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INFLUENCE OF NATURAL PERIOD AND DAMPING RATIO ERRORS ON WIND FORCES OF A TALL BUILDING ESTIMATED BY MODAL ANALYSIS

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ABSTRACT

Modern structures are more susceptible to larger dynamic excitation brought about by stronger wind forces. For this reason, accurate estimation of the actual wind loads acting on the structure has become greatly necessary. Also, important dynamic parameters of the building, particularly natural period and damping ratio are considered to be highly uncertain and may contribute additional errors in the estimation of wind forces. Hence, the effect of uncertainty in the evaluation of these dynamic parameters to the estimated wind forces must also be given more careful consideration. This paper investigates the accuracy of modal analysis in estimating the wind forces acting upon a 10 degree-of-freedom model and determines the sensitivity of these estimated wind forces when errors in the natural period and damping ratio are introduced.

Key Words: Wind force estimation, Modal analysis, Natural period, Damping ratio

1. INTRODUCTION

Tall and light buildings are susceptible to dynamic forces such as extreme winds, making wind load analysis an integral factor in the design process. As the building gets higher and lighter, the increase in wind velocity and force causes large structural vibrations that may affect the serviceability and habitability of the structure. For this reason, accurate estimation of the actual wind forces acting on the structure has become of great importance. Not only the wind forces, but also the dynamic properties of the structure, particularly the natural period and damping ratio for the fundamental mode, must be given more careful consideration. Natural period and damping ratio are considered greatly necessary but highly uncertain dynamic parameters of a structure 1). The uncertainty of the estimation of these two properties contributes additional amount of error in the depiction of the wind forces acting upon the structure.

This study uses modal analysis to estimate the wind forces acting upon a 10 degree-of-freedom (DOF) model. Also, the accuracy of using only the first three modes of vibration in the modal superposition to estimate the wind forces was investigated. Lastly, a numerical example was analyzed wherein errors in the natural period and damping ratio was applied to the fundamental mode in the estimation of the wind forces and the effect of these errors to the modal superposition including the other modes was also observed.

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2. THEORETICAL BACKGROUND

2.1 Multi-degree of Freedom (MDOF) Analysis

The equation of motion for an MDOF system subjected to external dynamic forces $\{P(t)\}$ is

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{P(t)\}. \tag{1}$$

Here, $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices, respectively. Also, $\{\ddot{x}(t)\}$, $\{\dot{x}(t)\}$ and $\{x(t)\}$ are the acceleration, velocity and displacement vectors, respectively. Note that these are the dynamic responses of the structure. This shows a system of N ordinary differential equations in terms of dynamic responses due to the forces applied where N depends on the number of degrees of freedom (DOFs) of the structure.

2.2 Single-degree of Freedom (SDOF) / Modal Analysis

The system of simultaneous equations shown in Equation (1) is not efficient for structures with more DOFs and it is more convenient to express this system in modal coordinates. For example, a 10-DOF model can be simplified by ten separate SDOF models. The dynamic responses of each DOF of an MDOF system can be expressed as the sum of the modal contributions of each SDOF model, e.g., $x(t)=\sum_{n=1}^{N}\phi_n q_n(t)$. Accordingly, the vector responses (i.e., $\{\ddot{x}(t)\}$, $\{\dot{x}(t)\}$ and $\{x(t)\}$) in Section 2.1 can be simplified as

$$\{\ddot{x}(t)\}=[\phi]\{\ddot{q}(t)\} \tag{2.a}$$
$$\{\dot{x}(t)\}=[\phi]\{\dot{q}(t)\} \tag{2.b}$$
$$\{x(t)\}=[\phi]\{q(t)\} \tag{2.c}$$

where $[\phi]$ = modal matrix, and $\{\ddot{q}(t)\}$, $\{\dot{q}(t)\}$, $\{q(t)\}$ = modal responses. Therefore, Equation (1) becomes

$$[M][\phi]\{\ddot{q}(t)\}+[C][\phi]\{\dot{q}(t)\}+[K][\phi]\{q(t)\}=\{P(t)\}. \tag{3}$$

Multiplying each term of Equation (3) to the transpose of the modal matrix will give

$$[sM]\{\ddot{q}(t)\}+[sC]\{\dot{q}(t)\}+[sK]\{q(t)\}=\{sP(t)\} \tag{4}$$

where $[{}_sM]$, $[{}_sC]$, $[{}_sK]$ and $\{{}_sP(t)\}$ are the generalized mass, generalized damping, generalized stiffness matrices and generalized force vectors, respectively. Solving Equation (4) will determine the acceleration, velocity and displacement per mode of vibration and substituting them to Equation (2) will determine the actual responses of the original MDOF system.

