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Integrated Public Transportation System with Shared Autonomous Vehicles and Fixed-Route Transits: Dynamic Traffic Assignment-Based Model with Multi-Objective Optimization

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Abstract

The shared autonomous vehicle (SAV) system is considered as an efficient transportation mode in the future. In the literature, optimization of SAV systems has been extensively studied. However, SAV systems could bring greater social benefits if we could use them with existing public transportation systems, such as bus rapid transit (BRT), in an integrated manner. This study proposes a model of SAV-BRT system, an integrated system that takes advantage of the flexibility of SAVs and the mass transport capability of BRT. The proposed model is based on a dynamic traffic assignment model so that it captures important features of SAV-BRT system, such as endogenous traffic congestion, detour and waiting of SAVs, BRT's dynamic scheduling. The model is formulated as a multi-objective optimization problem so that trade-off relations regarding the system's performance can be explicitly analyzed. The behavior of the model is investigated by conducting numerical experiments based on actual travel data obtained from an urban area in Japan. As a result, we confirmed that the model behaves reasonably, and several insights on SAV-BRT systems have been obtained.

Keywords Automated vehicle · Bus rapid transit ·
Aggregated vehicle routing and passenger pickup and delivery with time window · Infrastructure design

1 Introduction

1.1 Background

The *shared autonomous vehicle (SAV) system* is considered as an efficient transportation mode in the future. It transports people using SAVs, which are autonomous vehicles that are shared by society, with optimized routing and passenger pickup/delivery and ridesharing [6, 13]. Due to this nature, SAV systems can be considered as a new type of public transport. Compared with privately owned vehicles, SAV

systems would be efficient as many travelers can share one vehicle, resulting in less traffic congestion and parking demand. Compared with conventional fixed-route public transits, SAV systems would have high flexibility regarding routing and schedule and small passenger capacity. In the literature, optimization of operation of SAV systems has been extensively studied [6, 9, 17].

SAV systems could bring greater social benefits if we could use them with existing public transportation systems. Fixed-route public transits, such as bus rapid transit (BRT) and trains, have significantly larger passenger capacity than SAVs. Therefore, it would be possible to design integrated public transportation systems that take advantage of the advantages of both SAVs and fixed-route transits. For example, fixed-route transits could transport travelers on routes with high demand. SAVs could serve as an access/egress transportation mode to stations or a door-to-door transportation mode; because of the flexibility of the SAV system, the operation mode can be dynamically optimized depending on the demand profile. Such integrated public transportation systems might be efficient (in terms of operation cost, traveler cost, and infrastructure cost) for both

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of high demand urban areas and sparse demand rural areas. Hereafter, we call such an integrated transportation system as *SAV-BRT system*.

It has been pointed out that there exist strong trade-off relations in the design of SAV systems [17]. For example, an SAV system could be designed to maximize the passenger benefit or the operator benefit. The result of case studies [17] showed that the total travel time of passengers and the infrastructure construction cost vary by a factor of two or three depending on the design. This difference would be very significant for the society, and it is necessary to clarify what trade-off relations exist before designing an SAV system. SAV-BRT systems may have larger trade-off relations, because they have higher degree of freedom in design.

1.2 Literature Review

Optimization of SAV systems has been extensively studied in the literature. One of the most popular approaches is to model SAVs' routing problem as the vehicle routing with pickup and deliveries with time windows problem [1–3, 15, 20]. In this approach, the problem becomes mixed integer programming problem with a very large number of variables, making the problem very difficult to solve. To address this issue, some studies have proposed aggregated versions of the problem in which SAVs and travelers are considered as continuous flow [9, 17], in accordance to the common method in static and dynamic traffic assignments. Other approaches such as multi agent simulation and graph algorithms have been also used [6, 16].

Trade-off relations in SAV and related transportation systems have been pointed out. For example, travelers' total travel time and the number of SAVs would be in a trade-off relation: the former will be small when the latter is small, and vice versa. In addition, SAV systems can be operated to maximize various operation criteria, such as operator profit, travelers' welfare, and social welfare. The realized state would be significantly different depending on the choice of the operation criteria. Thus, it would be important to investigate such trade-off relations in the planning phase of SAV-BRT systems. Docherty et al. [4] studied general topics related to these trade-offs in the context of the governance of smart mobility. Aiko et al. [3] numerically investigated trade-offs between total travel time and total discomfort values of passengers due to ridesharing with strangers. Ruch et al. [16] numerically investigated trade-offs between the number of SAVs and the operational performance. Seo and Asakura [17] numerically investigated trade-offs among SAV systems' characteristics by using multi-objective optimization theory [5].

Models for integration of SAV systems and fixed-route transits have been proposed by some recent studies. Levin

et al. [10] proposed an optimization model for SAV systems with public transit. The specification of transit systems is given and not subject to optimization. Wen et al. [24] proposed an agent-based simulation method for transit-oriented SAV systems. The main idea is to model the demand-supply interaction. Pinto et al. [14] proposed a design problem for transit networks and SAV fleet size. Operation of SAVs is described by agent-based simulation. Gurumurthy et al. [8] investigated the role of SAV systems as first-mile and/or last-mile travel modes to existing transit systems. Shan et al. [18] proposed a framework for a transit network design where SAVs are used for first-mile travel. Operation of SAVs is modeled very simply: the travel time is fixed to the average travel time between regions. In summary, there have been no studies on full integration of SAVs and BRTs, in which they are modeled with equal granularity and optimized simultaneously. In addition, the trade-offs in SAV-BRT systems have not been explicitly investigated.

1.3 Objective

The objective of this research is to develop an optimization model for SAV-BRT systems (i.e., integrated public transportation system with shared autonomous vehicles and fixed-route transits) and investigate its behavior, especially trade-off relations, based on actual travel data. The proposed model is based on a dynamic traffic assignment (DTA) model [17] so that it captures important features of SAV-BRT system, such as endogenous traffic congestion, detour and waiting of SAVs, BRT's dynamic scheduling. The model is formulated as a multi-objective optimization problem [5] so that trade-off relations can be explicitly analyzed. The behavior of the model is validated by conducting numerical experiments based on actual travel data obtained from an urban area in Japan.

2 Model

In this section, the formulation and solution methods for dynamic system optimum (DSO)-based SAV and BRT system optimization problem are described. The outline of this model is explained in Section 2.1, the formulation is shown in Section 2.2, and the solution method of this problem is explained in Section 2.3.

2.1 Outline of the Model

The proposed model assumes that SAVs and BRTs are the only available transportation modes in the considered urban area. SAVs' movement can be flexibly adjusted depending on the demand, whereas BRTs' route is fixed as in the public

transit. Specifically, the following factors are determined simultaneously.

- aggregated SAVs' dynamic routing problem with passengers pickup and delivery
- passengers dynamic ridesharing matching
- fleet sizing
- road network design
- parking space assignment
- BRTs' routes and schedule design
- travelers' transportation mode assignment

This model is formulated as a multi-objective optimization problem [5] with 7 objective functions. The objective functions are

- T: total travel time of travelers
- $D, \widehat{D}$: total travel distance of SAVs and BRTs, respectively
- $N, \widehat{N}$: number of SAVs and BRTs, respectively
- C: total cost of the infrastructure construction
- G: total number of links determined as the BRT lanes

This study models SAV-BRT systems by considering its four sub-modules: network, SAV, BRT, and travelers. Their characteristics are explained in the following sub-sections.

2.1.1 Network

The concept of time-expanded network [17], shown in Fig. 1, is employed to describe the dynamic traffic
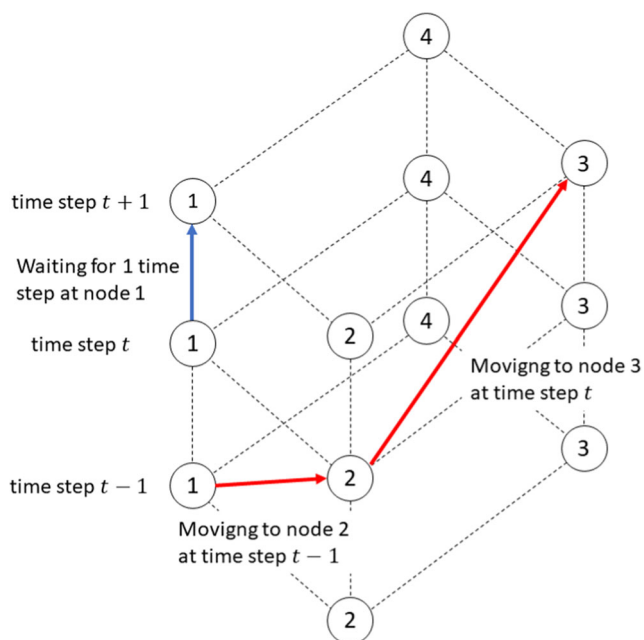

Fig. 1 An example of time-expanded network

assignment of SAVs, BRTs, and travelers in a network. A network is composed of nodes and links. SAVs can travel the network under capacity constraints. BRTs can only travel fixed routes in the network. Travelers can travel the network by riding SAV or BRT.

