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論 文 要 旨

THESIS SUMMARY

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要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

There have been many studies on time series analysis to predict future states, to identify governing equations, and to evaluate instabilities of nonlinear dynamical systems. In addition to various methods that have been developed in the past, machine learning has also been incorporated recently. However, when the system size is large, a method to investigate dynamical properties by time series analysis has not been established yet. This thesis is devoted to developing methods to investigate properties of large chaotic experimental systems by time series analysis. To accomplish this goal, I conducted two studies.

The first study is to extend a recurrence-based method so that it can be applied to large systems. In experimental systems, the most common approach to estimating the Lyapunov exponents from time series is to regard recurrences of the time series as perturbations on its trajectories, but one cannot apply this approach to large systems. This is because the recurrence frequency decreases exponentially with increasing dimensionality of phase space.

To address this problem, I focused on the fact that many of large systems are described by partial differential equations, neighbor coupled systems, and globally coupled systems, and the evolution of a local variable of such systems does not necessarily require a large number of variables. For example, the evolution of a local variable is determined only by the local variable and the mean field in the case of globally coupled systems or the consecutive local variables in the case of neighbor coupled systems. I found that one can effectively reduce the number of degrees of freedom related to recurrences by considering time series in a reduced space, composed only of such variables.

I numerically inspected this idea for systems with global and neighbor coupling. I detected recurrences in the reduced space from numerically generated time series data and estimated the parameters and functions needed to evaluate the dynamics in tangent space. After that, I evolved states and perturbations by interpolation from the given time series data and the estimated parameters and functions. By calculating the Lyapunov exponents, I demonstrated that this method is able to evaluate the Lyapunov spectrum precisely. One might consider that systems with global coupling are too special, but, in general, well-mixed systems are often considered to be globally coupled systems, and actually, there are real examples of such systems. In addition, the result for the neighbor coupled system suggests that the applicability of my method can be extended to systems of partial differential equations.

The second study is to measure the dimension of an inertial manifold from time series data using reservoir computing. Partial differential equations are formally infinite-dimensional dynamical systems, but in systems with dissipation, it is known that there tends to be a finite-dimensional manifold, called the inertial manifold, and since the solution trajectory is exponentially attracted to the inertial manifold, its trajectory can be described by a finite set of degrees of freedom. This, for example, makes it possible to determine the number of variables required to describe the dynamics of experimental systems.

Takeuchi et al. have proposed that there exists a structure in tangent space that seems to be a linear approximation of an inertial manifold by focusing on the tangencies of covariant Lyapunov vectors (CLVs). They found numerically that Lyapunov modes in dissipative systems can be divided into a finite number of ``physical modes" and the residual ``spurious modes." These modes can be distinguished by the existence or absence of tangencies of the CLVs. They argued that a finite-dimensional manifold composed of all physical modes, which they call the ``physical manifold," may give a local linear approximation of the inertial manifold in tangent space. In contrast to numerical calculations, there is still no reliable way to measure the dimension of an inertial manifold from time series data in experimental systems.

In this study, I trained a reservoir using time series data of the Kuramoto-Sivashinsky (KS) equation and calculated the CLVs of the reservoir dynamical system. I found that by checking the presence or absence of tangencies of the CLVs, the number of physical modes of the reservoir agrees with that of the KS equation, except for one physical mode. I identified this missing physical mode by calculating the angles between the physical manifold of the reservoir dynamical system and the perturbations corresponding to the three symmetries of the KS equation. These indicate that reservoir computing is able to capture an inertial manifold from time series data.

In this series of studies, I successfully procure the methods that can be applied to many large experimental systems, which can measure not only the Lyapunov spectrum but also the dimension of an inertial manifold. The proposed method in the second study, in combination with previous studies for discovering governing equations from time series data, has the potential to identify the governing equations of the experimental system. Finally, I discuss some open problems that should be solved in future applications to experimental systems.

備考 ： 論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。
Note : Thesis Summary should be submitted in either a copy of 2000 Japanese Characters and 300 Words (English) or 1copy of 800 Words (English).