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Prediction Formula of Superstructure for Isolated building Based on 2DOF

Part 2: Prediction formula of Superstructure

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MDOF Model, 2DOF Model, Isolated building,
Superstructure, Mass ratio, Prediction Formula

1. Introduction

In the part 1, the periods of two-degree-of-freedom (2DOF) and multi-degree-of-freedom (MDOF) models were mostly same for general isolated building (period ratio $T_0 / T_U \geq 2$ s) was verified. And the equivalent height ratio λ_{eq} (H_{eq}/H) = 0.5 was confirmed. In the part 2, the prediction formula based on 2DOF model is proposed. And the application range of Fu's [1] prediction equation is confirmed.

2. Analysis Model and Ground Motions

2.1. Analysis Model

Fig. 1 shows the analysis models of MDOF model for the base-isolated building, where the superstructure has N lump-masses m_i (i =1~N), and the isolation layer has a lump-mass m_0. k_i shows stiffness of i story, k_S and k_I are initial stiffness of steel damper and stiffness of isolator, respectively. c_V is viscous coefficient of oil damper. The bule circle is equivalent lump-mass of superstructure for 2DOF. As mentioned in chapter 4 of part 1, the equivalent height ratio λ_{eq} = 0.5. Therefore, the lump-mass m_U for 2DOF model position at half of height for MDOF model.

Fig. 2 shows restoring force characteristic of isolation layer. isolation layer is consisted by (a) natural-rubber isolator (elastic element), (b) displacement-dependent steel damper (elasto-plastic element) and (c) velocity-dependent oil damper (viscous element). Q is shear force. Subscript $_I$, $_S$, and $_V$ show isolator, steel damper, and oil damper, respectively.

Eq. 1-3 show the stiffness of isolator, steel damper, superstructure, and initial isolation layer, respectively. Eq. 4 shows the initial period of isolation layer.

$$k_I = \frac{4\pi^2}{T_I^2}\Sigma m \ , \quad k_S = \frac{\Sigma m \cdot g \cdot \alpha_S}{\delta_{S,y}} \ , \quad k_U = \frac{4\pi^2}{T_U^2} m_U \qquad (1, 2)$$

$$k_0 = k_I + k_S \ , \quad T_0 = 2\pi\sqrt{\frac{\Sigma m}{k_0}} \qquad (3, 4)$$

Where, Σm is total mass of building, $\Sigma m = m_U + m_0$. T_I and T_U are periods of isolator and superstructure. g is gravitational acceleration. And $\delta_{S,y}$ is the yield deformation of steel damper.

Eq. 5 and Eq.6 show the matrix of mass and stiffness for equivalent 2DOF model.

$$[M] = \begin{bmatrix} m_0 & 0 \\ 0 & m_U \end{bmatrix}, \quad [K] = \begin{bmatrix} k_0 + k_U & -k_U \\ -k_U & k_U \end{bmatrix} \qquad (5, 6)$$

2.2. Ground Motions

Two artificially ground motions are adopted in this paper. Earthquake recording at 1995 JMA Kobe (NS component) and Hachinohe Port from 1968 Tokachi-oki (EW component) are used as phase characteristics. They are named ART-HACHI and ART-KOBE, respectively.

Fig. 3 shows pseudo-velocity spectrum of ground motions. The mean value of pseudo velocity $_pS_v$ (ξ = 5%) is constant at 80 cm/s when period is longer corner period T_C = 0.64 s for artificial ground motions. The black line shows ART-KOBE, and the red line shows ART-HACHI.

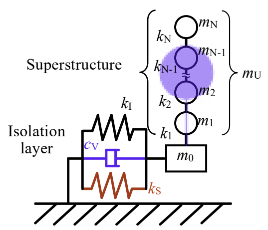
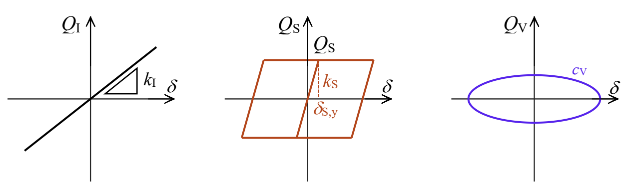


