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## 論文／著書情報 <br> Article／Book Information

| 題目（和文） | AdSCFT対応による $N=4 U(N)$ 超対称ヤンミルズ理論のシューア指数 |
| :---: | :---: |
| Title（English） | Schur index of the $N=4 U(N)$ supersymmetric Yang－Mills theory via the AdS／CFT correspondence |
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| 出典（和文） | 学位：博士（理学） <br> 学位授与機関：東京工業大学， <br> 報告番号：甲第12306号， <br> 授与年月日：2023年3月26日， <br> 学位の種別：課程博士， <br> 審査員：今村洋介，伊藤 克司，慈道 大介，須山 輝明，関澤 一之 |
| Citation（English） | Degree：Doctor（Science）， <br> Conferring organization：Tokyo Institute of Technology， <br> Report number：甲第12306号， <br> Conferred date：2023／$/ 26$ ， <br> Degree Type：Course doctor， <br> Examiner：，，，＂， |
| 学位種別（和文） | 博士論文 |
| Category（English） | Doctoral Thesis |
| 種別（和文） | 論文要旨 |
| Type（English） | Summary |

（博士課程）

# 論 文 要 旨 

THESIS SUMMARY

| 系・コース： <br> Department of，Graduate majpr in | 物理学物理学 | $\begin{gathered} \text { 系 } \\ \text { コース } \end{gathered}$ |
| :---: | :---: | :---: |
| 学生氏名： <br> Student＇s Name | 森 達也 |  |



## 要旨（英文 800 語程度）

Thesis Summary（approx． 800 English Words ）
In the thesis，we propose a new formula for the Schur index of the $\mathcal{N}=4 \mathrm{U}(N)$ supersymmetric Yang－Mills theory（SYM）via the AdS／CFT correspondence．

The AdS／CFT correspondence is a conjectural relation between a superstring theory in the anti－de sitter （AdS）space and a conformal field theory（CFT）．The simplest example which is discussed in this thesis is the correspondence between the Type IIB string theory in $A d S_{5} \times S^{5}$ and the four－dimensional $\mathcal{N}=4 \mathrm{U}(N)$
SYM．In the large $-N$ limit，the Type IIB string theory can be described by a classical supergravity，while the $\mathcal{N}=4 \mathrm{SYM}$ is in a strongly coupled region．Then，we can analyze quantities of the $\mathcal{N}=4 \mathrm{SYM}$ in the strongly coupled region via the corresponding classical supergravity．The AdS／CFT correspondence in the large $-N$ limit is useful to study strongly coupled CFTs．

How to tackle the AdS／CFT correspondence in the finite－$N$ region is also an important question because the rank of CFT that we are interested in is usually not large $N$ but finite $N$ ．If $N$ is finite，quantum gravity corrections are not negligible in the Type IIB string theory，and this is a difficulty in the finite $-N$ AdS／CFT correspondence．Fortunately，in recent years，there has been progress in the study of the finite $-N$ AdS／CFT correspondence．We can study the finite－$N$ AdS／CFT correspondence by using quantities that are protected from quantum gravity corrections．One of such quantities is the superconformal index， which is a kind of the supersymmetric partition function．The superconformal index can be calculated in Lagrangian gauge theories for the arbitrary rank $N$ and arbitrary coupling constant by using the localization method．The agreement of the index calculated on the gravity（AdS）side and that on the gauge theory（CFT） side has been confirmed at large $N$ in different AdS／CFT examples．For example，the large $-N$ index of $\mathcal{N}$ $=4 \mathrm{U}(N)$ SYM is the same as the index of the contribution from the Kaluza－Klein modes in $A d S_{5} \times S^{5}$ ．

The index on the gauge theory side can be calculated in principle as long as the Lagrangian is known， while contributions to the index on the gravity side are non－trivial．It was found by Arai and Imamura that on the gravity side not only the Kaluza－Klein modes in $A d S_{5} \times S^{5}$ but also giant gravitons，which are objects wrapped around three $-c y c l e s ~ i n ~ S^{5}$ ，contribute to the index．The contribution from giant gravitons is expected to be expressed as the sum over wrapping number $m=1,2$ ，．．．In previous works with the author and the collaborators only $m=1$ contribution was taken into account．To obtain the complete index we need to include $m \geq 2$ contributions．To calculate $m \geq 2$ contributions we need to carry out certain contour integrals，and it has not yet been well understood how we should choose contours in the integrals．

The difficulty of the choice of the integration contours is caused by an unual pole structure of the integrand．In particular，the existence of intersection strings makes the problem complicated．An intersection string is an open string appearing in the system of multiple－wrapping giant gravitons and stretches between two giant gravitons wrapped on different cycles．

In the thesis，we show that the problem of the intersection string can be avoided by taking the Schur limit，which is a specialization limit of the superconformal index．In the Schur limit，we find that the intersection string contribution in the integrand becomes a simple form and can be factor out of the integrals．Thanks to this，the multiple integrals factorize into integrals each of which is associated with coincident giant gravitons wrapped on a single cycle．This factorization makes the problem much simpler．

We also show a prescription for the choice of the integration contours．By using the prescription， we can calculate the contributions of the multiple－wrapping giant gravitons up to an arbitrary wrapping number in principle．As a consistency check，we confirm that the formula reproduces the correct index on the gauge theory side in the small $N$ cases．

Although we use the Schur index to simplify the problem，it is desirable to use the superconformal index because the superconformal index has more information rather than the Schur index．Even so，the Schur index itself has been attracted and great interesting，and plays an important role in the analysis
of superconformal field theories whose Lagrangian has not been known．We leave the analysis using such properties of the Schur index for future works．

備考：論文要旨は，和文 2000 字と英文 300 語を 1 部ずつ提出するか，もしくは英文 800 語を 1 部提出してください。
Note ：Thesis Summary should be submitted in either a copy of 2000 Japanese Characters and 300 Words（English）or 1 copy of 800 Words（English）．

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