Each link has its traffic capacity, and the excess of SAVs over the capacity cannot enter the link. Each node also has its storage capacity, and the excess of SAVs over the capacity cannot wait at the node. Both link traffic capacity and node storage capacity have maximum and minimum values, and they can be increased from the minimum to the maximum by infrastructure investment in the network planning phase.

BRTs travel the network using links with lanes for BRTs only (i.e., BRT lanes). BRT lanes are designated network planning phase, and they will not change during operation. BRT lanes limit the link traffic capacity because they occupy a part of the link.

2.1.2 SAV

SAVs move for travelers' transportation or parking, and SAVs stop when they are parked at the node or when they are caught in traffic congestion. The model for SAV is basically identical to that of Seo and Asakura [17].

The traffic congestion is represented by the point queue model with limited queue length. SAVs always run on the links at the free flow speed, and SAVs cannot enter the link when the traffic volume of SAVs reaches the link traffic capacity. SAVs which cannot enter a link must wait at the node which is the starting point of the link. When the volume of waiting SAVs at the node also reaches the node storage capacity, SAVs cannot enter the node. That is, congestion is expanded. The dynamic route is determined by solving the optimization problem.

SAVs have pre-determined passengers' capacity, and SAVs can transport one or more travelers with ridesharing under this constraint. How and which passengers are loaded by SAVs is determined by solving the optimization problem.

2.1.3 BRT

BRTs run along the BRT lanes. Thus, BRTs are free from traffic congestion and always run according to the schedule.

BRTs' routes design and decision of BRTs' schedules are determined by solving the optimization problems. BRTs' routes design means both the decision of the number of BRT routes and routing of BRTs. BRTs' schedules mean the decision of the frequency of BRTs and BRTs running intervals.

This model can describe a balance between the use of SAVs and BRTs. Because BRTs are mass transportation,

passengers' capacity of BRTs is larger than that of SAVs generally. Thus, passengers' capacity of SAVs and BRTs are given separately. However, considering that BRTs have to wait at every stop as the conventional transportation, the average speed of BRTs is slower than that of SAVs. Thus, the average speed of SAVs and BRTs are also given separately. In this way, the balance of SAVs and BRTs which have both merit and demerit is expressed in this model.

2.1.4 Travelers

Time-dependent origin-destination (OD) demand of all travelers is defined by origin and destination nodes, departure time, and allowable maximum travel time. All travelers need to finish their travel within their own allowable maximum travel time.

Travelers move to their destination by SAV or BRT, not on foot. Thus, when travelers cannot find both SAVs and BRTs or when SAVs and BRTs are filled with passengers, travelers must wait at the node without moving.

Travelers can mutually transfer between SAVs and BRTs at station nodes by spending given transfer time.

2.2 Formulation

DSO-based SAV-BRT system optimization problem is formulated according to the principles described in Section 2.1.

The definition of variables is summarized in the Table 1, and the formulation is as follows.

$$\min\{T, D, \widehat{D}, N, \widehat{N}, C, G\} \tag{1}$$

subject to

$$\sum_{ij,s,t,k} t_{ij} y_{s,ij}^{k,t} + \sum_{ij,s,t,k} \hat{t}_{ij} \hat{y}_{s,ij}^{k,t} + \sum_{i,s,t,k} y_{s,i}^{\prime k,t} t' + \sum_{i,s,t,k} \hat{y}_{s,i}^{\prime k,t} t' = T \tag{2}$$

$$\sum_{ij,i \neq j,t} d_{ij} x_{ij}^t = D \tag{3}$$

$$\sum_{ij,i \neq j,m,t} d_{ij} z_{ij}^{m,t} = \widehat{D} \tag{4}$$

$$\sum_i x_{0i}^0 = N \tag{5}$$

$$\sum_{i,t,m} z_{0i}^{m,t} = \widehat{N} \tag{6}$$

$$\sum_{ij} c_{ij}(\mu_{ij} - \mu_{ij}^{\min}) + \sum_i c_i(\kappa_i - \kappa_i^{\min}) = C \tag{7}$$

$$\sum_{ij,m} g_{ij}^m = G \qquad \forall m \tag{8}$$

$$\sum_j x_{ji}^{t-t_{ji}} - \sum_j x_{ij}^t = 0 \qquad \forall i, t \in (0, t_{\min}) \tag{9}$$

$$z_{0i}^{m,t} - z_{i0}^{m,t} + \sum_{j,i \neq j} z_{ji}^{m,t-\hat{t}_{ji}} - \sum_{j,i \neq j} z_{ji}^{m,t} = 0$$
$$\forall i, t, m \tag{10}$$

$$\sum_j y_{s,ji}^{k,t-t_{ji}} - \sum_j y_{s,ij}^{k,t} + y_{s,0i}^{k,t} - y_{s,i0}^{k,t} + w_{s,i}^{k,t-t'} - \hat{w}_{s,i}^{k,t} = 0$$
$$\forall i, s, k, t \in T_k \tag{11}$$

$$\sum_j \hat{y}_{s,ji}^{k,t-\hat{t}_{ji}} - \sum_j \hat{y}_{s,ij}^{k,t} + \hat{y}_{s,0i}^{k,t} - \hat{y}_{s,i0}^{k,t} + \hat{w}_{s,i}^{k,t-t'} - w_{s,i}^{k,t} = 0$$
$$\forall i, s, k, t \in T_k \tag{12}$$

$$\sum_{s,k} y_{s,ij}^{k,t} \leq \rho x_{ij}^t \qquad \forall ij, i \neq j, t \tag{13}$$

$$\sum_{s,k} \hat{y}_{s,ij}^{k,t} \leq \sum_m \hat{\rho} z_{ij}^{m,t} \qquad \forall ij, i \neq j, t \tag{14}$$

$$x_{ij}^t \leq \mu_{ij} - \hat{\mu}_{ij} g_{ij}^m \qquad \forall ij, i \neq j, t, m \tag{15}$$

$$x_{ii} \leq \kappa_i \qquad \forall i, t \tag{16}$$

$$y_{s,0r}^{k,k} + \hat{y}_{s,0r}^{k,k} = M_{rs}^k \qquad \forall rs, k \tag{17}$$

$$\sum_{t \in T_k} y_{s,s0}^{k,t} + \sum_{t \in T_k} \hat{y}_{s,s0}^{k,t} = \sum_r M_{rs}^k$$
$$\forall s, k \tag{18}$$

$$\mu_{ij}^{\min} \leq \mu_{ij} \leq \mu_{ij}^{\max} \qquad \forall ij \tag{19}$$

$$\kappa_i^{\min} \leq \kappa_i \leq \kappa_i^{\max} \qquad \forall i \tag{20}$$

$$\sum_i a_i^m = \sum_i b_i^m \qquad \forall m \tag{21}$$

$$\sum_i a_i^m \in \{0, 1\} \qquad \forall m \tag{22}$$

$$a_i^m + b_i^m \in \{0, 1\} \qquad \forall i, m \tag{23}$$

$$a_i^m \leq \sum_{j,i \neq j} g_{ij}^m \leq 1 - b_i^m \qquad \forall i, m \tag{24}$$

$$b_i^m \leq \sum_{j,i \neq j} g_{ij}^m \leq 1 - a_i^m \qquad \forall i, m \tag{25}$$

$$a_i^m + \sum_{j,i \neq j} g_{ji}^m - \sum_{j,i \neq j} g_{ij}^m - b_i^m = 0 \qquad \forall i, m \tag{26}$$

$$0 \leq z_{ij}^{m,t} \leq g_{ij}^m \qquad \forall ij, i \neq j, t, m \tag{27}$$

$$0 \leq z_{0i}^{m,t} \leq a_i^m \qquad \forall i, t, m \tag{28}$$

$$0 \leq z_{i0}^{m,t} \leq b_i^m \qquad \forall i, t, m \tag{29}$$

$$z_{ij}^{m,0} = 0 \qquad \forall ij, i \neq j, m \tag{30}$$

$$x_{ij} \geq 0 \qquad \forall ij, t \tag{31}$$

$$y_{s,ij}^{k,t} \geq 0 \qquad \forall ij, s, k, t \in T_k \tag{32}$$

$$\hat{y}_{s,ij}^{k,t} \geq 0 \qquad \forall ij, i \neq j, s, k, t \in T_k \tag{33}$$