Fig. 1 Analysis Model



(a) Isolator　(b) Steel damper　(c) Oil damper

Fig. 2 Restoring Force Characteristic of Isolation Layer

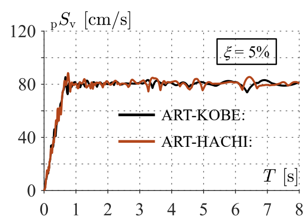


Fig. 3 Pseudo-velocity Spectrum

3. Prediction Formula

As mentioned in Chapter 3 of part 1, the 1st mode shape of 2DOF model could be used to predict 1st mode of MDOF model. And the 1st mode shape of 2DOF model could be derived as the formula. Therefore, a prediction equation of deformation for superstructure base on eigen analysis for 2DOF model is proposed.

3.1. Prediction Formula by 2DOF

The following equation Eq.7 is non-damping motion equation of 2DOF model. [3]

$$[M]\{\dot{x}\}+[K]\{x\}=-[M]\{1\}\ddot{x}_g \quad (7)$$

Where, x, $\dot{x}$ and $\ddot{x}_g$ are displacement, velocity, and acceleration of ground motions, respectively.

Then, Eq. 8 could be obtained by eigen equation.

$$\left|[K]-\omega^2[M]\right|\{\phi_{2\mathrm{DOF}}\}=0 \quad (8)$$

Where, $\{\phi_{2\mathrm{DOF}}\}$ is eigen vector.

Substituting Eq. 5&6 into Eq. 8, the equation is as follows:

$$\begin{vmatrix} k_0+k_U-\omega^2 m_0 & -k_U \\ -k_U & k_U-\omega^2 m_U \end{vmatrix}=0 \quad (9)$$

Furthermore, the 1st circle frequency ${}_1\omega$ is the smaller solution of Eq.9, and is expressed by following equation.

$${}_1\omega^2=\frac{\left(1+\frac{m_U}{m_0}\right)\left(\omega_0^2+\omega_U^2\right)-\sqrt{1+\frac{m_U}{m_0}}\sqrt{\left(\omega_0^2-\omega_U^2\right)^2+\frac{m_U}{m_0}\left(\omega_0^2+\omega_U^2\right)^2}}{2} \quad (10)$$

For 1st mode of eigen vector $\{{}_1\phi_{2\mathrm{DOF}}\}$, substituting Eq. 5, Eq. 6 into Eq. 8, and setting 1st eigen value of isolation layer ${}_1\phi_0$ = 1, the equation is as follows:

$$[K]\{{}_1\phi_{2\mathrm{DOF}}\}={}_1\omega^2[M]\{{}_1\phi_{2\mathrm{DOF}}\}$$

$$\begin{bmatrix} k_0+k_U & -k_U \\ -k_U & k_U \end{bmatrix}\begin{Bmatrix} 1 \\ {}_1\phi_{U,2\mathrm{DOF}} \end{Bmatrix}={}_1\omega^2\begin{bmatrix} m_0 & 0 \\ 0 & m_U \end{bmatrix}\begin{Bmatrix} 1 \\ {}_1\phi_{U,2\mathrm{DOF}} \end{Bmatrix} \quad (11)$$

The 1st eigen value of superstructure ${}_1\phi_{U,2\mathrm{DOF}}$ is the solution of Eq. 11 after substituting Eq. 10 into Eq. 11. And the 1st eigen vector $\{{}_1\phi_{2\mathrm{DOF}}\}$ is as follows:

$$\{{}_1\phi_{2\mathrm{DOF}}\}=\begin{Bmatrix} 1 \\ \dfrac{2}{2-\left(1+\frac{m_U}{m_0}\right)\left(\frac{T_U^2}{T_0^2}+1\right)+\sqrt{1+\frac{m_U}{m_0}}\sqrt{\left(\frac{T_U^2}{T_0^2}-1\right)^2+\frac{m_U}{m_0}\left(\frac{T_U^2}{T_0^2}+1\right)^2}} \end{Bmatrix} \quad (12)$$

For 2DOF model, assume only 1st mode is considered, meanwhile, the deformation ratio between superstructure and isolation layer $\delta_{U,2\mathrm{DOF}}/\delta_0$ is equal to 1st shape ratio between superstructure and isolation layer $({}_1\phi_{U,2\mathrm{DOF}} - {}_1\phi_0)/{}_1\phi_0$.

