Nov. 2012).

2. 計測自動制御学会 計測部門 リモートセンシング部会 部会奨励賞 (40th SICE Symposium on Remote Sensing, Mar. 2015).

3. IEEE Computational Intelligence Society Japan Chapter Young Researcher Award (8th SICE Symposium on Computational Intelligence, Dec. 2015).

4. IEEE Signal Processing Society Japan Student Best Paper Award (IEEE Transactions on Signal Processing, vol. 64, no. 8, pp. 2103–2118, Apr. 2016).