

論文 / 著書情報  
Article / Book Information

論題(和文)	
Title(English)	Effect of Structural Parameter Errors on Wind Force Estimation by Modal Analysis
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出典 / Citation	日本建築学会関東支部研究報告集, , , pp. 445-448
Citation(English)	, , , pp. 445-448
発行日 / Pub. date	2023, 2
権利情報	一般社団法人 日本建築学会

## Effect of Structural Parameter Errors on Wind Force Estimation by Modal Analysis

構造—振動

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Stiffness Error, Damping Error, Response identification

## 1. INTRODUCTION

## 1.1 Background

Modal analysis is one method that can be used to estimate wind forces acting upon a structure using the structural parameters (mass, stiffness, and damping) and known or recorded responses (acceleration, velocity, and displacement) [1]. Soriano et. al calculated the wind forces on a tall building using incomplete acceleration responses while also considering errors in the damping ratio estimates of the building model [2]. However, since modal analysis greatly depends on the structural parameters of the building, namely, mass, stiffness, and damping, it is also important to consider not only the sensitivity of the wind force estimation method on the errors on damping ratio estimates but also the errors that can be incurred from the estimation of the mass and the stiffness of the building. These parameters are also important but difficult to estimate. Also, errors obtained in estimating these structural parameters can occur simultaneously and understanding the effects of errors on these structural parameters when one or two parameters are erroneous is useful in determining how applicable modal analysis is in estimating wind forces even when errors in the structural parameters are present.

In lieu of the above, this paper estimates the wind forces on a numerical model by modal analysis by considering different cases wherein errors in the structural parameters are introduced. The accuracy of the response obtained from the estimated wind forces are also investigated.

## 1.2. Research objective

This study aims to investigate the effects of structural parameter errors on the estimation of wind forces by modal analysis. Also, in order to check the applicability of the estimated wind forces from modal analysis for response identification, the accuracy of the acceleration response from the wind forces estimated using the structural parameters with errors are also investigated.

## 2. THEORETICAL BACKGROUND

The equation of motion for a multi-degree of freedom system subjected to external dynamic forces  $\{F_e(t)\}$  is

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F_e(t)\} \quad (1)$$

where  $\{\ddot{x}(t)\}$ ,  $\{\dot{x}(t)\}$  and  $\{x(t)\}$  are the acceleration, velocity, and displacement vectors, respectively. Note that these are the dynamic responses of the structure. Also,  $[M]$ ,  $[C]$  and  $[K]$  are the structural parameter matrices, namely, mass, damping and stiffness matrices, respectively. These can be calculated using the following equations:

$$[M] = \begin{bmatrix} m_1 & 0 & \cdots & 0 \\ 0 & m_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_N \end{bmatrix} \quad (2)$$

$$k_i = \frac{{}_1\omega \cdot m_i \cdot {}_1\phi_i + k_{i+1}({}_1\phi_{i+1} - {}_1\phi_i)}{{}_1\phi_i - {}_1\phi_{i-1}} \quad (3a)$$

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 & \cdots & 0 \\ -k_2 & k_2 + k_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & k_N \end{bmatrix} \quad (3b)$$

$$\beta = \frac{2 \cdot {}_1\zeta}{{}_1\omega} \quad (4a)$$

$$[C] = \beta[K] \quad (4b)$$

where  $m_i$  and  $k_i$  is the mass and stiffness of each story, respectively, and  $i$  corresponds to the story level. The stiffness of each story,  $k_i$  is calculated using Eq. (3a) with the assumption that the first mode shape is linear where  ${}_1\phi_i$  is the 1<sup>st</sup> mode shape on the  $i^{th}$  story. The damping matrix is calculated using the stiffness matrix,  $[K]$  and the stiffness-proportional damping coefficient,  $\beta$  which is calculated from the damping ratio,  ${}_1\zeta$  and natural circular frequency,  ${}_1\omega$  of the 1<sup>st</sup> mode given by Equation (4a). Once all structural parameters are calculated and all dynamic responses are obtained, the wind forces are estimated using

