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# Maximum Deformation Prediction Method for Tall Buildings with Hysteretic Dampers based on Energy Balance Considering Plasticization of Main Frame

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## Abstract

In recent years, there has been an increase in the adoption of passive-controlled buildings. Specifically, evaluation by cumulative value is also required in addition to the maximum response value when long-period seismic ground motion is applied to passive-controlled buildings with hysteresis damper. An energy-based method that evaluates the total amount of energy input by seismic motion is an extremely effective method to evaluate the cumulative value. However, when the method is applied to a passive-controlled building, studies that consider the plasticization of the main frame are performed only in the first story. In this study, we focused on maximum deformation, and we propose a prediction method for maximum deformation in all stories by considering plasticization of the main frame.

**Keywords:** hysteretic damper; passive controlled building; energy-based method; elasto-plastic response.

## 1 Introduction

In recent years, the strong possibility of long-period earthquakes that significantly exceed expectations (for e.g., the Nankai Trough earthquake) has become evident. There is a demand for continuous use of high-rise buildings in Japan even in the event of the afore mentioned earthquakes. Most high-rise buildings in Japan employ passive control of buildings to adapt to the high demands. Various types of vibration dampers have been developed in Japan. Hysteretic dampers using steel are the most frequently used vibration dampers for passive-controlled buildings. When long-period ground motion is applied to a damping structure with a hysteresis damper, it is necessary to evaluate the maximum response and the accumulated value. The ideal design of a passive controlled building has a mainframe that maintains its elasticity when subject to ground motions. However, it is necessary

to consider the plastic behavior of the mainframe of a passive-controlled building when designing buildings that can resist seismic ground motions exceeding the assumptions. In this case, seismic evaluation based on energy balance is effective [1]-[4].

In this paper, we propose a method to predict the maximum response of buildings with hysteretic dampers based on energy balance by considering the plasticity of the main frame.

## 2 Prediction of maximum inter-story deformation using energy method

### 2.1 Energy balance

Figure 1 shows the energy time history of the passive-controlled structure during the earthquake. Specifically,  $t_m$  denotes time when the maximum

response value occurs, and  $t_0$  denotes the duration of the ground motion. Additionally,  ${}_fW_e$ ,  ${}_fW_h$ , and  ${}_fW_p$  denote the elastic vibration energy of the main frame, dissipated energy due to damping, and cumulative plastic strain energy of the main frame, respectively. Furthermore,  ${}_dW_e$  and  ${}_dW_p$  denote the elastic vibration energy and cumulative plastic strain energy of the damper, respectively. The energy balance equation when  $t = t_m$  is given below:

$${}_fW_e(t_m) + {}_dW_e(t_m) + {}_fW_h(t_m) + {}_fW_p(t_m) + {}_dW_p(t_m) = E(t_m) \quad (1)$$

where  $E$  denotes input energy. The input energy  $E$  is given by the following equation using  ${}_fW_h$ .

$$E_D(t) = E(t) - {}_fW_h(t) \quad (2)$$

Here,  $E_D$  denotes Energy that contributes to structural damage [1]. As shown in Fig. 1,  $E(t_m)$  is generally lower than  $E(t_0)$  in a large plasticized building or a building with a damper. Therefore,  $E(t_m)$  is replaced with  $E(t_0)$  and Eq. (2) is substituted into Eq. (1), and the energy balance equation for  $t = t_m$  is given as follows:

$${}_fW_e(t_m) + {}_dW_e(t_m) + {}_fW_p(t_m) + {}_dW_p(t_m) = E_D(t_0) \quad (3)$$

The elastic vibration energies of the frame and damper ( ${}_fW_e$  and  ${}_dW_e$ ) reach their maximum values at  $t = t_m$  and almost disappear at  $t = t_0$ . Therefore, the energy balance equation when  $t = t_0$  is expressed as follows:

$${}_fW_p(t_0) + {}_dW_p(t_0) = E_D(t_0) \quad (4)$$

It is noted that the maximum value (for e.g., the maximum inter-story deformation  $\delta_{\max}$ ) is obtained from the balance equation (Eq. (3)) at the maximum response occurrence time  $t_m$ . Additionally, cumulative values (for e.g., cumulative plastic deformation ratio  ${}_f\eta_i$  of the main frame) is obtained from the balance equation (Eq. (4)) at earthquake end time  $t_0$ . The velocity conversion value VD for ED is calculated as follows:

$$V_D = \sqrt{\frac{2E_D}{M}} \quad (5)$$

The maximum shear force coefficient  ${}_f\alpha_0$  and maximum deformation  ${}_f\delta_0$  of the no-damper elastic-frame model are calculated by the following equation using VD, total mass of building  $M$ , and elastic 1st natural period  ${}_fT$  of the main frame, respectively.

$${}_f\alpha_0 = \frac{2\pi \cdot V_D}{{}_fT \cdot g}, \quad {}_f\delta_0 = \frac{{}_fT \cdot V_D}{2\pi} \quad (6, 7)$$

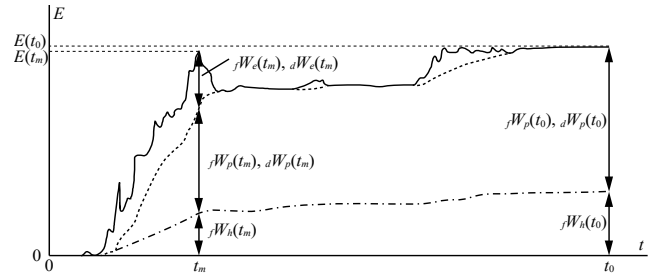


Fig.1 Energy time history

## 2.2 Prediction of maximum story deformation (Elastic frame)

The elastic vibration energy of the main frame, elastic vibration energy of the damper, and accumulated plastic strain energy of the damper during the elasticity of the main frame are denoted by  ${}_fW'_e$ ,  ${}_dW'_e$ , and  ${}_dW'_p$ , respectively.

