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RISK-AVERSE DYNAMIC SYSTEM OPTIMAL TRAFFIC ASSIGNMENT FOR RIDE-SHARING SYSTEMS

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Extended Abstract

1 INTRODUCTION

Continuous urbanization and population growth have caused serious traffic congestion worldwide. Recently, ride-sharing has emerged as a noteworthy solution to mitigate congestion by effectively utilizing surplus resources (i.e., spare seats). Ride-sharing refers to a travel mode in which passengers and drivers with similar itineraries complete their trips while sharing travel costs such as fuel, tolls, and parking fees (Furuhata et al., 2013). Additionally, drivers can qualify to use a high occupancy vehicle (HOV) lane for a quicker trip, whereas passengers can option a flexible, faster travel mode as an alternative to public transit (PT).

Dynamic assignment of drivers (vehicles) to passengers plays a central role in the success of ride-sharing (Agatz et al., 2012). Dynamic assignment problems assume that an online platform receives trip information from passengers and drivers and matches them in real-time. Compared with a static assignment, where a batch of trip information is provided before matching, the dynamic counterpart can increase flexibility in ride-sharing participants' itineraries. Moreover, further advances in communication technologies, such as mobile networks, enable ride-sharing platforms to develop strategies that not only mitigate traffic congestion from ride-shared HOVs but also achieve dynamic system optimum (DSO)—the minimization of social costs such as total travel time.

The optimization problems of the dynamic assignment can be classified into individual-based and flow-based models. Individual-based models usually assume road networks with exogenous travel time or virtual point-to-point networks, and are formulated, respectively, as vehicle routing problems (VRPs) (e.g., Agatz et al., 2012) or vehicle-passenger matching problems (e.g., Alonso-Mora et al., 2017). The extensive previous studies on individual-based models have mainly focused on scenarios where ride-sharing services do not significantly affect traffic congestion. Unfortunately, McKinsey & Company and Bloomberg (2016) predicted that by 2030, shared vehicles could account for almost half of passenger miles. Although several studies related to VRPs (e.g., Liang et al., 2020) incorporate endogenous congestion, they describe flow congestion, which is inappropriate for dealing with peak congestion.

Flow-based models aim to capture sophisticated traffic congestion by describing individual vehicles and passengers as continuous variables. There are two major research streams for flow-based models. The first numerically solves discrete-time DSO assignment problems in general networks as linear optimization problems by employing the relaxation of congestion flow propagation. Since Levin (2017) has developed linear programming for congestion-aware vehicle routing and passenger matching, this stream has incorporated time window constraints (Liu et al., 2020), integration with PT (e.g., Maruyama and Seo, 2023), and infrastructure design (e.g., Seo and Asakura, 2021). The second stream examines exact solutions to continuous-time DSO assignment problems in simple networks. This stream usually focuses on DSO assignments in parallel networks consisting of HOV and general purpose (GP) lanes (e.g., Huang et al., 2021).

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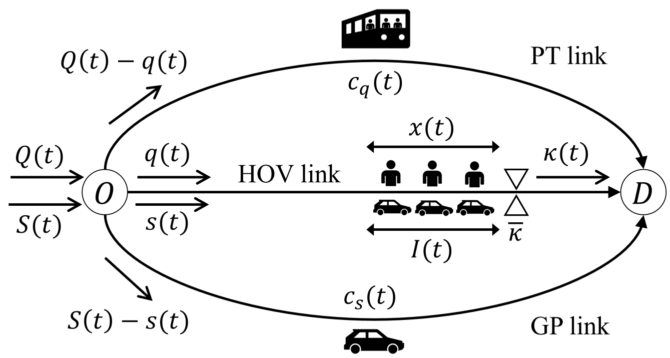


Figure 1: A parallel link network

Although the analytical methods may not be expandable to general networks, it is worth noticing that parallel networks are very common when studying the morning commute problem.

Although numerous powerful tools for dynamic vehicle-passenger assignment have been developed, they face a significant limitation in the lack of addressing stochasticity in traffic states (e.g., travel times). If the dynamics of traffic states would be deterministic, model outputs and observations are considered consistent. Unfortunately, traffic states are stochastic due to unexpected driving behavior and other factors, especially during peak congestion. Although real-time observation of uncertain traffic states helps mitigate their negative impacts, assignment strategies disregarding future risks may miss more effective solutions. Therefore, it is significant to incorporate stochastic traffic states into the dynamic assignment for ride-sharing systems.