2.3 Force Calculation

After determining the modal responses of the system (i.e., $\{\ddot{q}(t)\}$, $\{\dot{q}(t)\}$ and $\{q(t)\}$), it is now necessary to back substitute these calculated values to Equation (4) to determine the generalized force $\{{}_sP(t)\}$ applied to the model. The actual wind force $\{P(t)\}$ can be calculated as

$$\{P(t)\}=[\phi]^{T^{-1}}\{sP(t)\} \tag{5}$$

3. ANALYTICAL MODEL

The model in Figure 1 is a simplified lumped mass 10-DOF system with a height H = 100 m, density ρ_u = 180 kg/m^3 and each floor area A = 625 m^2. The structure has a natural period, T = 0.025H s. The wind force applied in the analysis was derived from a calculated typhoon simulation. The wind direction analyzed was the across wind direction. Stiffness-proportional damping is used in the analysis where damping ratio h = *2%*. The stiffness of the structure was calculated in such a way that the first mode of vibration is linear in shape.

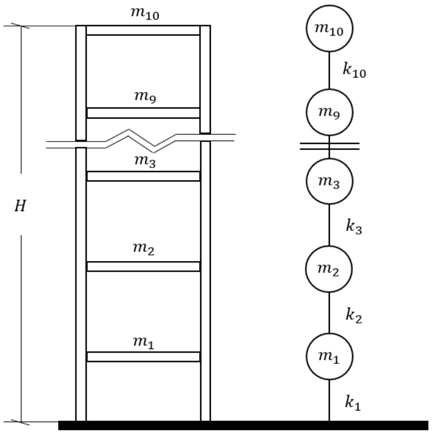


Figure 1. Analytical model.

3.1 Modal Analysis

The wind force in the across-wind direction was applied to ten simplified SDOF models (*Figure 2*). The modal wind forces are calculated by substituting the obtained modal responses, $\ddot{q}(t)\}$, $\{\dot{q}(t)\}$ and $\{q(t)\}$ to Equation (4) and Equation (5). The accuracy of the wind forces obtained by SDOF analysis is confirmed by calculating the correlation coefficient for each increment of mode in the modal superposition. Equation (7) shows the correlation formula used in this paper, where $\hat{y}$ is the theoretical or actual value and $\bar{y}$ is the mean of the calculated value y.

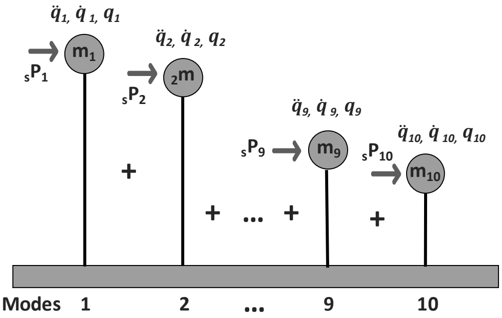


Figure 2. SDOF Analysis

$$Correlation = \left(1 - \frac{\sqrt{\sum_{k=1}^{N} \left(\hat{y}(k) - y(k)\right)^2}}{\sqrt{\sum_{k=1}^{N} \left(y(k) - \bar{y}\right)^2}} \right) \tag{7}$$

Figure 3 shows the comparison between the time-history of the actual wind forces applied on the model and the wind forces that were calculated by SDOF analysis for the bottom (1st) and top (10th) floor, respectively. From Figure 3a, it can be seen that using only one mode to calculate the wind forces is not effective especially for the bottom floor. This value, however, improves at the top floor of the structure as shown in Figure 3b. On the other hand, if all modes are included in the calculation of the wind forces, the time-history shows good agreement with the actual wind forces for both the bottom and top floors. These results are also manifested in the correlation values presented in Figure 4. It can be seen here that for the 1st floor and using only the first mode of vibration, the correlation coefficient has a value of almost -6 and increased to 0.3 for the top floor but this improvement is not enough to be considered as good correlation. On the contrary, if all modes are to be included in the modal superposition a correlation coefficient of 0.90 and 0.88 can be achieved for the bottom and top floor, respectively.