The energy that contributes to the damage of the structure  $E_D(t_0)$  is expressed as follows:

$$E_D(t_0) = \frac{1}{2} \cdot M \cdot V_D^2 \quad (8)$$

The elastic vibration energy of the system is only expressed by the elastic vibration energy  ${}_fW'_e$  of the main frame (Eq. (9)), and the elastic vibration energy of the damper ( ${}_dW'_e = 0$ ) is ignored. When the main frame is elastic,  ${}_fW'_e$  is expressed using the maximum shear force  ${}_fQ_{1\max}$  and maximum deformation  ${}_f\delta_{1\max}$  of the first story as follows:

$$\begin{aligned} {}_fW'_e(t_m) &= \sum_{i=1}^N {}_fW_{ei} = \sum_{i=1}^N \frac{{}_fQ_{i\max} \cdot {}_f\delta_{i\max}}{2} = \sum_{i=1}^N \frac{{}_fQ_{i\max}^2}{2 \cdot {}_f k_i} \\ &= \frac{1}{2} \frac{Mg^2 \cdot {}_fT^2}{4\pi^2} \sum_{i=1}^N \left\{ \left( \sum_{j=1}^N \frac{m_j}{M} \right)^2 \frac{1}{\kappa_i} \left( \frac{{}_f\alpha_i}{{}_f\alpha_1} \right)^2 \right\} \cdot {}_f\alpha_1^2 \\ &\cong \frac{1}{2} \frac{Mg^2 \cdot {}_fT^2}{4\pi^2} \cdot {}_f\alpha_1^2 = \frac{1}{2} \frac{{}_fQ_1^2}{k_{eq}} = \frac{1}{2} \kappa_1 \cdot {}_fQ_{1\max} \cdot {}_f\delta_{1\max} \\ &= \frac{1}{2} \cdot \kappa_1 \cdot \frac{({}_f\alpha_1 \cdot M \cdot g)^2}{{}_f k_1} = \frac{{}_fQ_{1\max}^2}{2k_{eq} \left( \frac{{}_f\alpha_1 \cdot \sum_{j=1}^N m_j}{M} \right)^2} \\ &= \frac{g^2 \cdot \kappa_1 \cdot M^2}{2 \cdot {}_f k_1 \cdot {}_f\alpha_1^2} \left( \frac{{}_fQ_{1\max}}{\sum_{j=1}^N m_j \cdot g} \right)^2 = \frac{M \cdot V_D^2}{2} \cdot \frac{{}_f\alpha_1^2}{{}_f\alpha_0^2} \cdot \left( \frac{{}_fT \cdot g}{2\pi \cdot V_D} \right)^2 \\ &= \frac{M \cdot V_D^2}{2} \cdot \frac{1}{{}_f\alpha_0^2} \cdot \left( \frac{{}_f\alpha_1}{{}_f\alpha_0} \right)^2 \end{aligned} \quad (9)$$

where

$${}_f\alpha_i = \frac{{}_fQ_{i\max}}{\sum_{j=1}^N m_j \cdot g} \quad (10)$$

$$\bar{\alpha}_i = \frac{\alpha_i}{\alpha_i} \cdot \frac{\left(\frac{{}_f k_i}{{}_f k_i + {}_d k_i}\right)}{\left(\frac{{}_f k_i}{{}_f k_i + {}_d k_i}\right)} = \bar{\alpha}_i \cdot \frac{\left(\frac{{}_f k_i}{{}_f k_i + {}_d k_i}\right)}{\left(\frac{{}_f k_i}{{}_f k_i + {}_d k_i}\right)} \quad (11)$$

$$\begin{cases} \bar{\alpha}_i = 1 + 1.5927x_i - 11.8519x_i^2 + 42.5833x_i^3 \\ \quad - 59.4827x_i^4 + 30.1586x_i^5 & (x_i > 0.2) \\ \bar{\alpha}_i = 1 + 0.5x_i & (x_i < 0.2) \end{cases} \quad (12)$$

where  ${}_f k_i$  denotes the initial stiffness of the main frame,  ${}_d k_i$  denotes the initial stiffness of the damper,  $\alpha_i$  denotes the story shear force coefficient of the whole frame, and  $\bar{\alpha}_i$  denotes the distribution of optimum yielding story shear force coefficient.