Equation (1). The accuracy of the estimated wind forces is verified by calculating the correlation between the actual and the estimated wind forces given by the following equation:

$$Correlation = 1 - \frac{\sqrt{\sum_{t=1}^N (\hat{F}_i(t) - F_i(t))^2}}{\sqrt{\sum_{t=1}^N (F_i(t) - \bar{F}_i)^2}} \quad (5)$$

where  $\hat{F}$  is the actual wind force value and  $\bar{F}_i$  is the mean of the estimated wind force value,  $F$ . To check the accuracy of the structure's dynamic response (acceleration response) to the estimated wind forces, modal analysis is carried out using the estimated wind forces. The accuracy of the obtained acceleration response is also determined using Equation (5), where the wind force variables are replaced with the appropriate acceleration response variables.

### 3. MODEL PROPERTIES AND ANALYSIS FRAMEWORK

A ten degree-of-freedom building model with height,  $H = 100$  m, density,  $\rho = 180$  kg/m<sup>3</sup> and floor area,  $A = 625$  m<sup>2</sup> is investigated in this paper. Furthermore, the first mode natural period,  $_1T$  and damping ratio,  $_1\zeta$  of the building model are 2.5 s and 2%, respectively. The actual wind force data,  $\{\hat{F}(t)\}$  used in this paper was a 10-minute wind record in the across-wind direction taken from a typhoon simulation in a wind tunnel experiment. The flowchart for the analysis discussed in this section is shown in Figure 1.

The structural parameters are calculated using the given model properties and Equations (2) – (4b) and are referred from hereon as the “structural parameters without errors”. Also, the dynamic responses obtained using these structural parameters and the actual wind data are referred to as the “actual responses”.

In order to determine how errors on the structural parameters can affect the wind force estimation by modal analysis, different error coefficients,  $v$  are multiplied on the structural parameter matrices without errors. The different cases considered in this paper are shown in Table 1. Each case (1, 2, 3, 4, and 5) is divided further into sub-cases (A, B and C). Cases 1 and 2 deals with errors on only one structural parameter for each sub-case. On the other hand, each sub-case of Cases 3, 4 and 5 introduces errors on two structural parameters simultaneously. Using the structural parameter

matrices with errors and the actual responses, the wind forces are estimated using Equation (1). The correlation between the actual and the estimated wind forces is calculated using Equation (5).

The applicability of using the estimated wind forces can be determined by checking how accurate the model responses are when subjected to the estimated wind forces. Therefore, the estimated wind forces are applied to the original building model (using structural parameters without errors) to calculate the new dynamic responses. The accuracy of the new acceleration response is then checked using Equation (5).

