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Title(English)	WKB periods and TBA equations in ODE/IM correspondence
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論 文 要 旨

THESIS SUMMARY

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申請学位（専攻分野）： Academic Degree Requested	博士 Doctor of （ 理学 ）
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要旨（和文 2000 字程度）
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In recent years, it has been studied the non-trivial relation between ordinary differential equations (ODE) and two dimensional integrable models (IM), which is called the ODE/IM correspondence. This correspondence relates the spectral analysis of ODE to the functional analysis of IM. For the second order ODE, the Stokes multipliers which appear in the asymptotic analysis of it are identified with the Y-functions in the corresponding IM. The Y-functions are the analytic functions satisfying a system of integral equations describing the models in a thermodynamic setting. These integral equations are useful to compute the free energies of the equilibrium states of IM and are called the thermodynamic Bethe ansatz (TBA) equations. The ODE/IM correspondence for the second order ODE with polynomial potential relates one ODE to several IM. This can be studied from the wall-crossing phenomena of the TBA equations, which are the analytic continuation of the TBA equations induced by the continuous deformations of the potential in the ODE. The form of the TBA equations deforms through the wall-crossing, thus the corresponding IM described by the TBA equations also changes. This relation may provide a new classification of IM and be useful to find new theories. By using the WKB analysis, we can relate the Stokes multipliers to the WKB periods that are, thus, also identified with the Y-functions. Applying the exact WKB analysis, in other words, the resurgence technique, to the ODE, it turned out that the TBA equations capture the whole information of the WKB periods, i.e. one can perform the calculation of them in all orders. A similar argument is expected to hold for the case of the higher order ODE, however, the theory of the exact WKB analysis has not been established yet.

In this thesis, we generalize the correspondence between the WKB periods and Y-functions to the higher order ODE with the polynomial potential. We first considered the case where the zeros of the potential are aligned near the real axis, and proposed the formula which relates the WKB periods to the Y-functions. In these arguments, we mainly used the WKB analysis, including the Stokes

graphs and the so-called abelianization trees. We then tested the formula in several ways, including the numerical tests at the level of the perturbative expansions. After establishing it, we also deformed the potential and studied the wall-crossing phenomena of the TBA equations. As the simplest examples, that for the third order ODE with cubic and quartic potentials were investigated in detail. Our study of the ODE/IM correspondence would provide some insight to establish the resurgence theory of the higher order ODE in future works.

The ODE/IM correspondence can also be connected with the four dimensional N=2 supersymmetric field theories. The kind of ODE we studied are regarded as the Seiberg-Witten curve of the Argyres-Douglas (AD) theory, which is a class of the simplest and strongly interacting four dimensional N=2 superconformal field theory with no known Lorentz covariant Lagrangian, in the Nekrasov-Shatashvili limit of a certain deformed background metric called the Ω-background. Furthermore, the classical parts of the WKB periods compute the central charges of the N=2 supersymmetric algebra for BPS states, which are the N=2 preserving states saturating the BPS mass bound, in the four dimensional theory. In the four dimensional N=2 theories, it has been well studied the spectrum of the BPS states and its abrupt changes that occur if one varies the parameters in the moduli space of the Coulomb vacua. In the context of the four dimensional theory, these changes of the spectrum are also called the wall-crossing phenomena. The AD theories enjoy the simplest type, the pentagon type, of the wall-crossing phenomena. The wall-crossing phenomena of the TBA equations are expected to be associated with that of the four dimensional theory, but this relation has not been well understood yet. From the detailed analysis of the wall-crossing of TBA equations for the third order ODE with the polynomial potential, we found that the wall-crossing of the corresponding TBA equations are related to the pentagon type wall-crossing in the four dimensional theory. Our study also shows the connections between the different types of the TBA equations, which are associated with the dualities of the AD theories. These observations are the pieces of evidence supporting the relation between the four and two dimensional theories. Our investigations and results would help us to uncover deeper connections among the exact WKB analysis of higher ODE, the classifications of IM, and the dynamics of four dimensional theories in strongly coupling phases in the future.