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論 文 要 旨

THESIS SUMMARY

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指導教員（主）： Academic Supervisor(main)	内藤 聡	
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要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

Quantum groups were introduced independently by Drinfel'd and Jimbo in the mid of 1980's in order to solve the quantum Yang-Baxter equation, which is important in mathematical physics. The quantum groups are also related to quantum invariants in knot theory. Here, we focus on the representation theory of quantum groups. Although the classification of finite-dimensional irreducible modules is done in a way similar to that in the classical case (namely, by highest weight theory), there are concepts peculiar to the quantum case. Below, we explain about Kashiwara's crystal bases, Lusztig's canonical bases, and Jimbo's q-Schur duality.

A crystal basis is a "local" basis of a module, which extract some combinatorial aspects of the module structure. Hence, we can reduce many problems in representation theory to those in combinatorics. It is well-known that the crystal basis of each finite-dimensional irreducible module can be "globalized" to a genuine basis, the global crystal basis (or, the canonical basis). The global crystal bases tell us more information about the module structure than the local crystal bases do.

Jimbo discovered that the quantum group associated with the special linear Lie algebra and the Hecke algebra associated with the symmetric group form a double centralizer. In particular, this shows that the representation theories of these algebras are closely related to each other. Later this duality was upgraded to the statement that the canonical basis of a module coincides with the Kazhdan-Lusztig basis. This result was then used to formulate the Kazhdan-Lusztig theory without the direct use of Hecke algebras.

Recently, the quantum symmetric pairs are getting attention. A quantum symmetric pair is a pair of a quantum group and its coideal subalgebra; the classical limit of the coideal subalgebra becomes the fixed-point subalgebra under an involution on a semisimple Lie algebra. Quantum symmetric pairs have played important roles in many branches of mathematics such as the representation theory of Lie superalgebras, low-dimensional topology, and integrable systems. Also, many counterparts of important concepts for quantum groups have been discovered in the setting of quantum symmetric pairs. Especially, the canonical basis for a quantum symmetric pair is influential. However, the classification of the irreducible modules, which is one of the most basic problems in representation theory, has not been settled yet. As a first step toward this problem, in this thesis we study the representation theory of a special class of quantum symmetric pairs.

Let us explain the main results obtained in this thesis. The first one is the multiparameter upgrade of the q-Schur duality in type B. This is an analog of Jimbo's q-Schur duality. Also, we prove that the canonical basis of a module over a quantum symmetric pair coincides with the Kazhdan-Lusztig basis of the Hecke algebra of type B. This result enables us to formulate the Kazhdan-Lusztig theory for the ortho-symplectic Lie superalgebras in terms of the canonical bases for quantum symmetric pairs.

The second one is the classification of finite-dimensional irreducible modules for quantum symmetric pairs. This is done by means of highest weight theory; it is a pleasing result that the highest weight theory, which is at the heart of the representation theory of quantum groups, is (after a modification) applicable to our setting.

The third one is the establishment of crystal basis theory for quantum symmetric pairs. Namely, we define the notion of j-crystal bases by extracting some combinatorial aspects of the module structure, and we prove the existence and uniqueness theorems for j-crystal bases. Also, we observe that we can analyze the module structure by means of these bases and their "globalized" ones.

備考 ： 論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。
Note : Thesis Summary should be submitted in either a copy of 2000 Japanese Characters and 300 Words (English) or 1copy of 800 Words (English).