$$x_{0i}^0 \geq 0 \qquad \forall i \tag{34}$$

$$y_{s,s0}^{k,t} \geq 0 \qquad \forall s, t, k \in T_k \tag{35}$$

$$\hat{y}_{s,s0}^{k,t} \geq 0 \qquad \forall s, t, k \in T_k \tag{36}$$

$$g_{ij}^m \in \{0, 1\} \qquad \forall ij, i \neq j, m \tag{37}$$

$$z_{0i}^{m,t} \in \{0, 1\} \qquad \forall i, m, t \tag{38}$$

Table 1 List of the mathematical notation

variable	definition
T	total travel time of travlers
D	total travel distance of SAVs
$\widehat{D}$	total travel distance of BRTs
N	total number of SAVs
$\widehat{N}$	total number of BRTs
C	total cost of the infrastructure construction
G	total number of links which are determined as the BRT lanes
$x_{i,j}^{t}$	number of SAVs starting to move the link ij at time step t
$y_{s,ij}^{k,t}$	number of travelers, who started to move to the node s at time step k, starting to move the link ij at time step t by SAV
$\hat{y}_{s,ij}^{k,t}$	number of travelers, who started to move to the node s at time step k, starting to move the link ij at time step t by BRT
$w_{s,i}^{k,t}$	number of travelers, who started to move to the node s at time step k, transferring from BRT to SAV at the node i at time step t
$\hat{w}_{s,i}^{k,t}$	number of travelers, who started to move to the node s at time step k, transferring from SAV to BRT at the node i at time step t
$z_{ij}^{m,t}$	number of BRTs starting to move the link ij at time step t $(m = 1, 2, \ldots, n)$
t_{ij}	SAV's free-flow travel time of of the link ij if $i \neq j$
t_{ii}	waiting time at the node i for one time step
$\hat{t}_{ij}$	BRT's travel time of the link ij
t'	time cost of tranxferring between SAVs and BRTs
d_{ij}	distance of the link ij
c_{ij}	unit cost of expanding traffic capacity of the link ij
c_i	unit cost of expanding storage capacity of the node i
ρ	passengers capacity of one SAV
$\hat{\rho}$	passenger capacity of one BRT
μ_{ij}	traffic capacity of the link ij
κ_i	storage capacity of the node i
$\hat{\mu}_{ij}$	number of SAVs which will be occupied by setting BRT lane at the link ij
$\mu_{ij}^{\max}$	maximum allowable value of μ_{ij}
$\kappa_i^{\max}$	maximum allowable value of κ_i
$\mu_{ij}^{\min}$	minimum allowable value of μ_{ij}
$\kappa_i^{\min}$	minimum allowable value of κ_i
n	maximum number of BRT routes in the network
a_i^m	if the node i is starting point of BRT route, this value is 1. if not, this value is 0. $(m = 1, 2, \ldots, n)$
b_i^m	if the node i is terminal point of BRT route, this value is 1. if not, this value is 0. $(m = 1, 2, \ldots, n)$
g_{ij}^m	if the link ij is determined as a part of BRT route, this value is 1. if not, this value is 0. $(m = 1, 2, \ldots, n)$
M_{rs}^k	time-depend demand of travelers with origin r, destination s, and departure time step k
T_k	subset of time step for travelers with departure time step k
$t_{\max}$	final time step

This model is formulated as a multi-objective mixed linear integer optimization problem. Equations (2) and (8) are the definitions of objective functions. Eq. (2) defines the total travel time of travelers. Equation (3) defines the total travel distance of SAVs, and Eq. (4) defines that of BRTs. Equation (5) defines the total number of SAVs, and Eq. (6) defines that of BRTs. Equation (7) defines the total cost

of infrastructure construction. Equation (8) defines the total number of the links determined as the BRT lanes, and this objective function is needed for the removal of subtour.

Equation (9) means the conservation of SAVs, Eq. (10) means that of BRTs, and Eqs. (11) and (12) mean that of travelers. The conservation of travelers is represented by two equations, Eq. (11) is for the SAV users and

Eq. (12) is for BRT users. Figures 2, 3 and 4 illustrate these conservation laws in the time-expanded network. In these figures, the circles show nodes and the arrows show movements of link. In addition, spatial change is shown by horizontal change, and temporal change is shown by the vertical change in these figures. The relation of inflow and outflow on node i at time step t is simply represented.

Equations (13) and (14) mean the restrictions of travelers' movement. Equation (15) means that the number of SAVs moving the link ij is limited to the link traffic capacity. Equation (16) means that the number of SAVs waiting at the node i is limited to the node storage capacity.

Equations (17) and (18) mean the restriction of the OD.

Equations (21)–(26) are the constraints about the determination of BRT routes. In these constraints, the binary variables a_i^m, b_i^m representing starting and terminal nodes of BRT routes and the binary variables g_{ij}^m which indicate the link is a part of the BRT routes or not are used.

BRTs are constrained to run on the links determined as a part of BRT routes by Eq. (27). In addition, Eqs. (28) and (29) are constraints defining the number of BRTs at the starting and terminal nodes, and BRTs are constrained to depart from the starting node and terminate at the terminal node.

The objective function G and Eq. (30) are needed for the removal of *subtour*, which is a technical concept used in routing optimization problems [22]. It is a circular route which doesn't have both the designated starting and terminal nodes. Thus, apart from the regular route which has the starting and terminal nodes defined by a_i^m and b_i^m, the subtour is formed as a circular route around a certain section; thus, a subtour is not appropriate solution to the considered problem and should be avoided. The subtour on which BRTs don't run can be eliminated by minimizing the objective function G because it doesn't affect the other objective functions. However, if BRTs run on the subtour and transport travelers, the other objective functions are affected by the subtour. Thus, the objective function G cannot handle this situation, and Eq. 30 is needed. Equation (30) means that the initial number of BRTs at every link is defined as zero. When BRTs run on the subtour, Eq. (39) holds at every time step in Eq. (10). Because the initial number of BRTs at every link is zero by the constraint of Eq. (30), the number of BRTs in the subtour is constantly zero.

$$\begin{cases} z_{0i}^{m,t} = z_{i0}^{m,t} = 0 \\ \sum_{j,i\neq j} z_{ji}^{m,t-\hat{t}_{ji}} = \sum_{j,i\neq j} z_{ji}^{m,t} = 1 \end{cases} \tag{39}$$

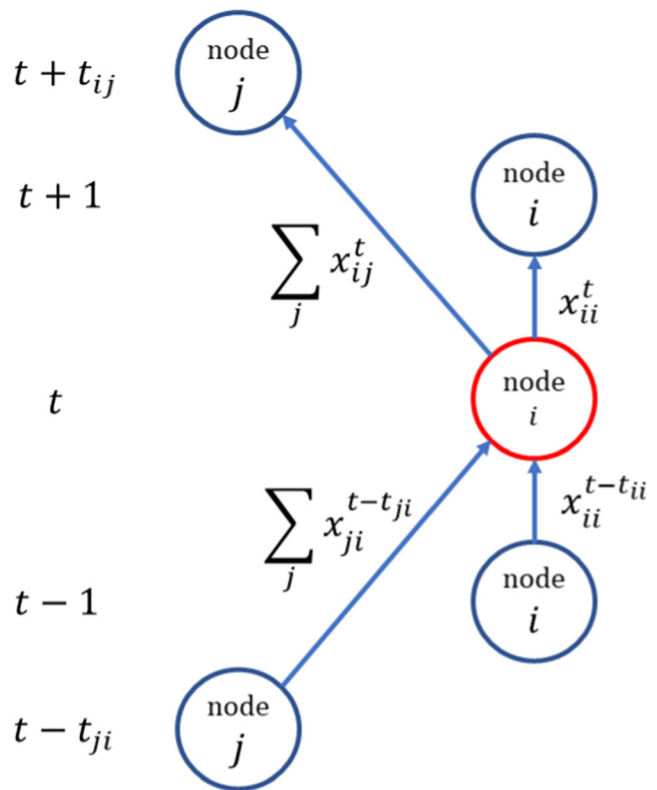Equations (31)–(38) are the non-negative constraints.