This study identifies the properties of the DSO assignment for ride-sharing with stochastic travel time in a parallel network. We focus on a continuous-time DSO assignment problem in a network with three parallel links; the first is an HOV link with a single bottleneck, the second represents a GP link that can be considered an aggregate of other road networks, and the third is a PT link for non-ride-sharing passengers. Only ride-sharing and non-ride-sharing vehicles are allowed to pass the HOV and GP links, respectively. This problem is formulated as a stochastic optimal control problem ([SOCP]). Our formulation can be considered a significant modification of control theoretic approaches for non-ride-sharing systems (e.g., Akamatsu and Nagae, 2007). Then, the optimality conditions of the [SOCP], Hamilton-Jacobi-Bellman equations, reveal DSO assignment patterns for vehicles and passengers, depending on the observed travel times and queue lengths.

2 STOCHASTIC DYNAMIC SYSTEM OPTIMAL ASSIGNMENT FOR RIDE-SHARING

2.1 System Specifications

This study focuses on a parallel network with a single bottleneck, connecting a single origin-destination pair, as illustrated in Fig. 1. The parallel network comprises three types of links: (1) an HOV link for ride-sharing vehicles and passengers, (2) a GP link for non-ride-sharing vehicles, and (3) a PT link for non-ride-sharing passengers.

We assume that the arrival rates at the origin for vehicles and passengers are known during a fixed planning horizon $[0,T]$. Let $S(t)$ and $Q(t)$ be the arrival rates at time t for traffic supply (vehicles) and demand (passengers), respectively. The platform can assign $s(t)$ vehicles and $q(t)$ passengers out of the traffic supply, $S(t)$, and demand, $Q(t)$, respectively, to the HOV link at each time t for the system optimum. Then, the remaining supply, $S(t)-s(t)$, and demand, $Q(t)-q(t)$, are assigned to the GP and PT links, respectively. Vehicles and passengers assigned to the HOV link meet at the origin and then travel together to the destination.

We make the following contrasting assumptions for the HOV link dedicated to ride-sharing services and other links in order to provide analytical insights into ride-sharing DSO assignments. Let $\overline{\kappa}_H$, $\overline{\kappa}_G$, and $\overline{\kappa}_P$ be the capacities of the HOV, GP, and PT links, respectively, and likewise, $c_H(t)$, $c_G(t)$, and $c_P(t)$ be their free-flow travel times (FFTTs). It is assumed that $\overline{\kappa}_H$ for the HOV link is small enough to cause bottleneck queueing, whereas $\overline{\kappa}_G$ and $\overline{\kappa}_P$ for the other links are sufficiently large compared to the traffic supply $S(t)$ and demand $Q(t)$. Then, throughout this study, let $\overline{\kappa}_H = \overline{\kappa}$ (constant) and $\overline{\kappa}_G = \overline{\kappa}_P = \infty$. Moreover, the FFTT on the HOV link, $c_H(t)$, is normalized to 0, and the relative FFTTs on the GP and PT links are defined as $c_s(t) \equiv c_G(t) - c_H(t) \geq 0$ and $c_q(t) \equiv c_P(t) - c_H(t) \geq 0$, respectively. The assumptions are plausible since the differences in FFTT on each link dominate the efficiency of dynamic assignments.

2.2 Network Dynamics

The vehicle and passenger queues, $I(t)$ and $x(t)$, increase in two cases; when no passengers or vehicles are assigned to the HOV link, or when their matching vehicles face the single bottleneck. The evolution of the queues is governed by the following equations:

$$\dot{I}(t) = s(t) - \kappa(t), \quad I(0) = I_0 \text{ and } I(T) \text{ is free,} \tag{1}$$

$$\dot{x}(t) = q(t) - \rho\kappa(t), \quad x(0) = x_0 \text{ and } x(T) \text{ is free,} \tag{2}$$

$$\kappa(t) = \begin{cases} \overline{\kappa} & \text{if } I(t) > 0 \text{ and } x(t) > 0 \\ \min[\overline{\kappa}, s(t)] & \text{if } I(t) = 0 \text{ and } x(t) > 0 \\ \min[\overline{\kappa}, q(t)/\rho] & \text{if } I(t) > 0 \text{ and } x(t) = 0 \\ \min[\overline{\kappa}, s(t), q(t)/\rho] & \text{if } I(t) = 0 \text{ and } x(t) = 0 \end{cases}, \tag{3}$$

