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A Study of Approximate Simultaneous Matrix Diagonalization and Robust Tensor Decomposition via Structured Low-Rank Approximation

[Outline]

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Outline

The matrix diagonalization is a problem for finding a similarity transformation to diagonalize a single n-by-n diagonalizable matrix, i.e., a matrix having n linearly independent eigenvectors. For a tuple of multiple diagonalizable matrices, the matrix tuple can be *diagonalized simultaneously* by a common similarity transformation if and only if every pair of matrices in the tuple commutes. The common similarity transformation or common diagonalizer of such a matrix tuple, called in this thesis a *simultaneously diagonalizable* tuple, is desired to be estimated as a key information in many branches of signal processing applications, e.g., Direction Of Arrival (DOA) estimation, Blind Source Separation (BSS), the *Canonical Polyadic Decomposition (CPD)* of a tensor. However, since, in reality, the observable matrix tuple is almost always only a perturbed version, influenced by noise, of the target simultaneously diagonalizable tuple, the desired common similarity transformation of the target matrix tuple has to be estimated not directly from the target tuple but from the observable matrix tuple as the perturbed version of the target matrix tuple. This estimation problem is called *Approximate Simultaneous Diagonalization (ASD)* in this thesis. Existing approaches, the so-called Jacobi-like methods, for ASD have been designed based on certain optimization models for an estimate, of such a common similarity transformation, which can suppress the magnitudes of all off-diagonal entries of every matrix in the tuple obtained by the similarity transformation, with the estimate, of the observed matrix tuple. This situation implies that the existing approaches do not succeed in denoising the observed matrix tuple by exploiting directly the simultaneous diagonalizability of the target matrix tuple. The goal of this thesis is to establish novel computational strategies for effective utilization of the simultaneous diagonalizability of the target matrix tuple in ASD and their signal processing applications.

To achieve this goal, we focus on a constructive proof for a classical relation between diagonalizability and commutativity regarding the simultaneous diagonalizability of a matrix tuple. Indeed, we can translate the constructive proof into an algebraic algorithm for exact simultaneous diagonalization of a simultaneously diagonalizable tuple. Such an algorithm, called in the *Diagonalize-One-then-Diagonalize-the-Other (DODO)* method, has been applied to an observed matrix tuple directly in ASD and neglected for many years as an unpractical algorithm due to its severe sensitivity against noise. This situation suggests the possibility to establish a new powerful strategy which can supersede the Jacobi-like methods for ASD if a simultaneously diagonalizable tuple somehow can be found as a good approximation of an observed matrix tuple. We propose such a strategy via new translations of such an approximation problem for a matrix tuple into certain *Structured Low-Rank Approximations (SLRAs)* for a single matrix in the transformed domain of nontrivial linear operators. We also propose to solve such SLRAs as nonconvex feasibility problems by applying iterative solvers, e.g., *Cadzow's algorithm* and *Douglas-Rachford splitting algorithm*.

In Chapter 2, we introduce notations and preliminaries for matrices and tensors. We also present the mathematical formulation of exact simultaneous diagonalization and exact CPD including their uniqueness conditions.

In Chapter 3, starting with a simple example indicating that the Jacobi-like methods in general have no guarantee to suppress the magnitudes of the off-diagonal entries even if the observed matrix tuple is simultaneously diagonalizable, we propose a novel two-step strategy called *Approximate-Then-Diagonalize-Simultaneously (ATDS)* algorithm for ASD . The ATDS algorithm decomposes ASD into (Step 1) finding a simultaneously diagonalizable tuple near the given one; and (Step 2) finding a common similarity transformation which diagonalizes exactly the tuple obtained in Step 1. The proposed approach to Step 1 is realized by solving an SLRA with Cadzow's algorithm. In Step 2, by using the DODO method, we obtain a common exact diagonalizer of the obtained tuple in Step 1 as a solution for the original ASD. Unlike the Jacobi-like methods, the ATDS algorithm has a guarantee to find a common exact diagonalizer if the given tuple happens to be simultaneously diagonalizable. Numerical experiments show that the ATDS algorithm achieves better performance than the Jacobi-like methods.

In Chapter 4, we propose a novel strategy to realize the noise suppression for the approximate CPD. The proposed strategy is also decomposed into two steps: (Step 1) finding a tensor satisfying a certain condition inspired by De Lathauwer et al. and (Step 2) solving the *exact* CPD of the obtained tensor. Step 1 can be realized by solving an SLRA with the Douglas-Rachford splitting algorithm and then Step 2 can be realized by solving the simultaneous diagonalization of a simultaneously diagonalizable matrix tuple constructed by the denoised tensor with the DODO method. Numerical experiments show that the proposed algorithm works well even in typical cases where a standard algorithm, called Alternating Least Squares (ALS) algorithm, and its certain variants for approximate CPD suffer from the so-called *bottleneck/swamp* effect.

Finally we conclude this thesis in Chapter 5.