Fig. 2 Conservation of SAVs in time-expanded network

2.3 Solution Method

Solving multi-objective optimization problem means drawing its Pareto frontier, which is a set of Pareto optimal points, of objective functions [5]. In this study, the weighted sum method is employed as a numerical solution method. The single objective optimization problem represented in
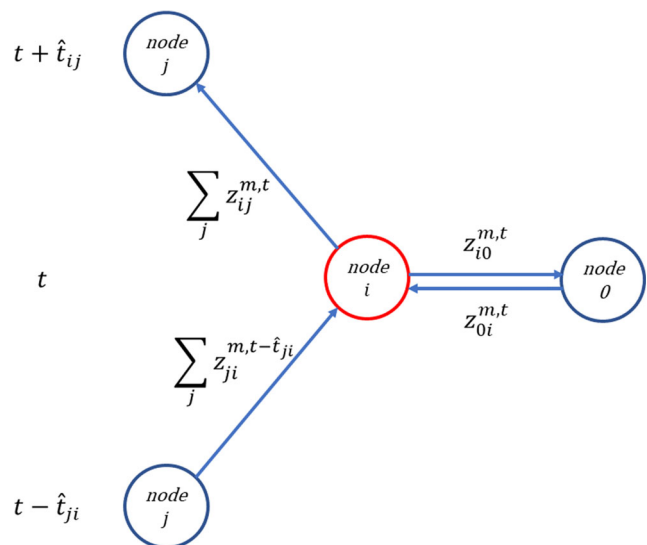

Fig. 3 Conservation of BRTs in time-expanded network
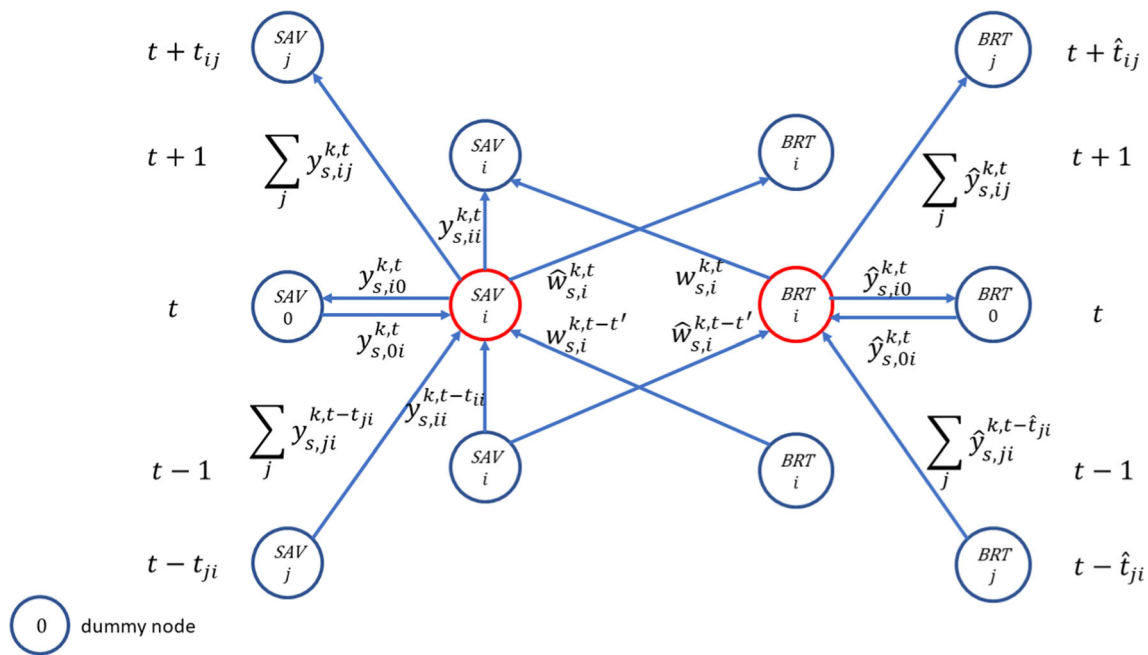
Fig. 4 Conservation of travelers in time-expanded network

Eq. 40 is calculated. The constraints are the same as the original problem: Eqs. (2)–(38).

$$\min \alpha_T T + \alpha_D D + \alpha_{\widehat{D}} \widehat{D} + \alpha_N N + \alpha_{\widehat{N}} \widehat{N} + \alpha_C C + \alpha_G G \quad (40)$$

The coefficients of each objective function are non-negative constants, and they are weights representing the priority of each objective function. The solution of problem Eq. (40) is Pareto optimal. Thus, the Pareto frontier is drawn approximately by obtaining the solution of problem Eq. (40) with different weight values. This is called the weighted-sum method and one of the standard methods to solve multi-objective optimization problems [5].

The model is classified as mixed integer linear programming, which is somewhat complicated but solution methodology is established. In this paper, the model has been solved by the branch and bound method of Gurobi Optimizer [7]. More efficient algorithms (e.g., application of Dantzig–Wolfe decomposition [23]) may be developed by considering the structure of the problem, and this is one of the future works.

2.4 Properties of the Model

The proposed model can treat important features of SAV-BRT systems, such as endogenous traffic congestion, detour and waiting of SAVs, and BRT's scheduling, but has limitations.

Firstly, the model does not identify which traveler uses which SAV and which traveler is doing ridesharing. In this model, we cannot trace the specific routes of SAVs

and travelers because the flows of SAVs and travelers are treated as the quantity on each link or node. Similarly, some travelers may experience too many transfers between SAVs and BRT in this model. This can be considered as a shortcoming of this model.

Secondly, the model cannot distinguish parking SAVs and waiting SAVs in traffic jams on nodes. SAVs located on nodes, which are waiting for parking or in congestion, are treated as the same in this model.

3 Numerical Experiment

In this section, the behavior of the proposed model is numerically investigated based on actual travel data. In order to investigate the model's behavior, the following scenarios are considered:

– SAV only case ($n = 0$) (in this case, the proposed model is reduced to that of Seo and Asakura [17])
– SAV and BRT case. The number of BRT routes is one ($n = 1$).
– SAV and BRT case. The number of BRT routes is two ($n = 2$).

For each scenario, the Pareto frontier is derived to investigate the trade-off relations in the SAV-BRT system. In addition, to investigate the detailed behavior of the model, spatial distribution of BRT routes, SAV flow, and traveler flow and dynamic traffic states are derived for some specific Pareto optimal states.

3.1 Data and Model Specification

In this numerical experiment, the network data and the OD data in Omiya-ku, Saitama City, Japan were used. This data is made by estimating traffic demand based on the augmented national digital road map database [19] and the sixth person trip survey in Tokyo metropolitan area [21] in high resolution with the method proposed by Mitani et al. [12].

The optimization problem assuming commuting hours in the morning was calculated. The range of data is 9 km in length and 10 km in width, centered in Omiya Station. The region of this numerical experiment is shown in Fig. 5. We discretize the area into grid-shaped meshes to aggregate the demand and network. The network data has information of the number of links connected between each mesh. The capacity of each link in the test network is set to be proportional to the number of actual roads at the corresponding location. Therefore, we consider that the test network reflects some properties of the actual road network.

For example, the central area of the test network (meshes 21 and 22 in Fig. 5) is the downtown (i.e., around Omiya station) of this area, and is connected by major roads to nearby meshes. Especially, the number of actual roads from the western areas is large. This information is reflected to the test network: the traffic capacity from meshes 15 and 16 is defined to be large.

The OD data is defined by origin and destination nodes and departure time of every trip. The OD data from 7:30 a.m. to 8:00 a.m. was used in this numerical experiment to assume the DSO assignment. The total number of trips was 42986. Because travelers' allowable maximum travel time is assumed 30 minutes, the flow for a total of one hour from 7:30 a.m. to 8:30 a.m. was calculated. In order to decrease the calculation cost, all destination nodes were aggregated into 5 nodes which are the most popular destinations.

The passenger capacity of a SAV is 2 persons, and that of a BRT is 50 persons. The speed of SAVs is 30 km per hour, and that of BRT is 20 km per hour. The values of the other model parameters are summarized in Table 2.

The number of variables and constraints, and computation time for this numerical experiment are listed in Table 3. The number of links including the ones for expressing waiting at nodes is 210, and the number of nodes is 42 in this network. The computation time strongly depends on the number of integer variables in the problem, which is almost proportional to the number of routes of BRT (variable n). For example, in the case study of Section 3, the number of decision variables are as follows: 82498 continuous variables for $n = 0$ cases, 164659 continuous and 15372 integer variables for $n = 1$ cases, and 164659 continuous and 30744 integer variables for $n = 2$ case. The number of constraints is as follows: 44613 for $n = 0$ cases, 85148 for $n = 1$ cases, and 113164 for $n = 2$ cases. The computation time is as follows: 9 seconds (exact solution) for $n = 0$ cases, 7095 seconds (2 hours) for $n = 1$ cases (approximate solution with 0.79% duality gap), and 235918 seconds (66 hours) for $n = 2$ cases (approximate solution with 2.69% duality gap).