Fig. 4 shows the equivalent deformation based on analysis model. Due to the equivalent height ratio λ_{eq} = 0.5, the equivalent deformation of superstructure $\delta_{U,eq}$ is difference between maximum displacement of middle floor for superstructure x_M (MDOF) and isolation layer δ_0. And the equation is calculated by Eq. (13). The middle floor position at half of height for MDOF model.

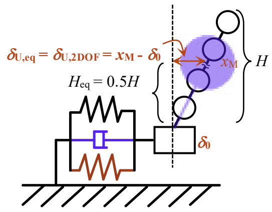


Fig. 4 Equivalent defo. based on analysis model

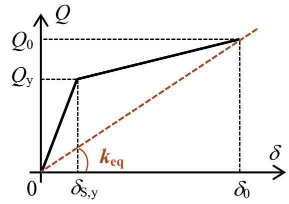


Fig. 5 Equivalent Stiffness of Isolation Layer

$$\delta_{U,eq}=\delta_{U,2\mathrm{DOF}}=x_M-\delta_0 \quad (13)$$

Fig. 5 shows the equivalent stiffness of isolation layer. Q_y and Q_0 are yield shear force and maximum elasto-plastic shear force of the isolation layer. The equivalent period T_{eq} is the period based on the equivalent stiffness k_{eq} when the deformation of isolation layer reaches the maximum δ_0. And T_{eq}, k_{eq} are as follows:

$$T_{eq}=2\pi\sqrt{\frac{\Sigma m}{k_{eq}}}\ ,\quad k_{eq}=k_I+k_S\frac{\delta_{S,y}}{\delta_0} \quad (14,\ 15)$$

For isolation layer of bilinear system, period of isolation layer T_0 is replaced as equivalent period of isolation layer T_{eq}. In summary, the prediction equation based on 2DOF model is expressed by the following equation.

$$\frac{\delta_{U,eq}}{\delta_0}=\frac{2}{2-\left(1+\frac{m_U}{m_0}\right)\left(\frac{T_U^2}{T_{eq}^2}+1\right)+\sqrt{1+\frac{m_U}{m_0}}\sqrt{\left(\frac{T_U^2}{T_{eq}^2}-1\right)^2+\frac{m_U}{m_0}\left(\frac{T_U^2}{T_{eq}^2}+1\right)^2}}-1 \quad (16)$$

3.2. Prediction Formula proposed by Fu et. al [1]

Fu et al. [1] proposed the prediction equation, which considered the equivalent period ratio between the isolation layer and the superstructure T_{eq}/T_U to predict the deformation of the superstructure. And Fu's equation is shown as Eq. (17) . The definition of notation is same with Eq.16.

$$\frac{\delta_{U,eq}}{\delta_0}=\left(\frac{T_{eq}}{T_U}\right)^{-2} \quad (17)$$

3.3. Comparation of Prediction Formulas

Compared with Eq.17, Eq.16 also considers the effect of mass ratio. However, Eq.16 is complex while Eq.17 is intuitive. Therefore, the application range of Eq.17 is confirmed in this paper by comparing with Eq.16.

Fig. 6 shows prediction formulas. The horizontal axis is the ratio between equivalent period of isolation layer and period of superstructure T_{eq}/T_U. And the vertical axis is the equivalent deformation ratio between superstructure and isolation layer

$\delta_{U,eq}/\delta_0$. For legend, various colorful dotted lines show various mass ratio m_U/m_0 based on Eq.16. With mass ratio m_U/m_0 increases, the gradient color become from red to blue. And black solid line is values based on Eq. 17.

According to Fig. 6, for proposed prediction equation in this paper (Eq.16), when period ratio T_{eq}/T_U is same, as the mass ratio m_U/m_0 increases, the displacement ratio $\delta_{U,eq}/\delta_0$ decrease. Furthermore, when mass ratio m_U/m_0 large enough ($m_U/m_0 \geq 2$), the prediction curves are apparently same. Besides, comparing with Eq. 16 and Eq.17, when period ratio T_{eq}/T_U large enough ($T_{eq}/T_U \geq 2$), Eq. 16 and Eq.17 are apparently same, although mass ratio is different