Based on these results it was confirmed that SDOF analysis can accurately estimate the wind forces and the maximum value of the force, provided that many modes are included in the modal superposition. However, considering all modes of vibration is not always possible especially when the data comes from observation records. The recorded data are sometimes limited only to the first three modes of vibration and it is important to determine how accurate will the result be if this limit is applied to the modal superposition. Figure 4 summarizes the correlation coefficient for each

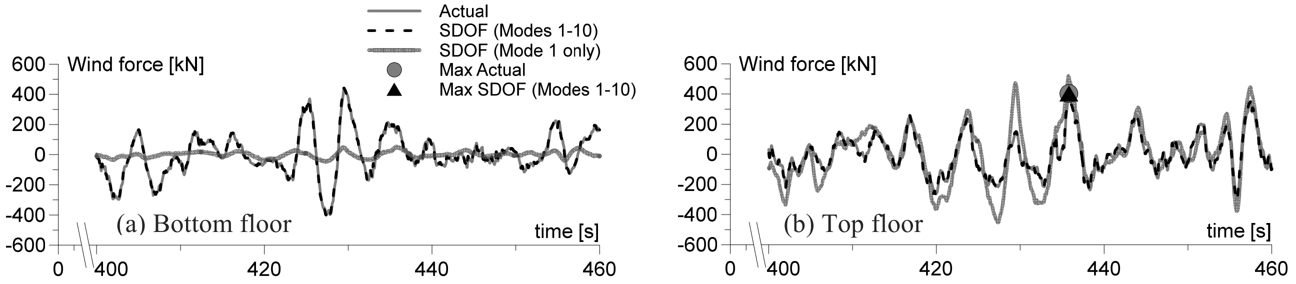
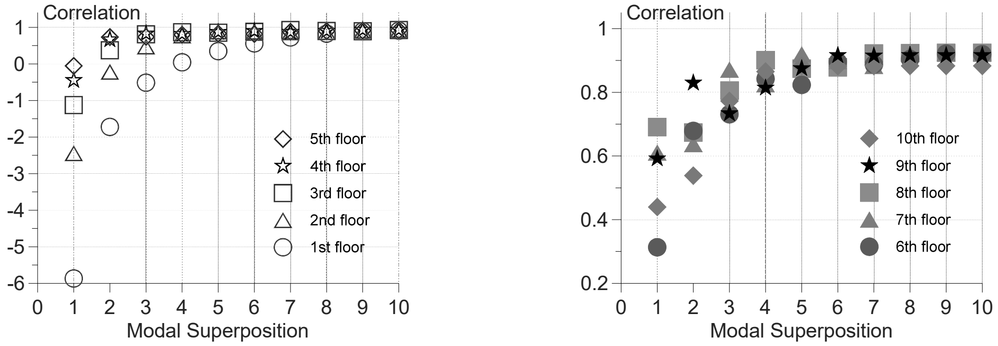


Figure 3. Time-history of wind forces (Actual vs SDOF analysis)



(a) 1st – 5th floors (b) 6th – 10th floors

Figure 4. Correlation of wind forces (Actual vs SDOF analysis)

increment of mode in the modal superposition. It is shown here that the lower (1st to 5th) floors have low correlation values for the modal superposition of the first 2 modes of vibration. The correlation moves closer to 1.0 if the third mode is added. However, this is not a major concern since the lower floors experience relatively low wind forces as compared to the top floor of the structure. Hence, the upper floors are more critical to wind forces and therefore will be the main focus of the paper. It can be observed that in order to obtain at least 70% correlation for the upper part of the structure, including at least the first three modes of vibration in the modal superposition of forces is required.

3.2 Numerical Example

Table 2. Given values

h_1, h_2, h_3
T_1, T_2, T_3
ϕ_1, ϕ_2, ϕ_3
$\ddot{x}_1(t), \ddot{x}_5(t), \ddot{x}_{10}(t)$

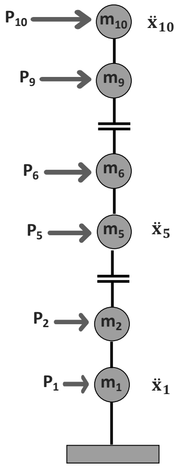


Figure 5. Sensor location

After determining that including the first three modes of vibration in the modal superposition can give at least 70% accuracy in the estimation of wind forces, the next concern is the effect of errors in the dynamic parameters of the model. The natural period and damping ratio are necessary but highly uncertain parameters in the dynamic behavior of a structure and there is no absolute theoretical method to estimate these values. Because of that, the effect of expected errors in these parameters in the estimation of the wind forces by modal analysis must be examined.

