論文 / 著書情報
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題目(和文)	シューベルト多様体の Newton-Okounkov 多面体と結晶基底
Title(English)	Newton-Okounkov polytopes of Schubert varieties and crystal bases
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種別(和文)	論文要旨
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論 文 要 旨

THESIS SUMMARY

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学生氏名： Student's Name	藤田 直樹	指導教員（主）： Academic Supervisor(main)	内藤 聡 教授
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要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

A Newton-Okounkov body is a convex body constructed from a polarized variety with a valuation on its function field; this generalizes the notion of Newton polytopes for toric varieties. The theory of Newton-Okounkov bodies was introduced by Okounkov, and afterward developed independently by Kaveh-Khovanskii and by Lazarsfeld-Mustata. If a Newton-Okounkov body is a convex polytope, then we call it a Newton-Okounkov polytope. A Newton-Okounkov polytope inherits information about algebraic, geometric, and combinatorial properties of the original projective variety. Indeed, the theory of Newton-Okounkov polytopes gives a systematic method of constructing toric degenerations of projective varieties.

In this thesis, we study Newton-Okounkov polytopes of Schubert varieties via Kashiwara's crystal bases in representation theory. Such researches were initiated by Kaveh, who proved that Berenstein-Littelmann-Zelevinsky's string polytope is identical to the Newton-Okounkov polytope of a Schubert variety associated with a specific valuation. The main results of this thesis are three-fold.

(1) Newton-Okounkov polytopes and polyhedral realizations of crystal bases:

The Kashiwara embedding gives a parametrization of a highest weight crystal basis, which yields an explicit description of Kashiwara operators. Under some technical assumptions, Nakashima described the image of the Kashiwara embedding as the set of lattice points in some explicit rational convex polytope; this description is called Nakashima-Zelevinsky's polyhedral realization of a crystal basis.

In this thesis, we relate the Kashiwara embedding with a specific valuation on the function field of a Schubert variety. From this, we deduce that Nakashima-Zelevinsky's polyhedral realization of a highest weight crystal basis is identical to the Newton-Okounkov polytope of a Schubert variety associated with a specific valuation. This result gives a new class of specific examples of Newton-Okounkov polytopes of Schubert varieties, which we can compute explicitly. As an application of this approach, we show without any assumptions that the image of the Kashiwara embedding is identical to the set of lattice points in some rational convex polytope. In addition, by combining our result with Kaveh's result, we see that Kashiwara's involution corresponds to a change of specific valuations. This is based on joint work with Satoshi Naito.

(2) Geometrically natural valuations and perfect bases with positivity properties:

The specific valuations used by Kaveh and in (1) are defined algebraically to be highest term valuations. Another kind of valuation, which is geometrically natural, is given by counting the orders of zeros along a sequence of subvarieties. In this thesis, we relate the highest term valuations used by Kaveh and in (1) with such geometrically natural valuations. More precisely, we show that, on a perfect basis with some positivity properties, the highest term valuations are identical to the valuations coming from sequences of specific subvarieties of a Schubert variety; the existence of such a perfect basis follows from Khovanov-Lauda-Rouquier's categorification of the negative part of the quantized enveloping algebra. From these, we deduce that the associated Newton-Okounkov polytopes coincide. This result gives new geometric interpretations of string polytopes and polyhedral realizations of crystal bases. This is based on joint work with Hironori Oya.

(3) Folding procedure for Newton-Okounkov polytopes of Schubert varieties:

Finally, we apply the folding procedure to Newton-Okounkov polytopes, which relates Newton-Okounkov polytopes of Schubert varieties of different Dynkin types. Since string polytopes and polyhedral realizations are realized as Newton-Okounkov polytopes of Schubert varieties, we can apply to these polytopes the folding procedure for Schubert varieties and also that for crystal bases. The folding procedure for Schubert varieties (resp., for crystal bases) relates these polytopes for a simply-laced

semisimple Lie algebra with those for its fixed point Lie subalgebra (resp., for its orbit Lie algebra); the orbit Lie algebra is the Langlands dual of the fixed point Lie subalgebra. Since the simple Lie algebra of type B (resp., type C) is a fixed point Lie subalgebra of that of type D (resp., type A), and also is an orbit Lie algebra of that of type A (resp., type D), we obtain relations among Newton-Okounkov polytopes of Schubert varieties of types A, B, C, and D. This leads to a new interpretation of Kashiwara's similarity between crystal bases in type B and those in type C.