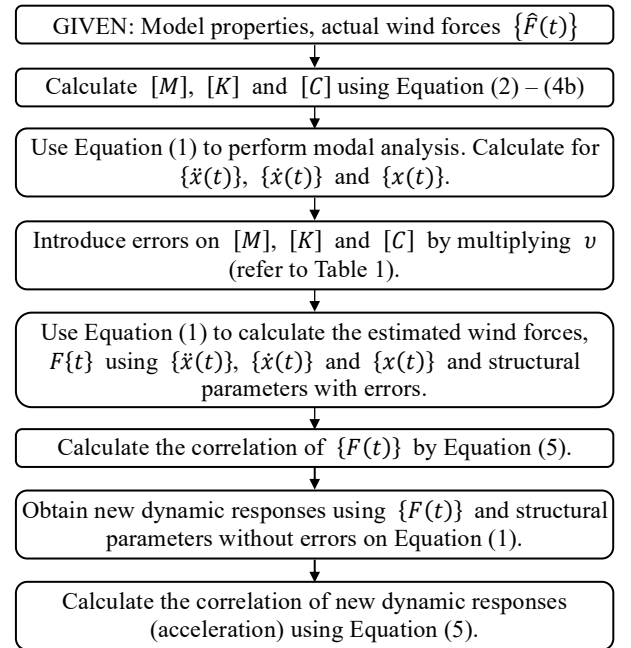


Figure 1. Analysis flowchart

Table 1. Cases of errors applied on structural parameters

Case	Error coefficient, $v$			Remarks
	$M_v$	$K_v$	$C_v$	
1	A	0.5	1	-50% error on each parameter
	B	1	0.5	
	C	1	0.5	
2	A	2	1	+100% error on each parameter
	B	1	2	
	C	1	2	
3	A	2	2	Errors on both [M] and [K]
	B	0.5	2	
	C	2	0.5	
4	A	2	1	Errors on both [M] and [C]
	B	0.5	1	
	C	2	0.5	
5	A	1	2	Errors on both [K] and [C]
	B	1	0.5	
	C	1	2	

## 4. RESULTS OF ANALYSIS

### 4.1 Error on one structural parameter (Cases 1 & 2)

In this section, errors are introduced on only one structural parameter at a time and the effect of these errors on the estimated wind forces and the new acceleration responses are investigated. Table 1 shows that an error of -50% and +100% are introduced for Case 1 and Case 2, respectively. Sub-case A corresponds to errors applied only on the  $[M]$  matrix, B on the  $[K]$  matrix, and sub-case C on the damping matrix.

The correlation values of the estimated wind forces for Case 1 are shown in Figure 2a. It can be seen here that a -50% error on the  $[M]$  matrix (Case 1A) caused a low correlation of wind forces only on the upper stories, whereas errors on the  $[K]$  matrix (Case 1B) caused a low correlation on all stories. Observing the power spectral density (PSD) plot of the estimated wind forces for Cases 1A and 1B shown in Figures 2b and 2c, respectively, it can be seen that both cases exhibit a high frequency peak error. However, errors in the  $[K]$  matrix (Case 1B) also caused errors in the low frequency range of the force PSD plot; thus, causing a lower correlation value. On the other hand, -50% error on the  $[C]$  matrix (Case 1C) did not cause a significant change on the wind forces shown in both the force correlation and the PSD plot. The acceleration responses obtained from the estimated wind forces for all sub-cases of Case 1 are significantly affected by the errors on the structural parameters and obtained low acceleration correlation values. (Figures 2e – 2h).

Increasing the value of the structural matrices by 100% on Case 2 shows a better result for the force correlation when errors on the  $[K]$  matrix (Case 2B) are introduced and a lower correlation for errors on the  $[M]$  matrix (Case 2A). In other words, a more accurate estimate of the wind forces can be obtained when the  $[K]$  matrix is overestimated rather than underestimated but the opposite can be said for the  $[M]$

matrix. However, similar with the results obtained in Case 1, the accuracy of the acceleration response for all sub-cases of Case 2 is also significantly affected by errors on the structural parameters.

### 4.2 Error on two structural parameters (Cases 3, 4 & 5)

For this next section, the effect of errors on the wind force estimation and new acceleration responses when errors on two structural parameters are introduced is investigated. For Case 3, errors on both  $[M]$  and  $[K]$  matrices are introduced, errors on  $[M]$  and  $[C]$  matrices are investigated for Case 4, and errors on  $[K]$  and  $[C]$  matrices are discussed for Case 5. Each sub-case has varying combinations of  $v$  values shown on Table 1.

The results of the analysis for Case 3 are shown on Figure 4. When both  $[M]$  and  $[K]$  were increased by 100% (Case 3A), the correlation of the estimated wind forces is higher than when one of the parameters has -50% error and the other one has +100% error (Case 3B and 3C) as can be seen in Figure 4a. This is because when both parameters were increased by the same amount, the PSD plot obtained has no high frequency errors and the only difference with the PSD of the actual wind force is the slight increase in amplitude (Figure 4b). For Case 3B, since  $[M]$  is decreased and  $[K]$  is increased, the result had a more similar behavior with Case 1A than Case 2B (Figure 4c) since it has already been observed that increasing the  $[K]$  matrix gave better force correlation. Therefore, most of the error will come from decreasing the  $[M]$  matrix. The opposite behavior can be observed in Case 3C (Figure 4d) since in this case, most of the errors came from decreasing the  $[K]$  matrix. For the acceleration responses obtained using the estimated wind forces, the highest correlation value comes from Case 3A which is around 50% correlation. The other cases obtained