The cumulative plastic strain energy of the damper  ${}_d W'_p$  is expressed using the equivalent repetition number  ${}_d n$  ( ${}_d n = {}_d n_i$ ) of the damper as follows:

$$\begin{aligned} {}_d W'_p(t_m) &= \sum_{i=1}^N {}_d W_{pi} = {}_d \gamma_i \cdot {}_d Q_{yi} \cdot {}_d \delta_{pi} \\ &= 4 \cdot {}_d n \cdot {}_d \gamma_i \cdot {}_d Q_{yi} \cdot {}_d \delta_{\max pi} \\ &= 4 \cdot {}_d n \cdot {}_d \gamma_i \cdot {}_d Q_{yi} \cdot ({}_{\delta_{i\max}} - {}_{\delta_{yi}}) \\ &= \frac{{}_d n \cdot {}_d \gamma_i \cdot {}_{f1} T^2 \cdot k_{eq}}{\pi^2 \cdot M} \cdot {}_d Q_{yi} \cdot \left( \frac{{}_f Q_{i\max}}{{}_f k_i} - \frac{{}_d Q_{yi}}{{}_d k_i} \right) \\ &= \frac{{}_d n \cdot {}_d \gamma_i \cdot {}_{f1} T^2}{\kappa_i \cdot \pi^2 \cdot M} \cdot {}_d \alpha_{yi} \cdot \left( {}_f \alpha_i - \frac{{}_f k_i}{{}_d k_i} \cdot {}_d \alpha_{yi} \right) \cdot \left( M \cdot g \cdot \sum_{j=1}^N \frac{m_j}{M} \right)^2 \\ &= \frac{M \cdot V_D^2}{2} (8 \cdot {}_d n \cdot {}_d \gamma_i \cdot c_i) \cdot \frac{{}_d \alpha_{yi}}{{}_f \alpha_0} \left( \frac{{}_f \alpha_i}{{}_f \alpha_0} - \frac{{}_f k_i}{{}_d k_i} \cdot \frac{{}_d \alpha_{yi}}{{}_f \alpha_0} \right) \end{aligned} \quad (13)$$

$${}_d \alpha_{yi} = \frac{{}_d Q_{yi}}{\sum_{j=1}^N m_j \cdot g} \quad (14)$$

$$c_i = \left( \sum_{j=1}^N \frac{m_j}{M} \right)^2 \cdot \frac{1}{\kappa_i} \quad (15)$$

We substitute Eqns. (8), (9), and (13) into Eq. (3) and divide both sides by  $(MV_D^2)/2$  to yield the following equation:

$$\frac{1}{{}_f \alpha_i^2} \cdot \left( \frac{{}_f \alpha_i}{{}_f \alpha_0} \right)^2 + 8 \cdot {}_d n \cdot {}_d \gamma_i \cdot c_i \cdot \frac{{}_d \alpha_{yi}}{{}_f \alpha_0} \left( \frac{{}_f \alpha_i}{{}_f \alpha_0} - \frac{{}_f k_i}{{}_d k_i} \cdot \frac{{}_d \alpha_{yi}}{{}_f \alpha_0} \right) = 1 \quad (16)$$

We solve Eq. (16) for  ${}_f \alpha_i / {}_f \alpha_0$ , and the following equation is then obtained.

$$\begin{aligned} \frac{{}_f \alpha_i}{{}_f \alpha_0} &= -4 \cdot {}_d n \cdot c_i \cdot {}_d \gamma_i \cdot \frac{{}_d \alpha_{yi}}{{}_f \alpha_0} \\ &\quad + \sqrt{8 \cdot {}_d n \cdot c_i \cdot {}_d \gamma_i \left( 2 \cdot {}_d n \cdot c_i \cdot {}_d \gamma_i \cdot \bar{\alpha}_i^2 + \frac{{}_f k_i}{{}_d k_i} \right) \left( \frac{{}_d \alpha_{yi}}{{}_f \alpha_0} \right)^2} + 1 \end{aligned} \quad (17)$$

At this time, the maximum inter-story deformation  $\delta_{i\max}$  is expressed as follows:

$$\begin{aligned} \delta_{i\max} &= \frac{{}_f Q_{i\max}}{{}_f k_i} = \frac{{}_f \alpha_i \cdot M \cdot g \cdot \sum_{j=1}^N \frac{m_j}{M}}{\kappa_i \cdot 4\pi^2 \cdot M} \\ &\quad \cdot \left( \frac{{}_{f1} T \cdot V_D}{2\pi} \right) \cdot \left( \frac{{}_{f1} T \cdot g}{2\pi \cdot V_D} \right) \cdot \sum_{j=1}^N \frac{m_j}{M} \cdot {}_f \alpha_i \\ &= \frac{{}_f \delta_0 \cdot \sum_{j=1}^N \frac{m_j}{M}}{\kappa_i} \cdot \frac{{}_f \alpha_i}{{}_f \alpha_0} \end{aligned} \quad (18)$$