A numerical example is provided for the analysis. The same analytical model was used but the acceleration sensors were assumed to be installed only on the 1st, 5th and 10th floors of the structure as shown in Figure 5. The parameters shown in Table 2 are considered to be given values. The subscript indicates the mode of vibration for h, T and ϕ and the floor level for the responses. Integration of the acceleration in the frequency domain was used to determine the velocity and displacement for the given floors and linear interpolation was applied to obtain the responses on the other floors.

The results of integration of the acceleration is shown in Figure 6. It is presented here that the obtained velocity and displacement from integration are in good agreement with the results of the MDF analysis both in correlation and time-history values. Figure 7 shows the results of linear interpolation which was used to determine the responses on the floors without sensors. Because of the interpolation process, a maximum error of 20% in the responses was incurred, particularly in the acceleration response. This occurs in the second floor of the model as shown in the correlation values presented in Figure 7a. The time-history of the responses by linear interpolation and by MDF analysis for the second floor is also shown in Figure 7b and it can be seen that 20% error in correlation is not at all evident in the time-history plot of the responses.

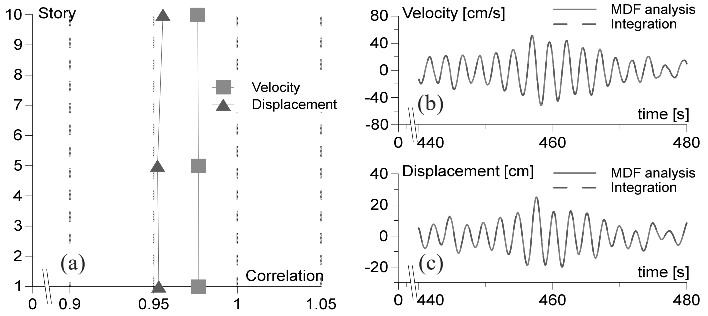


Figure 6. Velocity and displacement response calculated by integration and MDF analysis (a) *Correlation,* (b) *Velocity time-history (top),* (c) *Displacement time-history (top)*

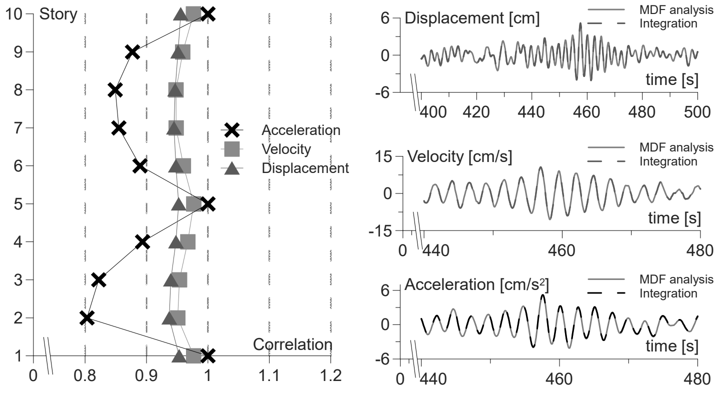


Figure 7. Responses of the floors without sensor calculated by linear interpolation (a) *Correlation*, (b) *Time-history of 2nd floor*

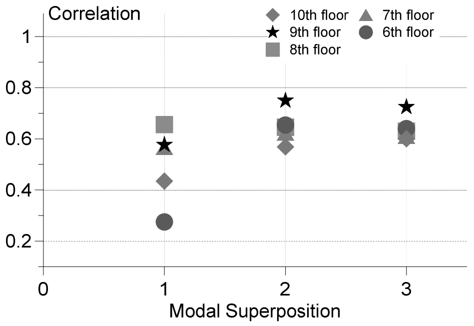


Figure 8. Correlation of wind forces (Actual vs SDOF analysis using interpolated responses)

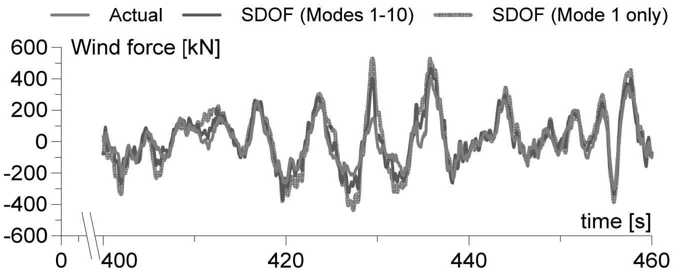


Figure 9. Time-history of wind forces for the top floor (Actual vs SDOF analysis using interpolated responses)