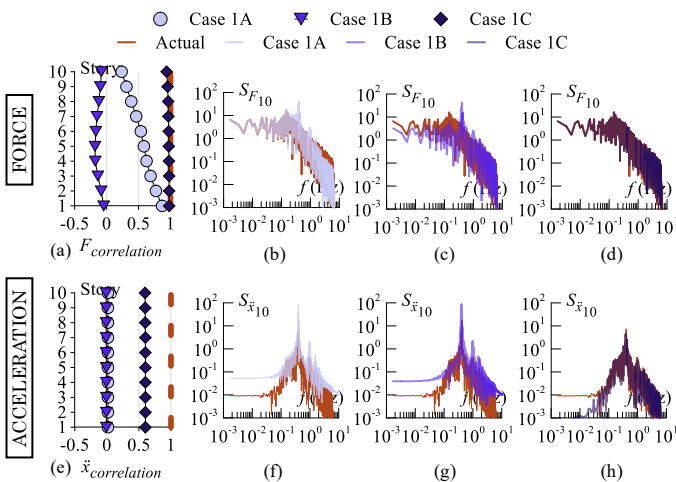


Figure 2. CASE 1 analysis results

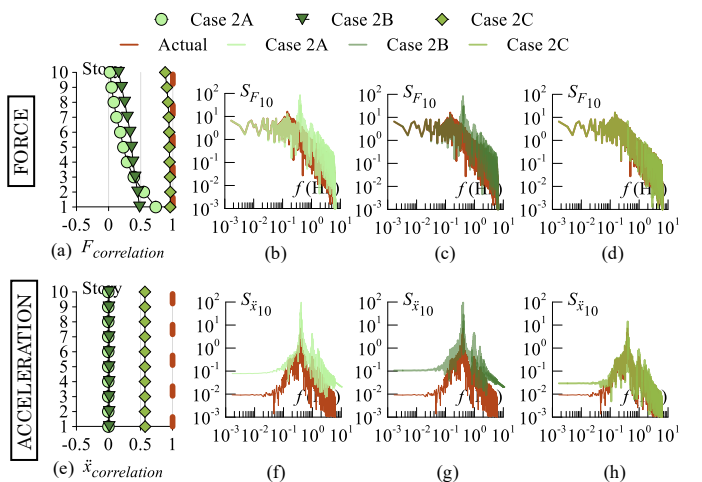


Figure 3. CASE 2 analysis results

low correlation values for the acceleration responses.

In Figure 5a, it can be seen that the behavior of the correlation values of the estimated wind forces when errors are applied on both  $[M]$  and  $[C]$  matrices (Case 4) depends on the errors introduced on the  $[M]$  matrix. This is because the errors on the  $[C]$  matrix do not affect the estimated wind