### 2.3 Prediction of maximum story deformation (Elasto-plastic frame)

In this section, the prediction method for the maximum interlaminar deformation  $\delta_{i\max}$  is derived from the energy balance equation when mainframe plasticization is considered. In order to determine the energy absorption of the mainframe, we focus on  $t = t_0$ . The cumulative plastic strain energy of the damper in all stories,  ${}_d W_p$ , is expressed using the cumulative plastic strain energy,  ${}_d W_{pi}$ , of the damper in the  $i$ -th story. The damage dispersion coefficient  ${}_d \gamma_i$ .  ${}_d W_p$  is expressed as the product of the accumulated plastic strain energy of the damper in the  $i$ -th story consumed at a constant amplitude of the maximum deformation  $\delta_{i\max}$  and equivalent repetition number of the damper  ${}_d n_{pi}$  when the main frame is plasticized. The equivalent repetition number of the damper  ${}_d n_{pi}$  is given by the product of the equivalent repetition number of the damper  ${}_d n_{ei}$  when the main frame is elastic and reduction rate of the equivalent repetition number of the damper due to the plasticization of the main frame  $\beta_{ni}$  as follows [3]:

$${}_d W_p(t_m) = {}_d \gamma_i \cdot {}_d W_{pi} = 4 \cdot \beta_{ni} \cdot {}_d n_{ei} \cdot {}_d \gamma_i \cdot {}_d Q_{yi} \cdot ({}_{\delta_{i\max}} - {}_{\delta_{yi}}) \quad (19)$$

The cumulative plastic strain energy of the main frame of all stories  ${}_f W_p$  is expressed using the cumulative plastic strain energy of the main frame of the  $i$ -th story  ${}_f W_{pi}$  and damage dispersion coefficient of the main frame  ${}_f \gamma_i$ . Additionally,  ${}_f W_p$  is

expressed as the product of the yield shear force of the main frame  $fQ_{yi}$ , accumulated plastic strain energy consumed by one loop of the maximum deformation  $\delta_{i\max}$ , and equivalent number of repetitions  $f n_{pi}$  of the main frame as follows:

$$fW_p(t_m) = f \gamma_i \cdot f W_{pi} = 4 \cdot f n_{pi} \cdot f \gamma_i \cdot f Q_{yi} \cdot (\delta_{i\max} - f \delta_{yi}) \quad (20)$$

As shown in the previous section, the elastic vibration energy of the damper is ignored ( $dW_e = 0$ ). It is only expressed by the elastic vibration energy of the main frame  $fW_e$  (Eq. (21)). When the main frame is plasticized,  $fW_e$  is expressed using the yield shear force  $fQ_{y1}$  and yield deformation  $f\delta_{y1}$  of the first story as follows:

$$\begin{aligned} fW_e(t_m) &= \sum_{i=1}^N fW_{ei} = \sum_{i=1}^N \frac{fQ_{yi} \cdot f\delta_{yi}}{2} = \sum_{i=1}^N \frac{fQ_{yi}^2}{2f k_i} \\ &= \frac{1}{2} \frac{Mg^2 f_1 T^2}{4\pi^2} \sum_{i=1}^N \left\{ \left( \sum_{j=1}^N \frac{m_j}{M} \right)^2 \frac{1}{\kappa_i} \left( \frac{f\alpha_{yi}}{f\alpha_{y1}} \right)^2 \right\} f\alpha_{y1}^2 \quad (21) \\ &\equiv \frac{1}{2} \frac{Mg^2 f_1 T^2}{4\pi^2} f\alpha_{y1}^2 = \frac{1}{2} \frac{fQ_1^2}{k_{eq}} = \frac{1}{2} \kappa_1 \cdot f Q_{y1} \cdot f \delta_{y1} \end{aligned}$$

where

$$f\alpha_{yi} = \frac{fQ_{yi}}{\sum_{j=1}^N m_j \cdot g} \quad (22)$$

The equivalent stiffness  $k_{eq}$  when the main frame is replaced with an SDOF system is set using the total mass  $M$  and  $f_1 T$  of the MDOF system model as follows [1]:

$$k_{eq} = \frac{4\pi^2 \cdot M}{f_1 T^2} \quad (23)$$

The ratio between  $f k_i$  and  $k_{eq}$  is defined as  $\kappa_i$  and expressed as follows:

$$\kappa_i = \frac{f k_i}{k_{eq}} \quad (24)$$

By substituting Eq. (19), (20), and (21) into Eq. (3), the following equation is obtained:

$$\begin{aligned} \frac{1}{2} \kappa_1 \cdot f Q_{y1} \cdot f \delta_{y1} + 4 \cdot f n_{pi} \cdot f \gamma_i \cdot f Q_{yi} \cdot (\delta_{i\max} - f \delta_{yi}) \\ + 4 \cdot \beta_{ni} \cdot d n_{ei} \cdot d \gamma_i \cdot d Q_{yi} \cdot (\delta_{i\max} - d \delta_{yi}) = E_D(t_0) \end{aligned} \quad (25)$$

By solving Eq. (25), the maximum inter-story deformation  $\delta_{i\max}$  is given as follows:

$$\delta_{i\max} = \frac{2E_D(t_0) - \kappa_1 \cdot f Q_{y1} \cdot f \delta_{y1} + 8 \cdot f n_{pi} \cdot f \gamma_i \cdot f Q_{yi} \cdot f \delta_{yi} + 8 \cdot \beta_{ni} \cdot d n_{ei} \cdot d \gamma_i \cdot d Q_{yi} \cdot d \delta_{yi}}{8 \cdot f n_{pi} \cdot f \gamma_i \cdot f Q_{yi} + 8 \cdot \beta_{ni} \cdot d n_{ei} \cdot d \gamma_i \cdot d Q_{yi}} \quad (26)$$