Regarding to accuracy, for $n = 0$ cases, we have always obtained exact solutions. For $n = 1$ cases, we generally obtained solutions with duality gap less than 1%. However, when α_D was large, solution time tends to be large. In such cases, we have accepted solutions with duality gap less than 3%. Likewise, for $n = 2$ cases, we generally obtained solutions with duality gap less than 3%, but we seldom accepted ones with 5%.
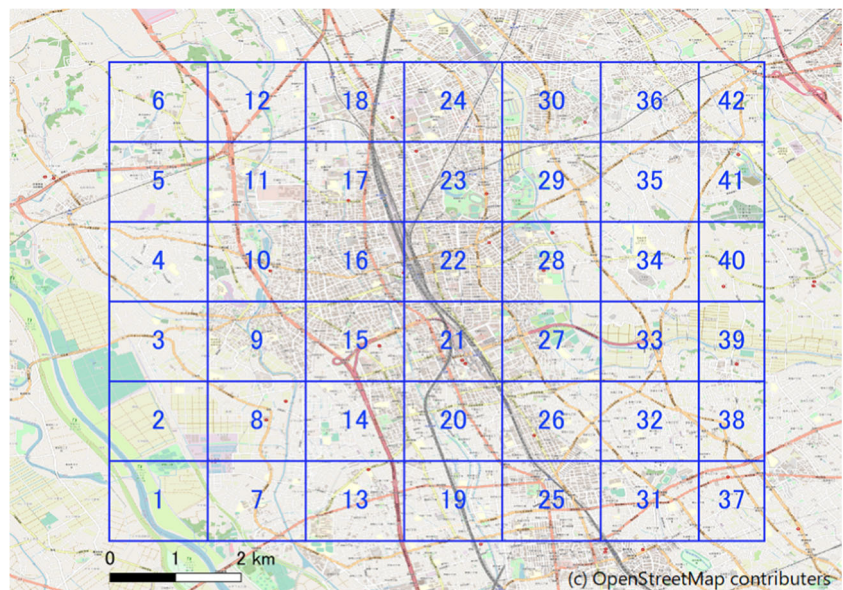
Fig. 5 The study area: Omiya, Saitama, Japan

Table 2 Model parameters

parameter	value
ρ	2 [person/veh]
$\hat{\rho}$	50 [person/veh]
c_{ij}	1 [/veh]
c_i	1 [/veh]
$\hat{\mu}_{ij}$	15 [veh]
t'	1 [time step]
t_{max}	60 [time step]
speed of SAVs	30 [km/h]
speed of BRTs	20 [km/h]

Because of the scale of the problem and integer variables, we cannot always obtain exact solutions. Instead, we obtain approximate solutions, and the accuracy of the approximation is evaluated by using the duality gap. We consider that we have successfully obtained reasonably accurate solutions, as the duality gaps of solutions were small (few percentages) and the shapes of Pareto frontiers were almost convex.

3.2 Results and Discussion

3.2.1 Pareto Frontiers

The Pareto frontiers of the three scenarios are shown in Fig. 6. Note that an actual Pareto frontier is 7-dimensional object (i.e., relation among $T, D, \widehat{D}, N, \widehat{N}, C, G$), which is impossible to illustrate. In order to show important characteristics of the Pareto frontiers, we show 2-dimensional relation (i.e., cross section of the 7-dimensional Pareto frontier) between D and another objective function. The objective function D, total distance traveled by SAVs, is chosen because the mode choice between SAVs and BRT is one of the important points in the model. Note that Pareto frontiers of the other two objectives also have similar convex shapes, and thus it is not very informative to show all of the possible combinations of two objectives out of seven objectives. Also note that the number of actual Pareto solution is infinity, thus the results shown in Fig. 6 is just an approximation.

The Pareto frontiers were obtained by the following procedure. The value of α_D, the weight parameter for D, was varied over a sufficiently wide range (roughly 0.0003–30), whereas the rest of weights were fixed to the values shown in Table 4 for all scenarios.

The meanings of the values of weights shown in Table 4 are not strictly realistic. The aim of the multi-objective optimization employed by our study is to derive the trade-off relations between different objectives, not to derive a single optimal solution with specific weights. The trade-off relation is represented as Pareto frontier, a set of Pareto efficient solutions. Pareto frontiers can be derived by changing the weight parameters from 0 to infinity. The Pareto frontiers we obtained are summarized in Fig. 6. The specific value for weights has little meaning in the context of investigating the trade-off relations, and thus we have not specified the weight values considering rigorous consistency to the reality.

Theoretically, it is possible to determine the values of weights and derive a single solution of the model. However, this approach has a limitation. That is, we cannot determine the weight values precisely, because it should be determined by the society's political decision making (some societies prioritize traveler's benefit, others prioritize environment, etc.). Our multi-objective approach avoids this issue by obtaining various possible solutions. By deriving such various possible solutions, future policy planners would be able to choose the most suitable solution that matches the society's goal.

The values of weight parameters shown in Table 4 are determined so that most of the objective functions are not negligible in the optimization problem. For example, T (total travel time of ten-thousands passengers) is always substantially larger than $\hat{N}$ (number of BRTs). Thus, if we choose similar values for their weights, we cannot obtain reasonable solutions as some of the elements become completely negligible. This is not useful to demonstrate the effects of other objectives. The weight values in Table 4 were determined in order to maintain balance among objective function values so that we can avoid such inappropriate solutions. Please note that Pareto frontiers of the other two objectives also have similar convex shapes, and thus it is not very informative to show all of the possible combinations of two objectives. Please also note

Table 3 The computational cost

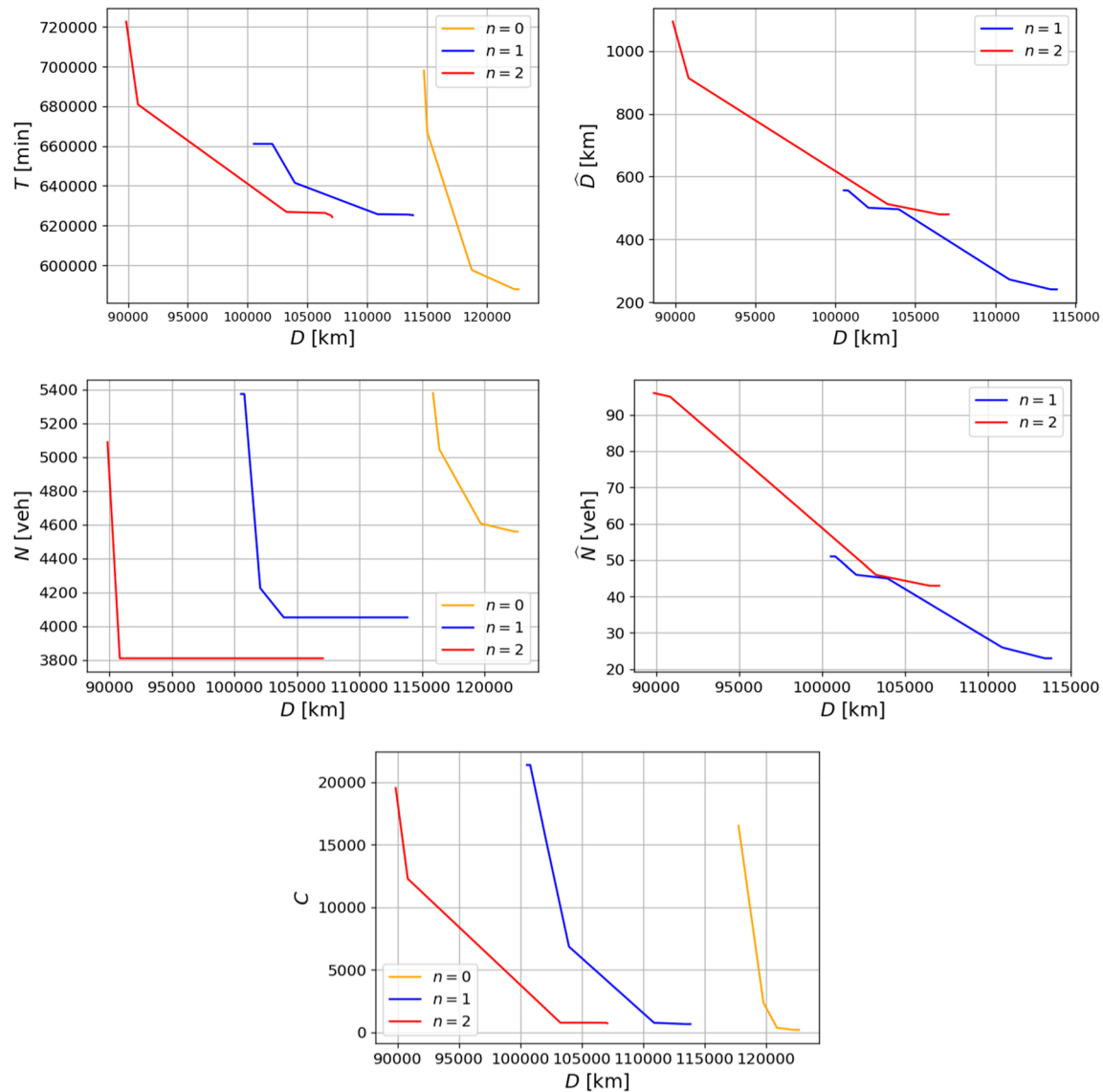|	$n=0$	$n=1$	$n=2$
number of continuous variables	82498	164659	164659
number of integer variables	0	15372	30744
number of constraints	44613	85148	113164
calculation time	9[s] (GAP: 0%)	7095[s] (GAP: 0.79%)	235918[s] (GAP: 2.69%)

Fig. 6 Comparison of Pareto frontiers

that the weight on G is exception and is set as small as possible. This is a technical objective function that eliminates mathematical errors called "subtour" and does not have physical meaning.