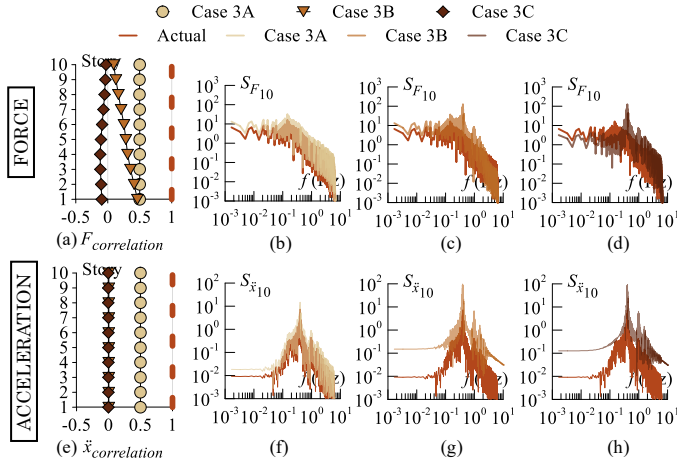


Figure 4. CASE 3 analysis results

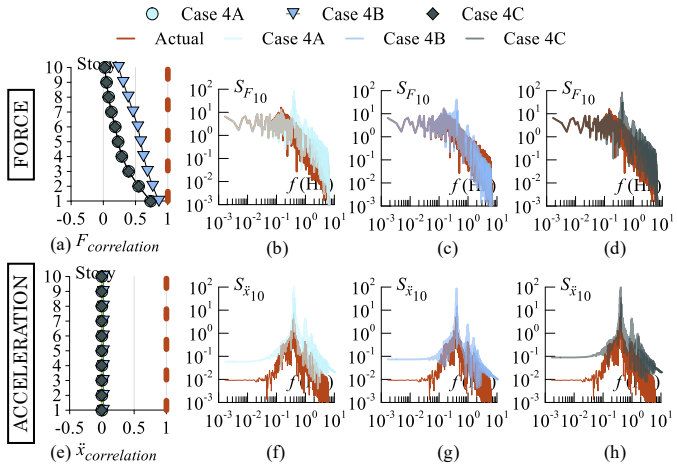


Figure 5. CASE 4 analysis results

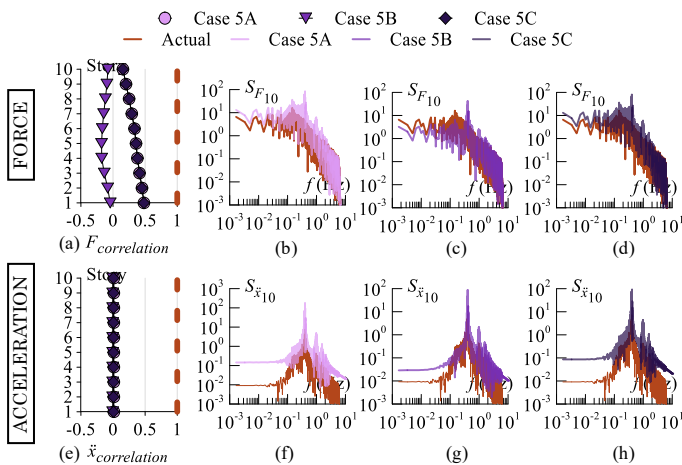


Figure 6. CASE 5 analysis results

forces significantly. Also, similar with Cases 1 and 2, underestimating  $[M]$  (Case 4B) gave better wind force estimates than overestimation of the said matrix (Cases 4A and 4C), and that errors on the estimates for the upper stories are more significant. Since large error was obtained in the upper stories, which are more critical, the acceleration responses for Case 4 all had low correlation values. This means that errors of -50% and +100% on the  $[M]$  matrix are very significant to the acceleration responses obtained from the estimated wind forces.

Lastly, for Case 5, which introduces errors on both  $[K]$  and  $[C]$  matrices, also shows that errors on the  $[C]$  matrix did not significantly affect the correlation values of the wind force estimates (Figure 6a) and that the values behave according to the changes on the  $[K]$  matrix. When the  $[K]$  matrix increases (Case 5A and 5C), the wind forces have better correlation values than when the  $[K]$  matrix is underestimated (Case 5B). Also, similar with Case 4, errors of -50% and +100% on the  $[K]$  matrix caused very low correlation on the obtained acceleration responses.

## 5. CONCLUSIONS

This study investigates the effect of structural parameter errors on the estimated wind forces by modal analysis and the obtained acceleration responses using the estimated wind forces. The results show that:

- Underestimation of the  $[M]$  matrix gives a more accurate wind force estimate as compared to overestimation of the  $[M]$  matrix. However, for the  $[K]$  matrix, it is the opposite.
- An error of -50% and +100% on the  $[M]$  and  $[K]$  matrices are very significant to both the wind force estimates and the acceleration responses obtained from the wind force estimates.
- Despite errors on the  $[C]$  matrix, a high correlation was obtained for the wind force estimates but not on the acceleration responses from the estimated wind forces.

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