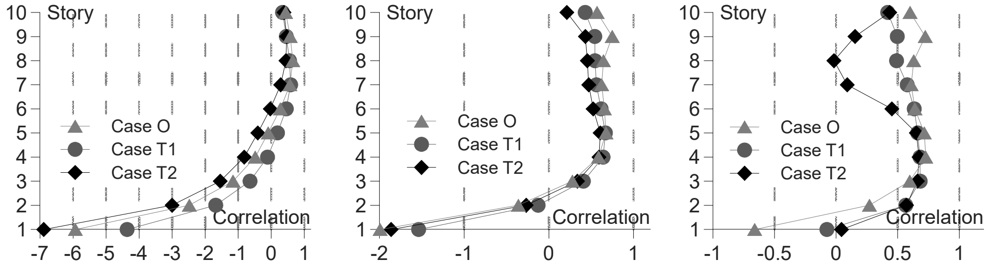
After obtaining the responses of the floors without sensors by linear interpolation, Equation (2) was used to obtain the modal responses and the same procedure as Section 3.1 was done in order to estimate the wind forces. In Figure 8, it can be seen that there is a significant change between the correlation values of the calculated wind forces using the interpolated responses and the actual wind forces. This may be attributed to the errors incurred from the acceleration response integration and linear interpolation. A 10% drop in the correlation of the wind forces in upper floors is observed for the modal superposition of the first 3 modes of vibration using the interpolated responses, decreasing the correlation from 70% to only 60%. The time-history comparison can be seen in Figure 9.

In the next part of the analysis, an error of ±10% in the natural period and damping ratio was applied in calculating the first mode forces to observe the effect of uncertainty of these dynamic parameters on the fundamental mode to the accuracy of the succeeding relevant modal forces. The cases considered in this paper are shown in Table 3.

Figure 10 shows the summary of the accuracy of the wind forces when ±10% error is applied to the natural period of the first mode forces. It can be observed here that errors that increase the value of the natural period obtain a lesser correlation of calculated forces with respect to the actual wind forces as compared to when the natural period is decreased by the applied error. Also, the sum of the first three modes of vibration is significantly affected by the error in the natural period as compared with including just the first mode or the first two modes in calculating the forces. The correlation on the top floor for Modes 1-3 decreased from 60% to almost 50% due to the error on the natural period. Figure 11 shows the correlation of the wind forces when an error of ±10% was introduced in the damping ratio of the first mode of vibration. In these figures, it can be seen that an error of either ±10% on the damping ratio did not cause any significant change in the correlation values of the modal superposition of the first three modes.

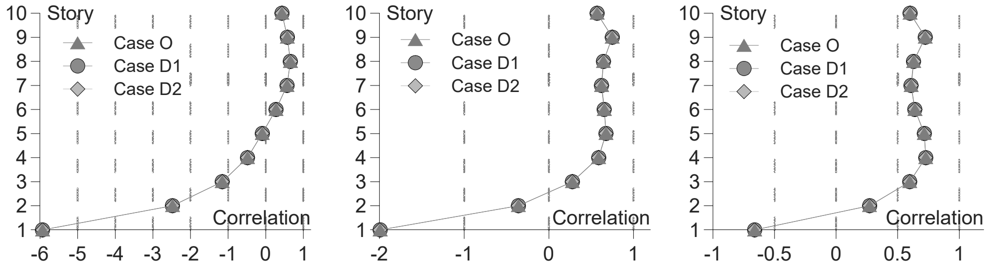
Table 3. Error in dynamic parameters

Cases	Error (%)	
	Natural period, *T*	Damping Ratio, *h*
O	0	0
T1	+10	0
T2	-10	0
D1	0	+10
D2	0	-10



(a) Mode 1 (b) Modes 1-2 (c) Modes 1-3

Figure 10. Correlation of wind forces (actual vs SDOF analysis) with natural period error



(a) Mode 1 (b) Modes 1-2 (c) Modes 1-3

Figure 11. Correlation of wind forces (actual vs SDOF analysis) with damping ratio error

4. CONCLUSIONS

Based on the results of this paper, modal analysis can be used to accurately estimate the actual wind forces on a structure. In order to obtain at least 70% accuracy in the estimated wind forces for the upper floors of the structure, at least the first three modes of vibration must be included in the modal superposition. Also in this method, natural period errors on the first mode of vibration have a more significant effect on the estimated wind forces as compared with errors of damping ratio applied to the first mode of vibration.

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