## 2.4 Prediction of maximum story deformation

In the section, we detail the conditions under which the prediction formula for maximum inter-story deformation in the case where the main frame is elastically and plastically deformed is used. Figure 2 shows a flowchart of the maximum inter-story deformation prediction, and the details are described as follows:

- ① We set the yield shear force coefficients  $f\alpha_{y1}$  and  $d\alpha_{y1}$ , initial stiffnesses  $f k_i$  and  $d k_i$ , equivalent number of repetitions  $f n_i$  and  $d n_i$ , and damage dispersion coefficients  $f\gamma_i$  and  $d\gamma_i$  of the main frame and damper, respectively. Additionally, we set the energy that contributes damage to the structure  $E_D$ .
- ② If the equivalent repetition number  $d n_i$  of the damper corresponds to 0 and the equivalent repetition number  $f n_i$  of the main frame corresponds to 0 (i.e., neither the damper nor the main frame is plastic), then it is not possible to use the prediction method because plastic strain energy (A) is absent. In this study, as a safety evaluation, the yield deformation  $d\delta_{yi}$  of the damper was determined as maximum inter-story deformation  $\delta_{i\max}$  (A).
- ③ If the equivalent number of repetitions  $d n_i$  of the damper does not correspond to 0 (i.e., the damper is plastic), then we first assume that the main frame is elastic. Furthermore, as the first prediction, the maximum inter-story deformation  $\delta_{i\max}^{(1)}$  is calculated using Eq. (18).
- ④ If  $\delta_{i\max}^{(1)}$  is lower than the yield deformation of the main frame  $f\delta_{yi}$ , then it is determined that the assumption in ③ is correct and  $\delta_{i\max}^{(1)}$  is determined as maximum inter-story deformation  $\delta_{i\max}$  (B).
- ⑤ If  $\delta_{i\max}^{(1)}$  exceeds the yield deformation of the main frame  $f\delta_{yi}$ , then the assumption in ③ is determined as incorrect, and the main frame is assumed as plastic. Therefore, the maximum inter-story deformation  $\delta_{i\max}^{(2)}$  is calculated as the second prediction by Eq. (26).

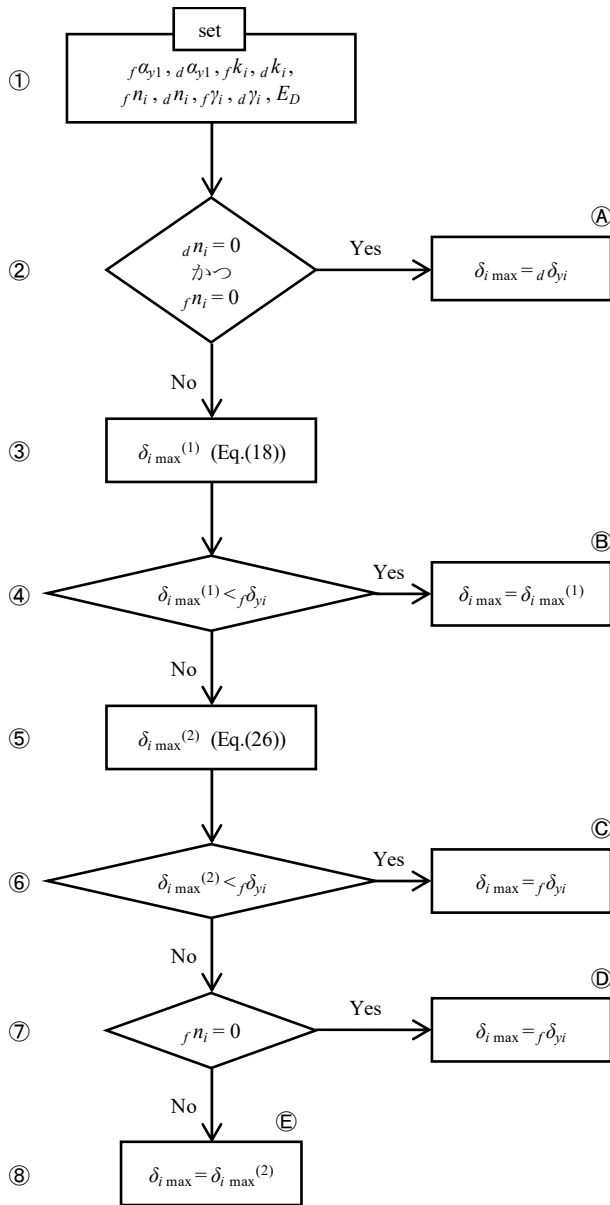


Fig. 2 Flow of maximum inter-story deformation prediction

- ⑥ If  $\delta_{i \max}^{(2)}$  is lower than the yield deformation of the main frame  $f\delta_{yi}$ , then assumption in ⑤ is determined incorrect. In this case, the energy method is unable to predict whether the main frame is elastic or plastic. At this time, the yield deformation  $f\delta_{yi}$  of the main frame is determined as the maximum inter-story deformation  $\delta_{i \max}$  (C) given that the maximum inter-story deformation  $\delta_{i \max}$  is very close to the yield deformation  $f\delta_{yi}$  of the main frame.
- ⑦ If the equivalent repetition number of the main frame  $f n_i$  corresponds to 0 (i.e., if the main

frame is not plastic), then the yield deformation of the main frame  $f\delta_{yi}$  is determined as the maximum inter-story deformation  $\delta_{i \max}$  for the same reason as ⑥ (D).