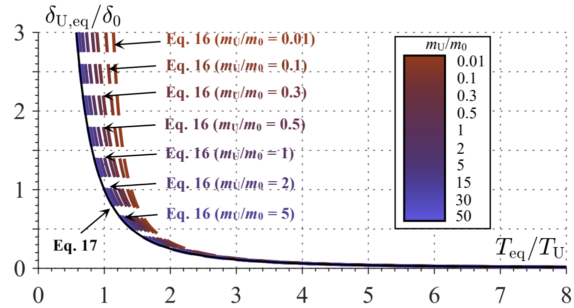


Fig. 6 Prediction Formulas

Fig. 7 shows equivalent displacement ratio of prediction formulas. The horizontal axis is the ratio between equivalent period of isolation layer and period of superstructure T_{eq}/T_U. And the vertical axis is the equivalent displacement ratio between Eq. 17 and Eq. 16 $\delta_{U,eq}$(Eq.17)/$\delta_{U,eq}$(Eq.16). For legend, various colorful solid lines show various mass ratio m_U/m_0, and dotted line is results for mass ratio $m_U/m_0 = 2$.

According to Fig. 7, with the mass ratio m_U/m_0 increases, equivalent displacement ratio $\delta_{U,eq}$(Eq.17)/$\delta_{U,eq}$(Eq.16) trends to 1. Besides, for the cases period ratio $T_{eq}/T_U \geq 2$ (general base-isolated building) [3], [4], and mass ratio m_U/m_0 large enough ($m_U/m_0 \geq 2$), the different between Eq.17 and Eq. 16 is less than 6 %. In the other word, for general base-isolated building, when mass ratio $m_U/m_0 \geq 2$, Fu et al. [1] proposed equation (Eq.17) is in high precision. However, for special base-isolated building, when mass of isolation layer m_0 is large enough ($m_0 > 0.5m_U$), Eq.17 is not safety enough. And Eq. 16 should be adopted.

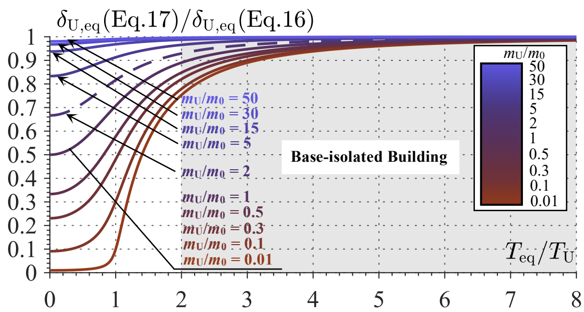


Fig. 7 Equivalent deformation Ratio

4. Verification of Prediction Formula

Various parameters are adopted to verify the prediction equations (Eq.16& Eq.17) by time-history-analysis (THA) results in this chapter. Table. 1 shows the parameters of the analysis model. Three types of the mass distribution for the superstructure (case A-C, shown in Fig.2 of Part 1) are adopted in this chapter.

Table.1 Parameters of Analysis Model

Items [Units]	Values
Period of isolator: T_I [s]	3, 4, 5, 6
Period of superstructure: T_U [s]	0.5, 1.0, 1.5, 2.0
Mass of superstructure: m_U [ton]	1000 (10 stories), 2000 (20 stories)
Mass of isolation layer: m_0 [ton]	170
Shear coefficient of steel damper: α_S	0.01 - 0.06 (@0.01)
yield deformation of steel damper: $\delta_{S,y}$ [cm]	3
Damping ratio of superstructure ξ_U	2%
Damping ratio of oil damper ξ_V	0%, 2%, 4% 10%, 20%, and 30%

Fig. 8 & 9 show the relationship between the period ratio T_{eq}/T_U and displacement ratio $\delta_{U,eq}/\delta_0$. Fig. 8 shows the THA results of ART-KOBE, and Fig. 9 shows the THA results of ART-HACHI. (a) shows the results of the 10 stories superstructure and (b) shows the results of the 20 stories superstructure. (1)-(3) are various mass distribution of superstructure (case A-C). The horizontal axis is the ratio between the equivalent period of the isolation layer and the period of the superstructure T_{eq}/T_U. And the vertical axis is the ratio between the equivalent displacement of the superstructure and the displacement of isolation layer $\delta_{U,eq}/\delta_0$. For legend, various icons show the damping ratio of oil dampers from 0%-30%. Blue and red solid lines show Eq. 17 and Eq.16, respectively.