where $I(t)$ and $x(t)$ implicitly describe both bottleneck and matching queueing. ρ represents a passenger capacity of a vehicle. $\kappa(t)$ is the outflow rate of the HOV link and describes the following flow; when the vehicle and passenger queues exist (i.e., $I(t) > 0$ and $x(t) > 0$), the outflow rate is equal to the bottleneck capacity $\overline{\kappa}$; otherwise, the assigned vehicle or passenger flow to the HOV link passes or faces the bottleneck. Therefore, Eq. (3) implies that the dynamics of $I(t)$ and $x(t)$ in Eqs. (1) and (2) are discontinuous.

The travel times of the GP and PT links, $c_s(t)$ and $c_q(t)$, are supposed to evolve unpredictably due to many factors such as traffic accidents and timetable disruptions. We express the stochastic dynamics of $c_s(t)$ and $c_q(t)$ as follows:

$$dc_s(t)/c_s(t) = \mu_s(t)dt + \sigma_s(t)dz_s(t), \quad c_s(0) = c_{s0}, \tag{4}$$

$$dc_q(t)/c_q(t) = \mu_q(t)dt + \sigma_q(t)dz_q(t), \quad c_q(0) = c_{q0}, \tag{5}$$

where $z(t)$ is an independent standard Wiener process; $\mu(t)$ and $\sigma(t)$ are the drift rate and volatility parameters, respectively. The subscripts of $z(t)$, $\mu(t)$, and $\sigma(t)$ indicate the corresponding links. Each first term in Eqs. (4) and (5) indicates the deterministic dynamics of the travel times, while each second term indicates the stochastic dynamics (i.e., travel time uncertainty).

2.3 Stochastic Optimal Control Formulation

A platform operator pursues minimizing social costs spent in the network during the planning horizon $[0,T]$. The simplest expression of the social cost is the expected total travel time, given by

$$\mathbb{E}_0\left[F\middle|(I_0, x_0, c_{s0}, c_{q0})\right],$$

$$F \equiv \int_0^T \{f_H(t) + f_G(t) + f_P(t)\}\, dt,$$

$$f_H(t) \equiv I(t) + x(t), \quad f_G(t) \equiv c_s(t) \cdot (S(t) - s(t)), \quad f_P(t) \equiv c_q(t) \cdot (Q(t) - q(t)), \tag{6}$$

where $\mathbb{E}_t\left[\cdot \middle| (I, x, c_s, c_q)\right]$ denotes the expectation operator given the information available $\{I(t), x(t), c_s(t), c_q(t)\} = (I, x, c_s, c_q)$ at time t. F is the total travel time and consists of the time-integrands of the queue length on the HOV link, $f_H(t)$, and the total FFTT on the GP and PT links, $f_G(t)$ and $f_P(t)$, at time t. However, this expression allows for risky assignment strategies where the worst-case F is very large despite the small expectation.

We assume a risk-averse operator and formulate the DSO assignment problem as the following stochastic optimal control problem (SOCP):

[SOCP]

$$\min_{s(t), q(t)} \mathbb{E}_0 \left[\int_0^T \{U(f_H(t)) + U(f_G(t)) + U(f_P(t))\}\, dt + U(f_H(T)) \middle| (I_0, x_0, c_{s0}, c_{q0}) \right] \tag{7}$$

$$\text{subject to Eqs.}(1)-(6), \;\; 0 \leq s(t) \leq S(t), \text{ and } \;\; 0 \leq q(t) \leq Q(t),$$

where $U(\cdot)$ is a risk-averse function (i.e., $\partial U / \partial f > 0$ and $\partial^2 U / \partial f^2 \geq 0$); the highter its curvature, the higher the risk-aversion of the operator represents. The [SOCP] yileds a feedback control where the optimal solutions in $s(t)$ and $q(t)$ are functions of observations $\{I(t), x(t), c_s(t), c_q(t)\}$ at time t. We denote the optimal controls by $s^*(t) \equiv s(t, I, x, c_s, c_q)$ and $q^*(t) \equiv q(t, I, x, c_s, c_q)$.

