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論 文 要 旨

THESIS SUMMARY

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要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

We present structure and applications of the differential game framework with an infinite time horizon. The differential game can be viewed as a repeated game (or a dynamic game) in which the stage game is played repeatedly in continuous time and agents' strategies dynamically manipulate the evolution of the external state over time. In order to derive optimal solutions to the game, we generalize the differential game at each instantaneous moment of the continuous time horizon, and show a long-run optimal solution to the differential game can be characterized by clarifying an optimal control path of each instantaneous moment. This result suggests both the differential game and the long-run optimal solution can be expressed in the recursive form. Making use of the recursive structure, we verify how the Hamilton-Jacobi-Bellman equation (or HJB equation for short) and Pontryagin's maximum principle work in characterizing an optimal control path (i.e., an optimal solution) of the differential game. In particular, the HJB equation provides a sufficient condition for the optimal control of the differential game, whereas Pontryagin's maximum principle only yields a necessary condition. Therefore, the HJB equation is the main technique that is used to characterize the equilibria in differential game models.

There exist two kinds of strategies in the differential game: the open-loop strategy and the Markovian strategy. In the open-loop strategy, each agent determines a control path (i.e., a sequence of actions over the infinite time horizon) before the starts of the game, and then sticks to the control path throughout the entire duration of the game. In this sense, none of the agents revises its action according to the external state after the start of the game. Different from the open-loop strategy, when the Markovian strategy is chosen, the information on the external state is always available to the agents, and each agent dynamically conditions its action on the external state for each instantaneous moment. We use the HJB equation to show the Markovian strategy can form a Markov perfect equilibrium (MPE) in which each agent's dynamic actions satisfy the subgame perfectness (i.e., each agent dynamically reoptimizes its strategy along the control path of the equilibrium). This result suggests the MPE is more acceptable than the open-loop equilibrium, and thereby we are concerned with precise characterization and examination of analytic MPEs in applications of the differential game.

In theoretical research in economics, the differential game with quadratic utility function and linear transition function of the external state receives large attention, because such a

linear-quadratic form (or LQ-form) permits the characterization of analytic MPEs. In the rest of the thesis, we employ the LQ-form differential game to formulate the following three subjects. In particular, we restrict attention to the stable MPE in which the equilibrium strategies guide the external state to converge to a steady state.

Public goods provision under dynamic budgets: We formulate the public goods provision problem subject to dynamic budget constraints. Each individual (i.e, each agent) is assumed to earn income over time, and allocates the income between the private consumption and the public goods provision subject to dynamic budgets. Individuals' contribution of the public good results in a positive stock externality of the game. We characterize all stable MPEs including a linear one and infinitely many nonlinear ones, and show in the set of the stable MPEs, the level of provision of public goods is increasing in the income. Thus, it can be concluded that in order to enhance the level of public goods at the steady state, the individuals need to choose a stable MPE with higher income and higher public goods provision.

A dynamic policy game of the combination use of emission taxation and a pollution-removal technique: The game comprises multiple polluting oligopolistic firms and a government who takes charge of the pollution management. The government seeks to optimally control the pollution by means of emission taxation together with a pollution-removal technique. During the play of the game, we assume the government first collects the emission tax from the firms, and then spends tax revenue on the expense of the pollution-abatement efforts or equally redistributes the tax revenue to each firm. We show there only exists a unique stable linear MPE in the game. In the MPE, when the pollution level is sufficiently close to the steady state, the tax revenue exceeds the cost of the pollution-removal efforts, so that the government can reimburse the surplus of the tax revenue to the firms.

The best stable Markov perfect equilibrium with the highest long-run utility: In the dynamic renewable resource duopoly, we propose a new approach to show that for each feasible initial state, the long-run profit is increasing in the steady-state resource. This result implies the MPE with the highest steady state provides the highest long-run profit, and thereby is the best equilibrium of the game. Moreover, we generalize the LQ-form differential game framework, and state the above new approach is possibly valid to other LQ-form differential game model such as the transboundary pollution control model and the dynamic public goods provision problem.