Approximated Pareto optimal solution can be obtained with high accuracy by using the weighted-sum method. The reasons are as follows. First, the continuous relaxation of the problem is a linear programming (which is convex), and therefore we can expect that a lower envelope (i.e., Pareto frontier) of the solutions is also convex. Thus, an accurate solution obtained by the weighted-sum method can be considered as an accurate Pareto optimal solution. We confirmed that the obtained solution was actually accurate (i.e., small duality gap). Second, the true Pareto frontier is

Table 4 The weight parameters for the Pareto optimal state

	α_T	α_D	$\alpha_{\widehat{D}}$	α_N	$\alpha_{\widehat{N}}$	α_C	α_G
for Pareto frontiers in Fig. 6	0.05	30	0.0003–30	8	80	1	0.001
for the other analysis	0.05	30	30	8	80	1	0.001

expected to be convex. The obtained solutions in Fig. 6 are almost convex. This is supporting evidence that the solution is reasonable.

According to Fig. 6, various trade-off relations in the SAV-BRT system are clarified. Pareto frontiers are obtained as monotonic curves with negative slopes. This means that the two variables are in a trade-off relation.

In the T–D relation, the total travel time of travelers T and the total travel distance by SAVs D are in trade-off relation for each scenario. It means that the more actively SAVs are used, the faster transportation systems can be realized. By comparing the scenarios, it is clear that the travel time generally increased by introduction of BRTs, while the distance traveled by SAVs decreased. This is because the more BRTs are used, the fewer travelers use SAV, but BRTs are generally slower than SAVs; this usage pattern is evident in $\hat{D}$–D relation. Thus, introduction of BRT enables efficient transportation systems in terms of traffic congestion, but it slightly increases average travel time of travelers.

The N–D relation shows that the introduction of BRTs decreases the number of required SAVs N. In SAV only case ($n = 0$), the allowable range to the number of SAVs is narrow: roughly 4600–5400 SAVs are required regardless of operation pattern. When BRT routes are introduced, the allowable ranges are significantly increased: roughly 4050–5300 SAVs for 1 route scenario, and 3800–5100 SAVs for 2 route scenario. There are 2 implications from this result: 1) the number of SAVs can be significantly decreased by introducing 1 or 2 BRT routes; 2) the operational flexibility of the SAV system also is increased by introducing BRTs.

The $\hat{N}$–D relation confirms that the distance traveled by SAVs is significantly decreased by increasing the number of BRTs $\hat{N}$. This is a reasonable result.

The C–D relation shows that the range of the infrastructure cost C is almost the same for all scenarios. However, the distance traveled by SAVs can be significantly decreased by BRTs without increasing the infrastructure cost. It means that the introduction of BRTs is a cost effective approach to reduce traffic congestion.

According to the above results, the introduction of BRT enables flexible operation of SAVs and efficient development of infrastructure, but tends to increase the travel time of travelers.

3.2.2 BRT Routes

Figure 7 (a) and (b) show BRT routes in a Pareto optimal state in two scenarios with the same weight values shown in Table 4; hereafter, we use these parameter values, and the weight value for D is set to demonstrate situations where BRT is intensively utilized by travelers. In Fig. 7 (a) and (b), the red links and the blue links represent the BRT

route, and the red nodes represent the destination nodes of travelers; especially the two red nodes in the center of the map corresponds to the area near the Omiya station, which attract many travelers.

According to the result, the BRT routes are determined to pass the central area regardless of the scenario. In addition, in the two routes scenario, both of the BRT routes go through the central area from the west. It means that the model automatically identifies where should be transfer point of the public transit. We considered this result reflects the actual road and travel patterns in the area.

Figure 7 (c) and (d) show BRT routes with the same weight values as Fig. 7 (a) and (b) except for α_D. In these two plots, α_D was set to 0.03, meaning that usage of SAVs is strongly encouraged. By comparing these results, we can confirm that the route length, the number of BRT vehicles, and passing nodes are significantly different. The length and number of BRT vehicles (roughly 20 vehicles per route in $\alpha_D = 30$ case, whereas roughly 50 vehicles per route in $\alpha_D = 0.03$ case) are smaller in $\alpha_D = 0.03$ cases due to increased usage of SAVs. This is an intuitive result. On the other hand, the difference of passing nodes is not intuitive. In fact, this difference came from the passenger transportation capabilities of both modes and road capacity. In $\alpha_D = 30$ case, BRT routes are placed at the central area of Omiya city where travel demand and road capacity are large. And by operating BRT and SAVs very frequently, it transports the large demand efficiently. In $\alpha_D = 0.03$ case, however, BRT is not frequently operated. Such low frequency operation would not be sufficient to transport the large demand in the central area and decreases traffic capacity for SAVs (because we assume that BRT route blocks one of the lanes). Thus, BRT route is placed to low demand areas.

In summary, selection of BRT routes is affected by the mechanism that decreases traffic capacity of a road. Thus, if operational frequency of BRT is low, BRT routes are not placed at high demand areas, because otherwise BRT blocks SAV traffic too significantly. Contrarily, if operational frequency of BRT is high, BRT routes are placed at high demand areas so that roads can be utilized more efficiently.

3.2.3 SAV Flow

In Fig. 8, the comparison of SAVs' flow between the DSO assignment with and without BRTs is shown. In this comparison, the DSO assignment with BRT means the result of $n = 2$. The flow of SAVs means the sum of the SAVs through all time on each link. In Fig. 8, the red nodes mean the destination nodes. In Fig. 8a and b, the thickness of links means the traffic volume of SAVs. In Fig. 8c, the links on which the sum of SAVs in the
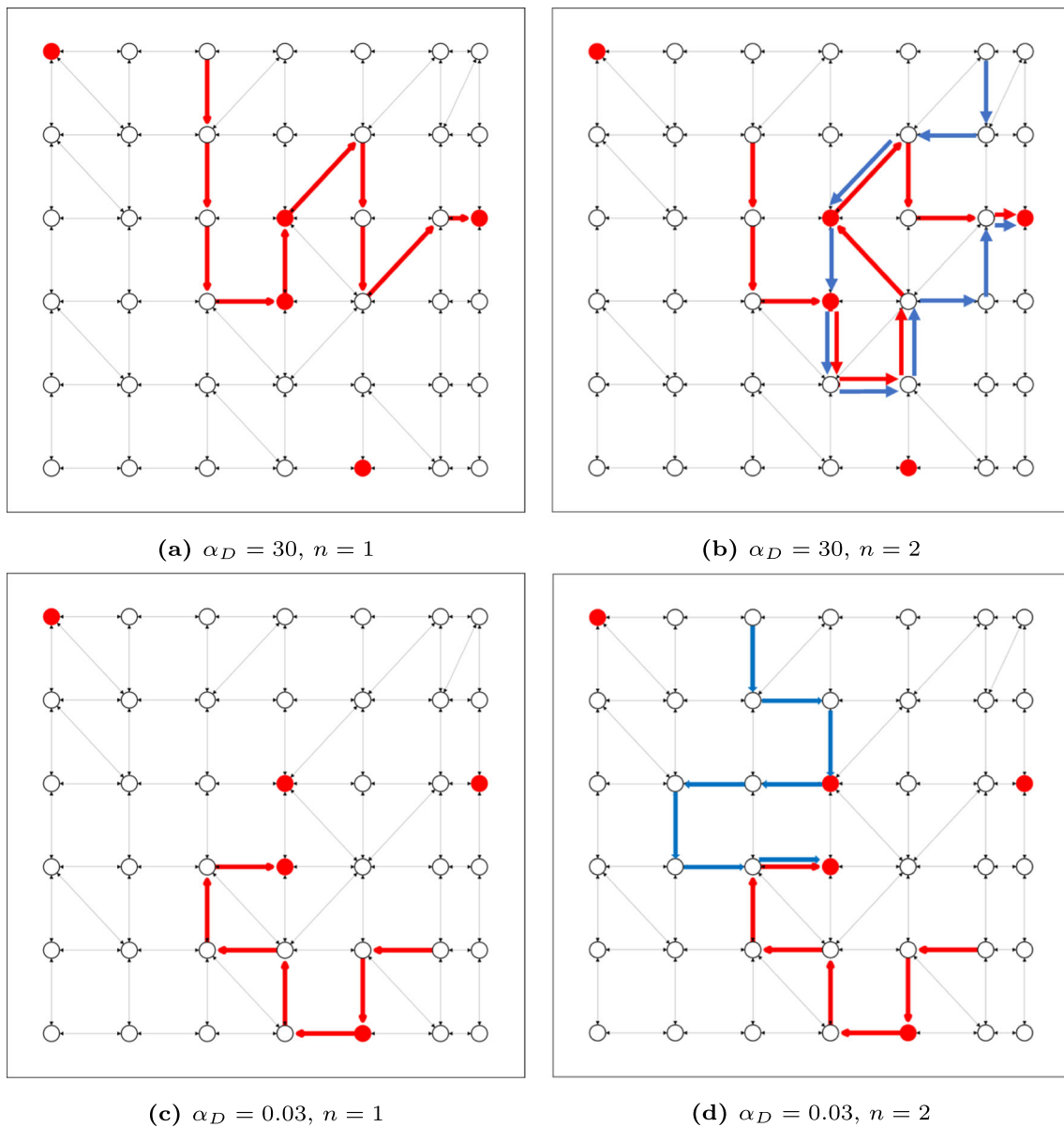
(a) $\alpha_D = 30$, $n = 1$