3 SYSTEM OPTIMAL ASSIGNMENT STRATEGIES

3.1 Optimality Conditions

The optimal solutions to the [SOCP] can be derived from the maximum principle. The differential equations (1)–(5) and control constraints are linear with respect to state and control variables; thus, the [SOCP] satisfies the constraint qualifications (Arrow et al., 1961). Accordingly, the stochastic maximum principle (Peng, 1990) is the first-order optimality condition. The stochastic maximum principle is given by

$$0 = \min_{s(\cdot), q(\cdot)} H(s(\cdot), q(\cdot)) + \mathscr{D}_0 V(\cdot), \tag{8}$$

$$H(s(\cdot), q(\cdot)) \equiv U(f_H(t)) + U(f_G(t)) + U(f_P(t)) + \{\mathscr{D}_I + \mathscr{D}_x\} V(\cdot), \tag{9}$$

$$-\frac{d}{dt} V_I(\cdot) = \frac{\partial U}{\partial f_H} > 0, \quad V_I(\cdot)\Big|_{t=T} = \frac{\partial U}{\partial f_H}\Big|_{t=T} > 0, \tag{10}$$

$$-\frac{d}{dt} V_x(\cdot) = \frac{\partial U}{\partial f_H} > 0, \quad V_x(\cdot)\Big|_{t=T} = \frac{\partial U}{\partial f_H}\Big|_{t=T} > 0, \tag{11}$$

where $(\cdot)$ is a compact expression of traffic state (t, I, x, c_s, c_q) observed at time t. We define the value function $V(t, I, x, c_s, c_q)$ as follows:

$$V(t, I, x, c_s, c_q) \equiv \min_{s(\cdot), q(\cdot)} \mathbb{E}_t \left[\int_t^T \{U(f_H(\tau)) + U(f_G(\tau)) + U(f_P(\tau))\}\, d\tau + U(f_H(T)) \middle| (I, x, c_s, c_q) \right], \tag{12}$$

where the subscript of V denotes the partial derivative (e.g., $V_I = \partial V / \partial I$). Let $\mathscr{D}_0$, $\mathscr{D}_I$, and $\mathscr{D}_x$ be partial differential operators as follows:

$$\mathscr{D}_0 \equiv \frac{\partial}{\partial t} + \mu_s(t) \cdot c_s(t) \frac{\partial}{\partial c_s} + \frac{1}{2} \sigma_s^2(t) \cdot c_s^2(t) \frac{\partial^2}{\partial c_s^2} + \mu_q(t) \cdot c_q(t) \frac{\partial}{\partial c_q} + \frac{1}{2} \sigma_q^2(t) \cdot c_q^2(t) \frac{\partial^2}{\partial c_q^2},$$

Table 1: DSO assignment patterns for vehicles

Queues $(I(t), x(t))$		Supply $S(t)$ and Demand $Q(t)$	Travel Time $c_s(t)$	$s^*(t)$	ID
$I(t)=0$	$x(t)=0$	$\min\{Q(t)/\rho, S(t)\} = S(t) \leq \overline{\kappa}$	—	$S(t)$	$\mathrm{A_s}$
		$\min\{Q(t)/\rho, S(t)\} = Q(t)/\rho \leq \overline{\kappa}$	$V_I > c_s(t)\frac{\partial U}{\partial f_G}$	$Q(t)/\rho$	$\mathrm{B'_{s_0}}$
			$V_I = c_s(t)\frac{\partial U}{\partial f_G}$	$\hat{s}$	$\mathrm{C_s}$
			$V_I < c_s(t)\frac{\partial U}{\partial f_G}$	$S(t)$	$\mathrm{D_s}$
		$\min\{Q(t)/\rho, S(t)\} > \overline{\kappa}$	$V_I > c_q(t)\frac{\partial U}{\partial f_G}$	$\overline{\kappa}$	$\mathrm{B_{s_0}}$
			$V_I = c_s(t)\frac{\partial U}{\partial f_G}$	$\hat{s}$	$\mathrm{C_s}$
			$V_I < c_s(t)\frac{\partial U}{\partial f_G}$	$S(t)$	$\mathrm{D_s}$
	$x(t)>0$	$S(t) \leq \overline{\kappa}$	—	$S(t)$	$\mathrm{A_s}$
		$S(t) > \overline{\kappa}$	$V_I > c_s q(t)\frac{\partial U}{\partial f_G}$	$\overline{\kappa}$	$\mathrm{B_{s_0}}$
			$V_I = c_s(t)\frac{\partial U}{\partial f_G}$	$\hat{s}$	$\mathrm{C_s}$
			$V_I < c_s(t)\frac{\partial U}{\partial f_G}$	$S(t)$	$\mathrm{D_s}$
$I(t)>0$	—	—	$V_I > c_s(t)\frac{\partial U}{\partial f_G}$	0	$\mathrm{B_s}$
			$V_I = c_s(t)\frac{\partial U}{\partial f_G}$	$\hat{s}$	$\mathrm{C_s}$
			$V_I < c_s(t)\frac{\partial U}{\partial f_G}$	$S(t)$	$\mathrm{D_s}$