- ⑧ If the equivalent repetition number of the main frame  $f n_i$  does not correspond to 0, (i.e., the main frame is plastic), then the assumption in ⑤ is determined as correct, and  $\delta_{i \max}^{(2)}$  is determined as the maximum inter-story deformation  $\delta_{i \max}$  (E).

### 3 Outline of analysis model and input ground motions

#### 3.1 Specification of main frames and hysteretic dampers

In this study, we used 10-story equivalent shear models with the elastic 1st natural period  $f_1 T = 1.0$  s of the main frame [2], [4]. Figure 3 shows the analysis model used in the study. The mass distribution is uniform ( $m_i = 9.8 \text{ kN}\cdot\text{s}^2/\text{cm}$ ), and the stiffness distribution of the main frame  $f k_i / f k_1$  and the yield shear force distribution of the main frame  $f Q_{yi} / f Q_{y1}$  are trapezoidal (The top story corresponds to 1/2 of the 1st story). The initial stiffness of the main frame  $f k_i$  is obtained as follows [5]:

$$f k_1 = \frac{{}_s \omega^2 \cdot m_1 \cdot {}_s \phi_1 + k_2 ({}_s \phi_2 - {}_s \phi_1)}{{}_s \phi_1} \quad (27a)$$

$$f k_i = \frac{{}_s \omega^2 \cdot m_i \cdot {}_s \phi_i + k_{i+1} ({}_s \phi_{i+1} - {}_s \phi_i)}{{}_s \phi_i - {}_s \phi_{i-1}} \quad \{i = (N-1) \sim 2\} \quad (27b)$$

$$f k_N = \frac{{}_s \omega^2 \cdot m_N \cdot {}_s \phi_N}{{}_s \phi_N - {}_s \phi_{N-1}} \quad (27c)$$

Here,  ${}_s \omega$  denotes the  $s$ -th mode natural frequency, and  ${}_s \phi_i$  denotes the mode shape of the  $s$ -th mode and  $i$ -th story. First, an arbitrary stiffness  $f k_i / f k_1$  is set such that it exhibits a trapezoidal distribution (Top story is 1/2 of 1st story, See Fig. 3) and the 1<sup>st</sup> mode shape  ${}_1 \phi_i$  is obtained from eigenvalue analysis. Subsequently, the stiffness  $f k_i$  is calculated from Eq. (27) using  ${}_1 \omega$  and  ${}_1 \phi_i$ . The structural damping is set to the stiffness-proportional damping that corresponds to 2% for 1<sup>st</sup> natural period of the main frame. The yield shear force of the main frame  $f Q_{yi}$  is obtained as follows:

$$f Q'_{y1} = f Q_{y1} \quad (f_1 T = 0.5 \text{ s}) \quad (29a)$$



$${}_f\alpha'_{y1} = \frac{{}_f\alpha_{y1}}{{}_fT} \quad ({}_fT = 1.0, 2.0 \text{ s}) \quad (29b)$$

The yield shear force of the damper  ${}_dQ_{yi}$  is based on the yield shear force of the first story damper  ${}_dQ_{y1}$ , and yield shear force distribution of dampers  ${}_dQ_{yi}/{}_dQ_{y1}$  is divided into three groups based on the optimum yield shear coefficient distribution  $\bar{\alpha}_i$ , as proposed by Akiyama [1] (Eq. (30)).

$$\bar{\alpha}_i = 1 + 1.5927x' - 11.8519x'^2 + 42.5833x'^3 - 59.4827x'^4 + 30.1586x'^5 \quad (30a)$$

In case of  $x' < 0.2$ , the following expression is obtained:

$$\bar{\alpha}_i = 1 + 0.5x' \quad (30b)$$

where

$$x' = \frac{i-1}{N} \quad (31)$$

Subsequently,  ${}_dQ_{yi}$  in the case of 10-DOF model is obtained as follows:

$${}_dQ_{yi} = {}_d\alpha'_{y1} \cdot \bar{\alpha}_i \cdot \sum_{j=1}^N m_j \cdot g \quad (i = 1 \sim 4) \quad (32a)$$

$${}_dQ_{yi} = {}_d\alpha'_{y1} \cdot \bar{\alpha}_5 \cdot \sum_{j=5}^N m_j \cdot g \quad (i = 5 \sim 8) \quad (33b)$$

$${}_dQ_{yi} = {}_d\alpha'_{y1} \cdot \bar{\alpha}_9 \cdot \sum_{j=9}^N m_j \cdot g \quad (i = 9, 10) \quad (33c)$$

