

論文 / 著書情報
Article / Book Information

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種別(和文)	論文要旨
Type(English)	Summary

論文要旨

THESIS SUMMARY

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要旨 (英文 800 語程度)

Thesis Summary (approx.800 English Words)

The theory of the value-distributions of zeta and L -functions has been developed since the classical work of Bohr and his collaborators in the early 20th century. The major modern method is to compare an L -function with a probabilistic model. For example, we study mean values of L -functions via the expected values of suitable random variables. Ihara and Matsumoto also introduced the notion of M -functions which are certain probability density functions closely related to the value-distributions of L -functions. The main terms of mean values of L -functions are represented by integrals involving the attached M -functions. Furthermore, many researchers studied error terms arising from the comparisons of L -functions with their probabilistic models. The upper bounds of such error terms are called the *discrepancy bounds*. This thesis contains five issues on the probabilistic studies of the value-distributions of zeta and L -functions.

First, we consider the value-distributions of L -functions as the complex variable $s = \sigma + it$ varies on vertical lines, namely in the t -aspect. The Riemann zeta-function is the most basic example of L -functions possessing Euler product representations, and the attached M -function was classically studied. Matsumoto further determined M -functions for Dedekind zeta-functions of Galois number fields. It should be noted that his method relied on a special property of the Euler products called the convexity, although we know many L -functions with non-convex Euler products, e.g. Dedekind zeta-functions of non-Galois number fields and automorphic L -functions of holomorphic primitive cusp forms. The first result of this thesis presents a general method of constructing M -functions by the axiomatic treatments of L -functions. Several axiomatic classes of L -functions were introduced by Selberg, Matsumoto, Steuding, and others. Then we prove that an L -function has a suitable M -function if it belongs to the intersection of Selberg's class and Steuding's class. It enables us to construct M -functions for L -functions with non-convex Euler products. In addition, we obtain sharp discrepancy bounds for such L -functions. We further prove an asymptotic formula related to the large deviations of the values of the Riemann zeta-function.

It is also worth considering the value-distributions of L -functions in the other aspects. For example, the value-distribution of Dirichlet L -functions as the Dirichlet character varies has been studied. As the second issue, we

consider Artin L -functions arising from non-Galois cubic fields. The main difference from the case of Dirichlet L -functions is that such Artin L -functions have Euler products of degree two, which causes combinatorial complexity of the treatment of Dirichlet coefficients. We overcome this difficulty by applying the results on counting cubic fields achieved by Bhargava-Shankar-Tsimerman and Taniguchi-Thorne in 2013. We show the existence of an M -function for the value-distribution of the Artin L -functions as the cubic field varies. Furthermore, we apply the result to derive a limit formula involving class numbers of cubic fields. It can be regarded as a cubic analogue of the Gauss-Siegel formula of class numbers of quadratic forms.

Automorphic L -functions of holomorphic primitive cusp forms also have Euler products of degree two. The third issue concerns the value-distributions of such automorphic L -functions as the cusp form varies. Recently, Matsumoto-Umegaki and Lebacque-Zykin tried to construct an M -function in this case, and their attempts were partially succeeded. The difficulty to complete their study was again combinatorial complexity of the treatment of Dirichlet coefficients. In this thesis, we construct the desired M -function by using the Eichler-Selberg trace formula which can be regarded as a relation between the Dirichlet coefficients and certain random variables. We further obtain discrepancy bounds in this case, which is similar to the result on the first issue.

As the fourth issue, we study Hurwitz-Lerch zeta-functions with transcendental parameters, which do not have Euler product representations. It is classically known that such Hurwitz-Lerch zeta-functions have infinitely many zeros off the critical line, while the Riemann Hypothesis asserts that all non-trivial zeros of the Riemann zeta-function lie on the critical line. Furthermore, several inequalities on upper and lower bounds for the densities of these zeros of the Hurwitz-Lerch zeta-functions were proved by Garunkštis-Laurinčikas. Then we improve the inequalities by proving asymptotic formulas when the parameters are S -numbers in the sense of Mahler's classification of transcendental numbers.

Finally, we consider the value-distributions of certain functions represented by iterated integrals of the logarithm of the Riemann zeta-function as the fifth issue. Such functions were introduced by Inoue to study the value-distributions on the critical line precisely. Then we prove a probabilistic limit theorem of Inoue's functions. It refines the recent result of Endo-Inoue which ensures that the sets of the values of Inoue's functions on the critical line are dense in the complex plane.

備考：論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。

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