(b) $\alpha_D = 30$, $n = 2$

(c) $\alpha_D = 0.03$, $n = 1$

(d) $\alpha_D = 0.03$, $n = 2$

Fig. 7 The BRT routes

DSO assignment with BRTs is more than that in the DSO assignment without BRTs are red, the links on which the value in the DSO assignment with BRTs is less than that in the DSO assignment without BRTs are blue, and the links on which the values are the same are black. Then, the thickness of red and blue links means the absolute value, and the thickness of the black links is fixed. Thus, the thicker the blue link is, the more the number of SAVs in the DSO assignment without BRTs is.

The number of SAVs in the area where the destination nodes are gathered, or the center of the network, is large in Fig. 8a and b. In Fig. 8, the flow of SAVs is large in the central areas, and the inflow to the central area from the west is especially large. We considered these results reflect the actual road and travel patterns in the area, at least to some extent.

Figure 8c indicates that the number of SAVs in the DSO assignment with BRTs is decreased in most links. Thus, less congestion is expected by introducing BRTs. Comparing Figs. 8c and 7b, the amount of decrease is especially larger in the links determined as the BRT routes. In addition, there are some links that are not determined as BRT routes in which the number of SAVs is increased. That's because movements to and from the station nodes are increased. This result shows that the use of BRTs increases the number of SAVs in some links.

Fig. 8 The flow of SAV. A red node denotes destination, a red link in (c) means that the number of SAVs in $n = 2$ is larger than that in $n = 0$, a blue link in (c) means that the number of SAVs in $n = 2$ is smaller than that in $n = 0$, and a black link in (c) means that the number of SAVs in $n = 2$ is equal to that in $n = 0$


(a) $n = 0$

(b) $n = 2$

(c) Difference of SAV flow
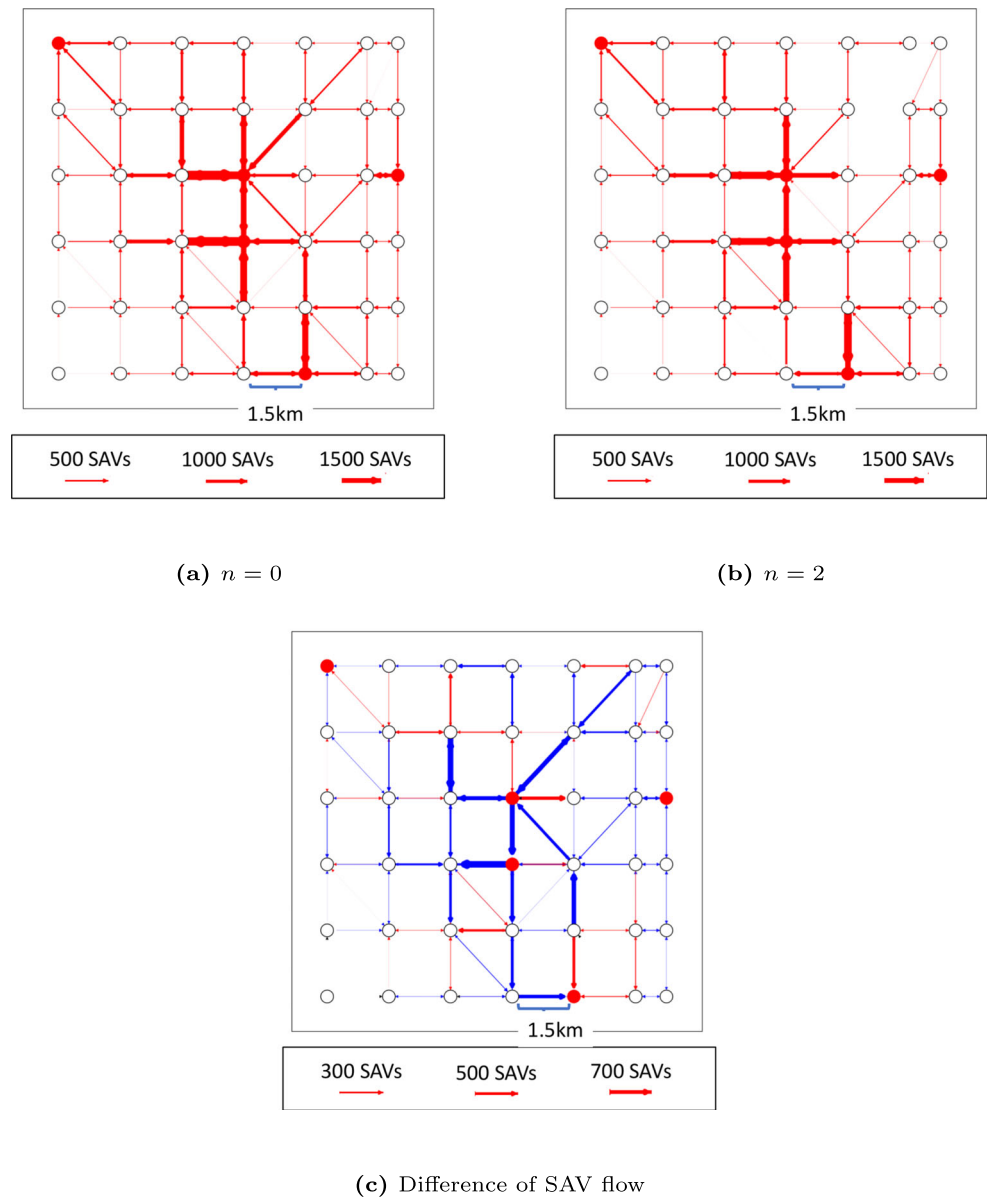
3.2.4 Traveler Flow

In Fig. 9, the comparison of travelers' flow between the DSO assignment with BRTs and the DSO assignment without BRTs is shown. In this comparison, the DSO assignment with BRT means that of $n = 2$. The flow of travelers means the sum of the travelers through all time on each link. The color of links and the thickness of links in Fig. 9c have the same meaning as Fig. 8c. Thus, in Fig. 9c, the thicker the blue link is, the more the number of travelers in the DSO assignment without BRTs is.

Similar to Fig. 8a and b, the number of travelers is large in the center of the network in which the destination nodes are gathered in Fig. 9a and b. We considered the actual road and travel patterns in the area are reflected to the results because the flow of travelers is large in the central areas, and the inflow to the central areas from the west is especially large. Figure 9c indicates that the number of travelers in most links is decreased a little by introducing BRTs, whereas that of travelers in a part of links, which are determined as BRT routes or connected with the station nodes, is considerably increased by introducing BRTs. This result shows that travelers' routing is gathered by introducing BRTs.

According to SAV and traveler flows in Figs. 8 and 9, the introduction of BRT causes traffic to be concentrated

Fig. 9 The flow of travelers. The red nodes denotes destination, a red link in (c) means that the number of SAVs in $n = 2$ is larger than that in $n = 0$, a blue link in (c) means that the number of SAVs in $n = 2$ is smaller than that in $n = 0$, and a black link in (c): the number of SAVs in $n = 2$ is equal to that in $n = 0$.