The yield shear force coefficient of the first story damper  ${}_d\alpha'_{y1}$  is defined as follows [6]:

$${}_d\alpha'_{y1} = \frac{{}_d\alpha_{y1}}{{}_fT} \quad (34)$$

The yield story deformation  ${}_d\delta_{yi}$  of the damper is constant for all stories,  ${}_d\delta_{y1} = 0.42$  cm when  ${}_fT$  corresponds to 0.5 s and  ${}_d\delta_{y1} = 0.64$  cm when  ${}_fT$  corresponds to 1.0 and 2.0 s [2], [4]. The initial stiffness of the damper  ${}_d k_i$  is given as follows:

$${}_d k_i = \frac{{}_d Q_{yi}}{{}_d \delta_{yi}} \quad (35)$$

As shown in Fig. 4, we focused on the restoring force characteristics when the main frame is elastic and plastic. In this study, the combination of the main frame and damper is termed as “system.” Further, it is assumed that there is no influence of the bending deformation of the building and the deformations of connections that hinder the transmission of the deformation to the

damper [6] and that all the maximum inter-story deformation of each story contributes to the deformation of the damper.

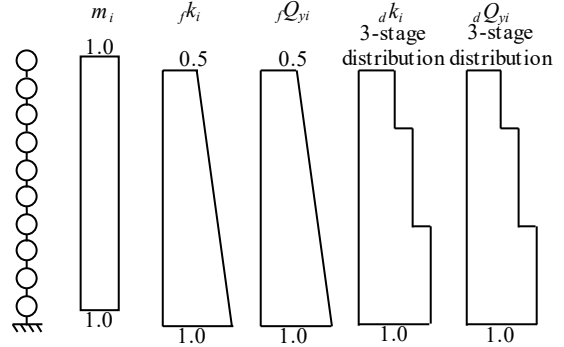


Fig. 3 Analysis model

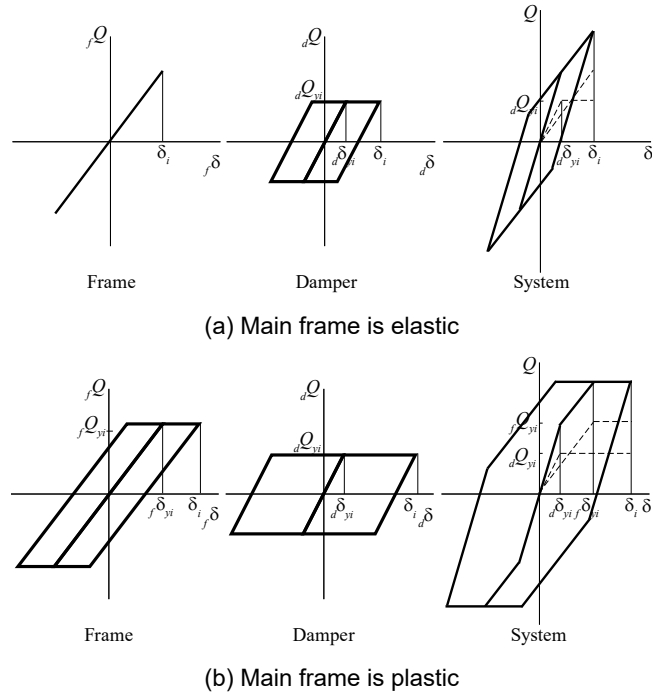


Fig. 4 Restoring force characteristics

### 3.2 Outline of input earthquake

The ground motions for the study were simulated using the simulated ground motion ART HACHI (phase characteristic: HACHINOHE 1968 EW) and ART KOBE (phase characteristic: JMA KOBE 1995 NS) where the pseudo velocity response spectrum  $\rho_{SV}$  is constant in the region after the corner period  $T_c = 0.64$  s. In this study, the input level was changed, and ground motions with  $\rho_{SV} = 100, 150$  cm/s ( $h = 5\%$ ) were positioned as Level 2 and Level 3. The analysis time interval corresponds to  $\Delta t =$

0.01 s for both ground motions. Figures 5 (a) and (b) show the pseudo velocity response spectrum  ${}_{\rho}S_V$  and energy spectrum  $V_E$ , respectively.

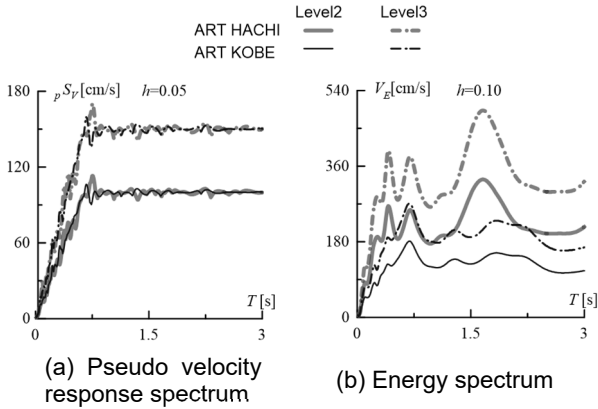


Fig.5 Input wave characteristics

#### 4 Validity of prediction method

Figure 5 shows a comparison between the predicted value and analysis value of the maximum inter-story deformation. Figure 6 (a) shows the results when the yield shear coefficient of frame corresponds to  $f\alpha_{y1} = 0.2$  for the main frame, and Fig. 6 (b) shows the results for  $f\alpha_{y1} = 0.3$ . Each figure shows the yield shear coefficient of damper  $d\alpha_{y1} = 0.01, 0.02, 0.04, 0.10, 0.16, 0.20$  from the top and HACHI100, HACHI150, KOBE100, KOBE150 from the left. The upper right of each figure shows the absolute error between the analysis value and predicted value at the point where the maximum inter-story deformation occurs. Subsequently, the values of the equivalent repetition numbers of the main frame and damper ( ${}_f n_{pi}$  and  ${}_d n_{pi}$ ) are calculated from the accumulated plastic strain energy at time  $t = t_0$  (analysis value) as follows [2]:

$${}_f n_{pi} = \frac{{}_f W_{pi}(t_0)}{4 \cdot {}_f Q_{yi} \cdot (\delta_{imax} - {}_f \delta_{yi})} \quad (36)$$

$${}_d n_{pi} = \frac{{}_d W_{pi}(t_0)}{4 \cdot {}_d Q_{yi} \cdot (\delta_{imax} - {}_d \delta_{yi})} \quad (37)$$

The results confirmed that the predicted values accurately capture the analysis values irrespective of the yield shear coefficient of frame  $f\alpha_{y1}$ , yield shear coefficient of damper  $d\alpha_{y1}$ , and the type and level of input.

#### 5 Conclusions

In this paper, we proposed an extended method to predict the deformation of the maximum inter-story deformation. The validity of the prediction method was examined by comparing it with time history response analysis results.

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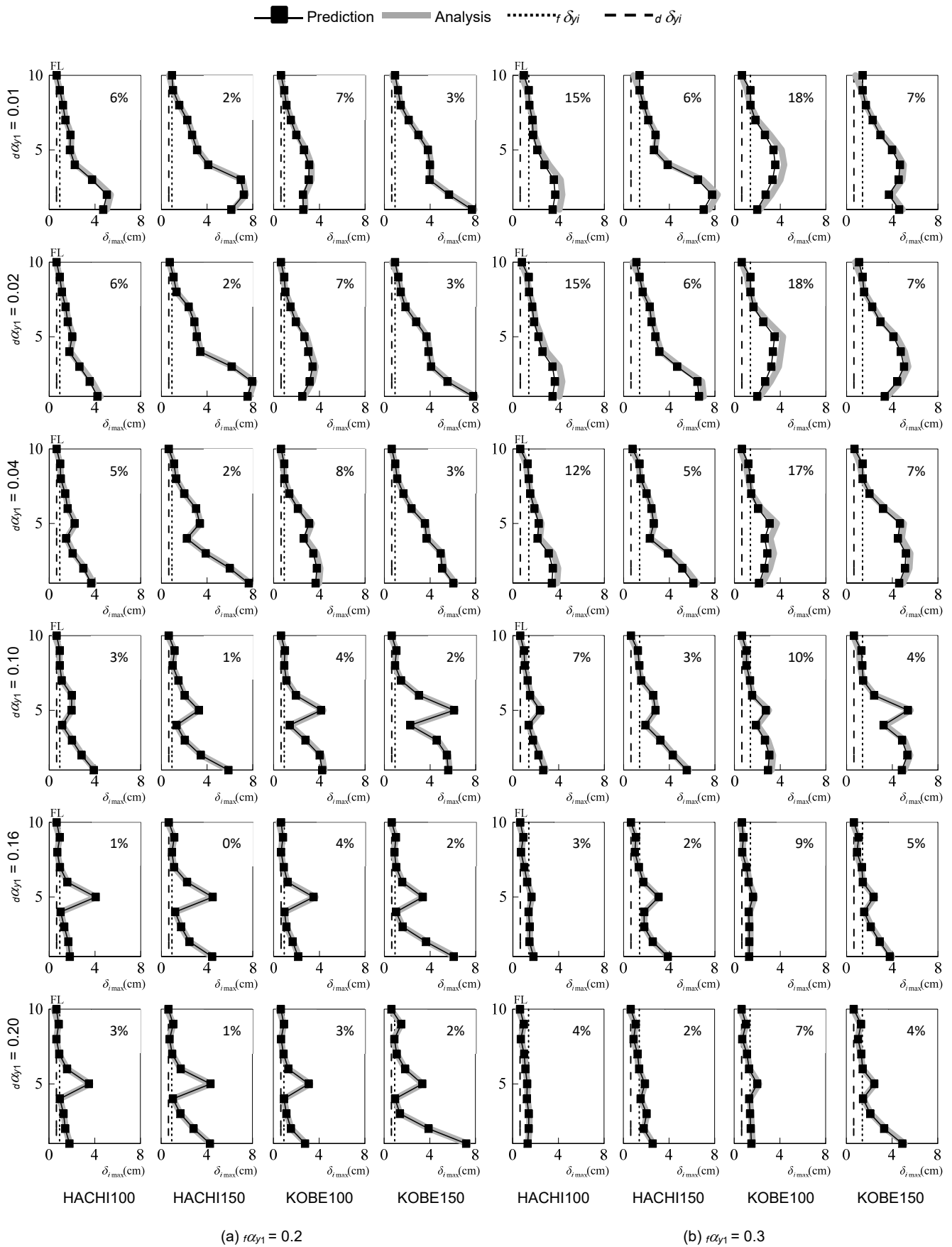


Fig. 6 Comparison of predicted value and analysis value of maximum inter-story deformation