According to Fig. 8 & 9, the THA results match the prediction curves very well not only for different ground motions but also for various stories of the superstructure (10 and 20 stories). Also, the THA results match the prediction curves for various mass distribution of superstructure (case A-C). Although the damping ratio of oil damper is different.

5. Conclusions

In the part 2, the prediction formula based on 2DOF model was proposed. And the application range of Fu's [1] prediction equation was confirmed. The conclusion can be summarized as follows:

(1) For general base-isolated building ($T_{eq}/T_U \geq 2$), when mass ratio $m_U/m_0 \geq 2$, Fu et al. [1] proposed equation (Eq.17) was in high precision. However, when mass of isolation layer m_0 is large enough ($m_0 > 0.5m_U$), proposed equation (Eq.16) in

this paper should be adopted.

(2) According to the THA results, the THA results matched the prediction curves very well not only for different ground motions (ART-KOBE & ART-HACHI) but also for various stories of the superstructure (10 and 20 stories). Also, the THA results matched the prediction curves for various mass distribution of the superstructure (case A-C). Although the damping ratio of the oil damper was different.

Reference:

[1] Fu HX., Sato D., Kitamura H., Matsuda Y., Miyagawa K., Ueki T., Murakami Y.: Response Prediction Formula for Base-Isolated Building by Using Period Ratio Between Superstructure and Seismic Isolation Layer, *AIJ Journal of Technology and Design*, No.17(5), pp.951-956, 2018, 02. (In Japanese)

[2] Li J., Sato D.: Effect of Viscous Damping of Isolation Layer on Period Ratio Between Superstructure and Seismic Isolation Layer, *AIJ Summaries of technical papers of annual meeting*, 2021.09.

[3] AIJ: Recommendation for the Design of Seismically Isolated Buildings, pp.131-132,2013, 10.

[4] Miyuki O., Kitamura H.: Investigation and Analysis on the Structural Characteristics of the Recent Base Isolated Buildings Corresponding to Long-Period Ground Motion, *AIJ Journal of Technology and Design*, No.59(25), pp.61-66, 2019, 02. (In Japanese)