(a) $n = 0$

(b) $n = 2$

(c) Difference of travelers flow
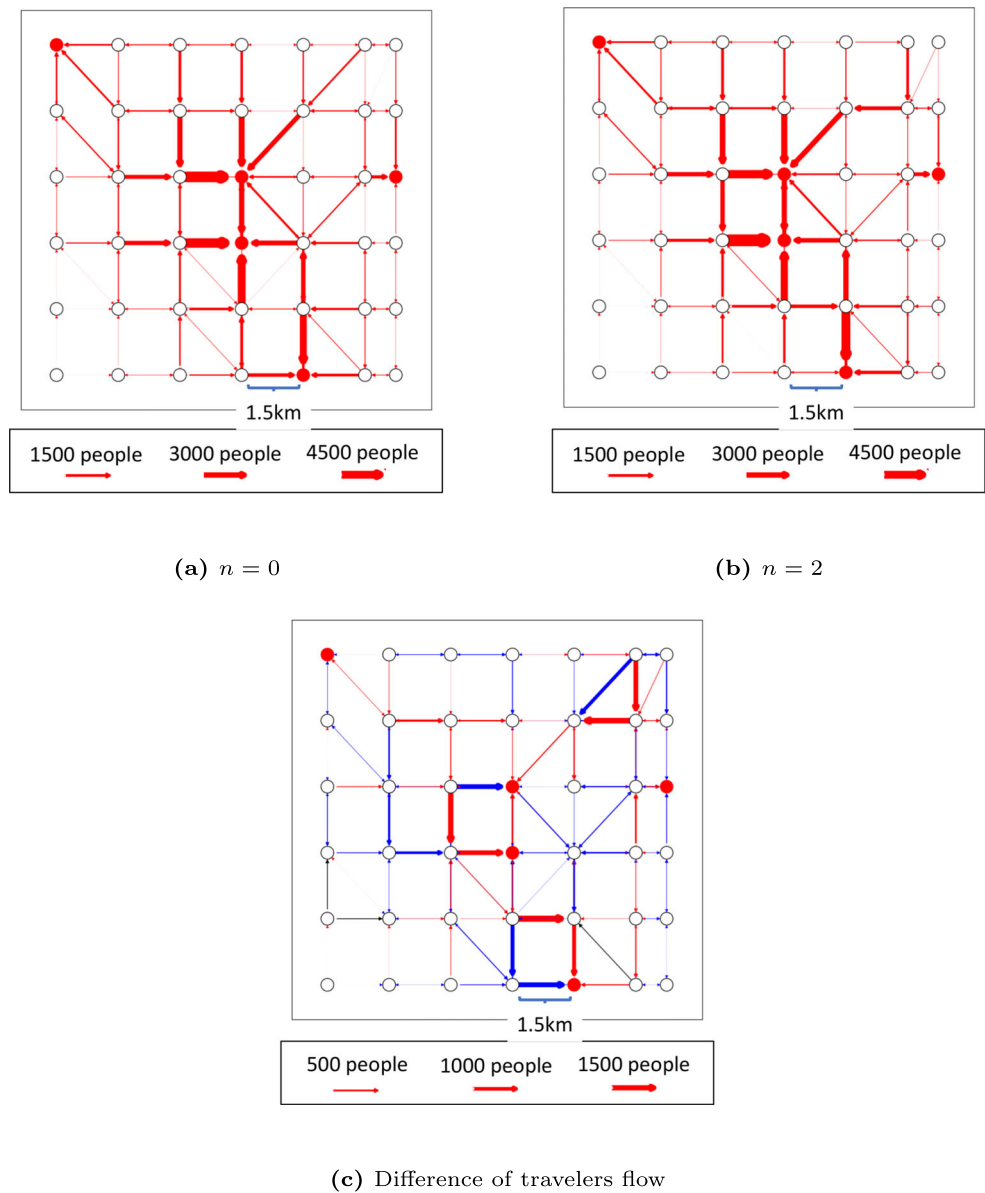
at BRT stations. It makes transfer at stations and rideshare matching easier. In addition, only limited links will be used intensively, making infrastructure investment efficient.

3.2.5 Average Speed and Cumulative Curves of Travelers

In order to investigate the dynamical features of the proposed model, time-dependent average speed and cumulative curves of travelers are derived. Figure 10 shows the average speed of travelers at each time step. The horizontal axis represents time, and the vertical axis represents the average speed of travelers.

Because the speed of BRTs is slower than that of SAVs, the average speed in the network with more BRT routes is basically smaller. However, the minimum speed is larger in

the network with more BRT routes; this is because the no BRT case ($n = 0$) was suffered by traffic congestion caused by SAVs. Thus, SAV-BRT systems would be slower than SAV systems when the traffic congestion was not severe, but it would be faster when severe congestion was possible.

According to the above results, the introduction of BRT stabilizes travelers' speed, possibly due to the mitigation of traffic congestion.

4 Conclusion

An optimization model for the SAV-BRT system is proposed. It is aimed to utilize unique advantages and disadvantages of SAVs and BRTs in a complementing
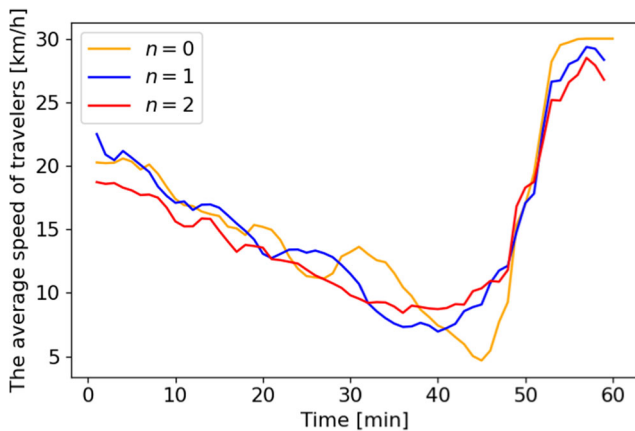
Fig. 10 Comparison of the average speed of travelers

manner: SAVs are fast and flexible, but the passenger capacity is small; BRTs are slow and fixed-route, but the passenger capacity is large. The model is based on the dynamic system optimal assignment method, so that important dynamical features of transportation systems are captured. Furthermore, the model is formulated as a multi-objective optimization problem, in order to investigate the trade-off relations in SAV-BRT systems. This is one of the unique features of the proposed model and could be important for policy planners is that the model can output diverse, various optimal solutions based on the multi-objective optimization. The model outputs the Pareto frontier, in which various states with various priorities to objective values are included. Policy planners would able to choose the most suitable state from the Pareto front depending on the society's future decision making.

Numerical experiments based on actual travel demand data in Japan are conducted to investigate the quantitative behavior of the proposed model. According to the results, the model behaves reasonably. Some specific findings are as follows. First, introduction of BRT enables efficient transportation systems in terms of traffic congestion, but it slightly increases average travel time of travelers. Second, the number of SAVs can be significantly decreased by introducing 1 or 2 BRT routes (e.g., decreased from 5700 vehs to 3800 vehs), and operational flexibility of the SAV system also be increased by introducing BRTs. Third, in terms of infrastructure construction, the introduction of BRTs is a cost effective approach to reduce traffic congestion. The proposed model and these experimental findings would be useful to plan SAV-BRT systems in the real world. However, the experimental findings depend on the parameter setting. We have not investigated the sensitivity to the parameter setting, but it will be important

for planning purposes and can be investigated by using the model in the future.

The proposed model has some other future works. Firstly, we cannot consider realistic situations, such as cities with private vehicles. This model considers an ideal situation where all vehicles are controlled to achieve the system optimal. If private vehicles exist, traffic congestion will be intensified, and BRT will be more advantageous. Secondly, the development of efficient algorithms for the proposed model is important as mentioned. This can be done by applying the technique called Dantzig–Wolfe decomposition [23]. Thirdly, this model considers SO only and may not be realistic. The comparison between UE-type models would be very interesting. In fact, the authors are working on such analyses (i.e., SAV system in which travelers' route choice is determined by the dynamic user optimal principle) and have obtained a preliminary result [11].

Declarations

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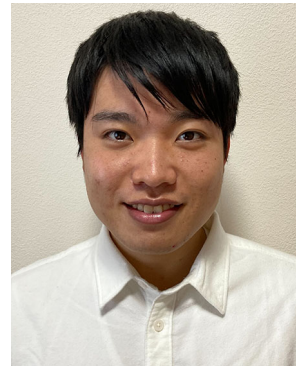
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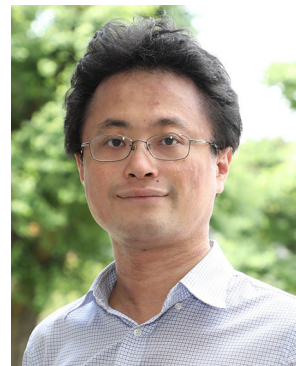
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