$$\mathscr{D}_I \equiv (s(\cdot) - \kappa(t))\frac{\partial}{\partial I}, \quad \mathscr{D}_x \equiv (q(\cdot) - \rho\kappa(t))\frac{\partial}{\partial x}.$$

Eq. (8) denotes the Hamilton-Jacobi-Bellman equations and indicates that the optimal controls $(s^*(t), q^*(t))$ minimize the Hamiltonian H defined as Eq. (9). Eqs. (10) and (11) represent the dynamics of the social marginal cost of $I(t)$ and $x(t)$, respectively. The monotonic decrease and the termination conditions lead to $V_I > 0$ and $V_x > 0$. Note that the dynamics of V_I and V_x are discontinuous due to the non-differentiability in $I(t)$ and $x(t)$ shown in Eqs. (1)–(3).

3.2 Optimal Control

The optimality conditions of the [SOCP] provide the optimal assignments as a combination of functions of observations (I, x, c_s, c_q) at time t. Although the proofs are not provided due to space limitations, we obtain the following propositions regarding the risk-averse DSO assignment:

Proposition 1. *The DSO assignment patterns consist of upper and lower bound solutions (i.e., corner solutions) and an interior solution that smoothly bridges them. The upper bound solutions, $\overline{s} = S(t)$ and $\overline{q} = Q(t)$, are independent of the traffic states. The lower bound solutions $(\underline{s}, \underline{q})$ are classified depending on the traffic states as follow:*

$$\underline{s} = \begin{cases} 0 \\ \min[\overline{\kappa}, S(t)] \\ 0 \\ \min[\overline{\kappa}, Q(t)/\rho, S(t)] \end{cases}, \quad \underline{q} = \begin{cases} 0 & \text{if } I(t) > 0 \text{ and } x(t) > 0 \\ 0 & \text{if } I(t) = 0 \text{ and } x(t) > 0 \\ \min[\rho\overline{\kappa}, Q(t)] & \text{if } I(t) > 0 \text{ and } x(t) = 0 \\ \min[\rho\overline{\kappa}, Q(t), \rho S(t)] & \text{if } I(t) = 0 \text{ and } x(t) = 0 \end{cases}. \tag{13}$$

Proposition 2. *The interior solutions to the [SOCP], $(\hat{s}, \hat{q})$, are given by*

$$\hat{s} = S(t) - \frac{1}{c_s(t)} G_s\left(\frac{1}{c_s(t)} V_I\right), \quad \hat{q} = Q(t) - \frac{1}{c_q(t)} G_q\left(\frac{1}{c_q(t)} V_x\right), \tag{14}$$