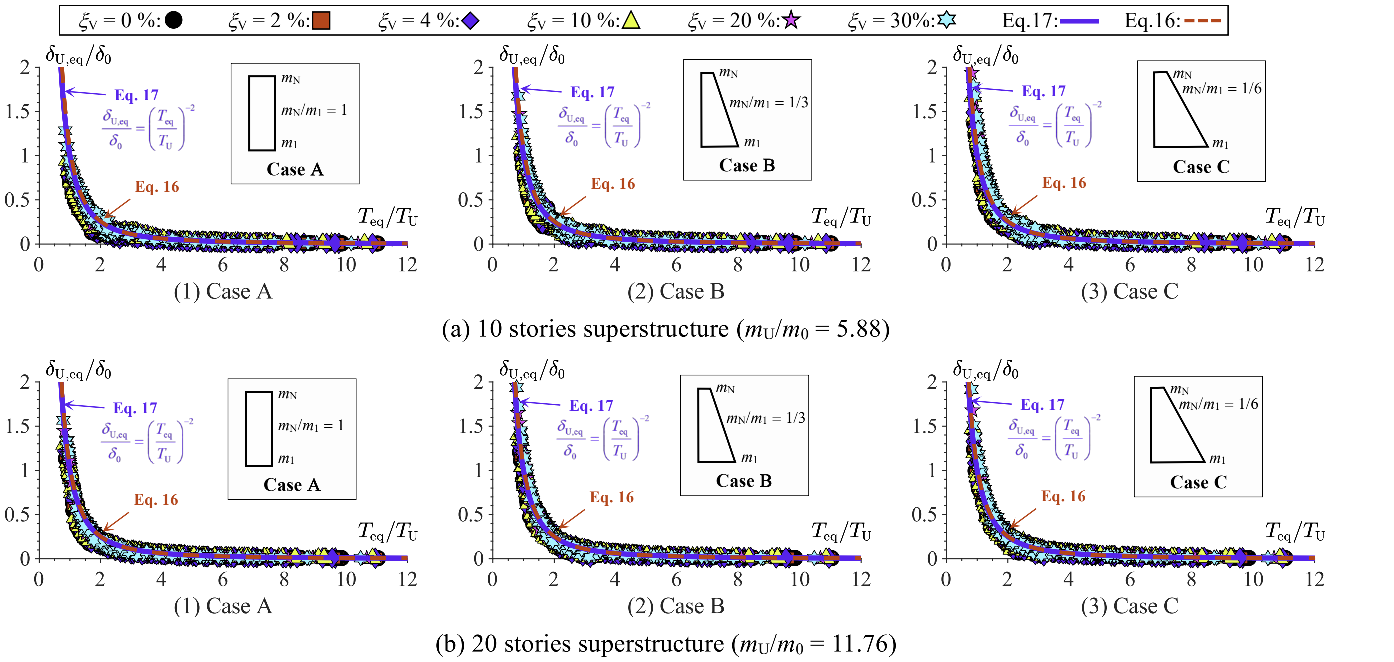


Fig. 8 The relationship between T_{eq} / T_U and $\delta_{U,eq}$ / δ_0 (ART-KOBE $_sP_v$ = 80 cm/s)

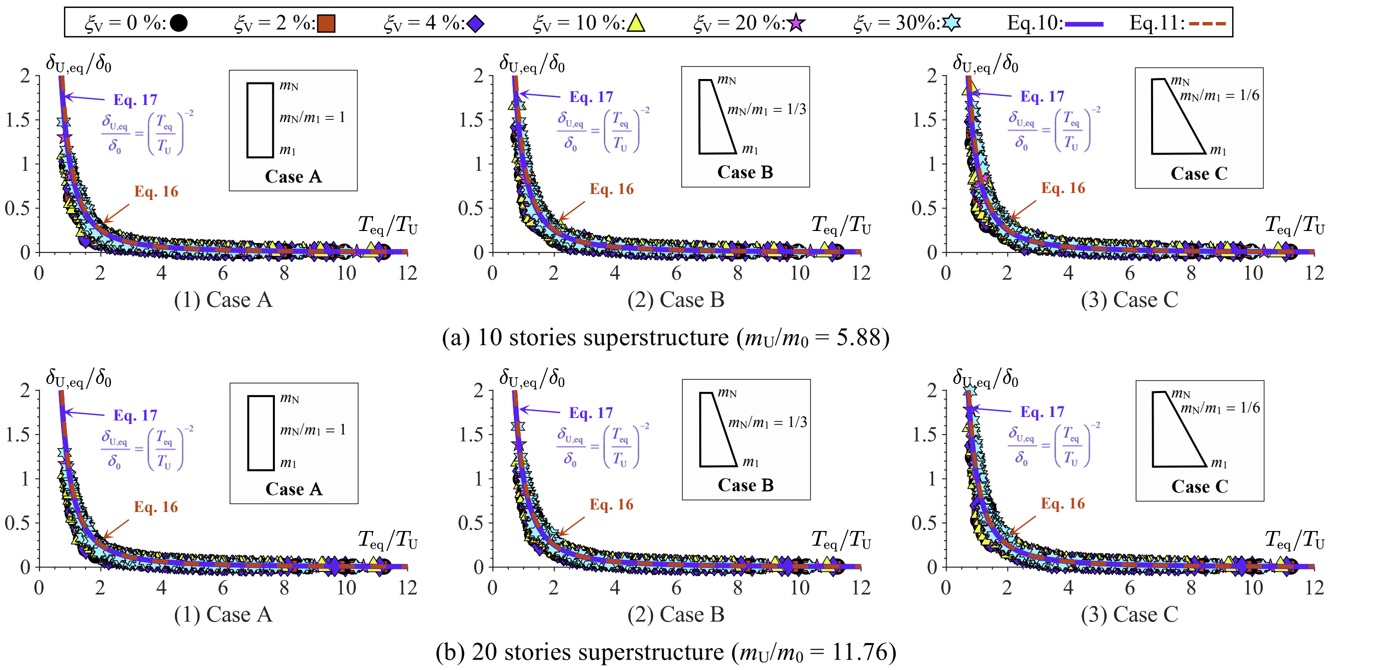


Fig. 9 The relationship between T_{eq} / T_U and $\delta_{U,eq}$ / δ_0 (ART-HACHI $_sP_v$ = 80 cm/s)

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