Table 2: DSO assignment patterns for passengers

Queues $(I(t),x(t))$		Supply $S(t)$ and Demand $Q(t)$	Travel Time $c_q(t)$	$q^*(t)$	ID
$x(t)=0$	$I(t)=0$	$\min\{Q(t)/\rho, S(t)\} = Q(t)/\rho \le \overline{\kappa}$	—	$Q(t)$	$\mathrm{A_q}$
		$\min\{Q(t)/\rho, S(t)\} = S(t) \le \overline{\kappa}$	$V_x > c_q(t)\frac{\partial U}{\partial f_P}$	$\rho S(t)$	$\mathrm{B'_{q0}}$
			$V_x = c_q(t)\frac{\partial U}{\partial f_P}$	$\hat{q}$	$\mathrm{C_q}$
			$V_x < c_q(t)\frac{\partial U}{\partial f_P}$	$Q(t)$	$\mathrm{D_q}$
		$\min\{Q(t)/\rho, S(t)\} > \overline{\kappa}$	$V_x > c_q(t)\frac{\partial U}{\partial f_P}$	$\rho\overline{\kappa}$	$\mathrm{B_{q0}}$
			$V_x = c_q(t)\frac{\partial U}{\partial f_P}$	$\hat{q}$	$\mathrm{C_q}$
			$V_x < c_q(t)\frac{\partial U}{\partial f_P}$	$Q(t)$	$\mathrm{D_q}$
	$I(t)>0$	$Q(t)/\rho \le \overline{\kappa}$	—	$Q(t)$	$\mathrm{A_q}$
		$Q(t)/\rho > \overline{\kappa}$	$V_x > c_q(t)\frac{\partial U}{\partial f_P}$	$\rho\overline{\kappa}$	$\mathrm{B_{q0}}$
			$V_x = c_q(t)\frac{\partial U}{\partial f_P}$	$\hat{q}$	$\mathrm{C_q}$
			$V_x < c_q(t)\frac{\partial U}{\partial f_P}$	$Q(t)$	$\mathrm{D_q}$
$x(t)>0$	—	—	$V_x > c_q(t)\frac{\partial U}{\partial f_P}$	0	$\mathrm{B_q}$
			$V_x = c_q(t)\frac{\partial U}{\partial f_P}$	$\hat{q}$	$\mathrm{C_q}$
			$V_x < c_q(t)\frac{\partial U}{\partial f_P}$	$Q(t)$	$\mathrm{D_q}$

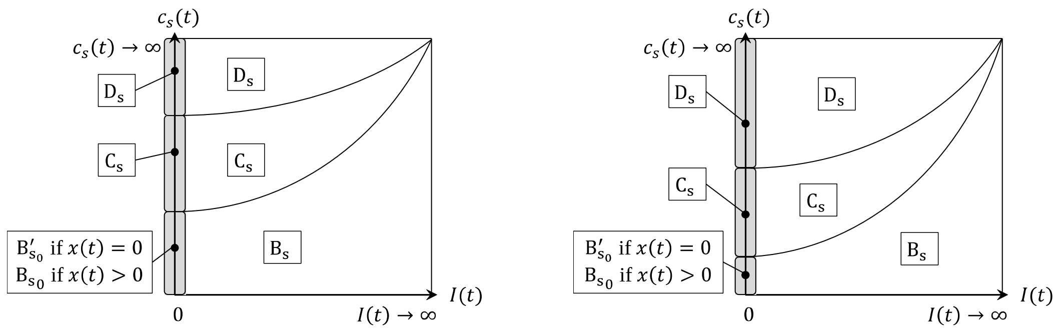
where $G_s(\cdot)$ and $G_q(\cdot)$ are inverse functions of $\frac{\partial U}{\partial f_G}$ and $\frac{\partial U}{\partial f_P}$, defined as $G_s\left(\frac{1}{c_s(t)}V_I\right) \equiv f_G(t)$ and $G_q\left(\frac{1}{c_q(t)}V_x\right) \equiv f_P(t)$, respectively. The interior solutions, $\hat{s}$ and $\hat{q}$, both monotonically decrease with respect to the queues $(I(t),x(t))$, whereas monotonically increase with respect to the travel times $c_s(t)$ and $c_q(t)$, respectively.

Proposition 3. *The relationship between the social marginal cost of the HOV link, V_I and V_x, and that of anothor link, $c_s(t)\partial U/\partial f_G$ and $c_q(t)\partial U/\partial f_P$, characterizes the DSO assignment patterns for vehicles and passengers, respectively. The DSO assignment patterns $(s^*(t), q^*(t))$ are*

$$s^*(t) = \begin{cases} \underline{s} & \text{if } c_s(t)\partial U/\partial f_G < V_I \\ \hat{s} & \text{if } c_s(t)\partial U/\partial f_G = V_I \\ \overline{s} & \text{if } c_s(t)\partial U/\partial f_G > V_I \end{cases}, \quad q^*(t) = \begin{cases} \underline{q} & \text{if } c_q(t)\partial U/\partial f_P < V_x \\ \hat{q} & \text{if } c_q(t)\partial U/\partial f_P = V_x \\ \overline{q} & \text{if } c_q(t)\partial U/\partial f_P > V_x \end{cases}. \tag{15}$$

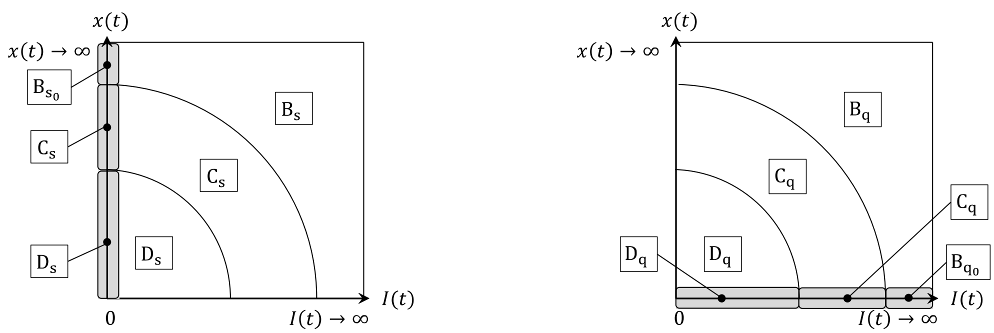
Tables 1 and 2 summarize the properties of the DSO assignment in Propositions 1–3. Tables 1 and 2 show all the optimal control patterns for traffic supply (i.e., vehicles) and demand (i.e., passengers), respectively, when observing the feasible traffic states. For example, in extreme cases, when some moderate length of queues $(I(t),x(t))$ and sufficiently large travel times $(c_s(t), c_q(t))$ are observed, Eq. (15) illustrates that all traffic supply $S(t)$ and demand $Q(t)$ should be assigned to the HOV link. The social marginal costs (V_I, V_x) in Tables 1 and 2 can be calculated as functions of time and exogenous parameters. They allow us to draw boundary curves, as shown in Fig. 2. The [SOCP] with corner solutions as optimums can be amenable to the generalized complementarity model-based efficient algorithm (Nagae and Akamatsu, 2008), thereby providing more detailed properties of the boundary curves.

A few qualitative properties of the DSO assignment are in order here: first, the marginal social costs of $(I(t),x(t))$ decrease over time (i.e., $\frac{d}{dt}V_I \le 0$ and $\frac{d}{dt}V_x \le 0$). From Eq. (15), this indicates that the optimal controls $(s^*(t), q^*(t))$ increase over time. Fig. 2 shows a typical pattern of the dynamics of the DSO assignment for vehicles. This property implies employing a risk-free strategy (i.e., assignment to the HOV link),



(a) $t = t_1$ (b) $t = t_2 > t_1$

Figure 2: Dynamics of the DSO assignment for vehicles in the state space $(I(t), c_s(t))$



(a) Optimal supply control $s^*(t)$ (b) Optimal demand control $q^*(t)$

Figure 3: Risk-averse DSO assignment at time t in the state space $(I(t), x(t))$

because significant losses cannot be recouped in a shorter period $[t, T]$. Second, risk-averse preferences lead to monotonically increasing social marginal costs (V_I, V_x) with respect to $(I(t), x(t))$. This property gives rise to a monotonical decrease of the optimal controls $(s^*(t), q^*(t))$ with respect to $(I(t), x(t))$, as shown in Fig. 3. Third, the smaller the risk aversion (i.e., the smaller $\partial^2 U/\partial f^2$), the smaller the regions of the interior solutions, $\mathrm{C_s}$ and $\mathrm{C_q}$. Eventually, the regions disappear in a risk-neutral assignment, because when the risk-averse function $U(\cdot)$ is linear, we cannot define the inverse functions, $G_s(\cdot)$ and $G_q(\cdot)$. We can conclude from the monotonicity of $(s^*(t), q^*(t))$ that risk-averse preferences smooth out the DSO assignment.

4 CONCLUDING REMARKS

This paper explores dynamic assignment strategies for ride-sharing that achieve DSO on a network with three parallel links in stochastic environments. The control theoretical analysis provides DSO assignment patterns, as summarized in Table 1 and 2. Furthermore, the qualitative properties of the risk-averse DSO assignment were discussed in Propositions 1–3.

It should be noted that this study is designed as an initial step towards analyzing the properties of the DSO assignment for ride-sharing systems. Our analysis may be expandable to a more complex model (e.g., that with departure time choice) without any severe difficulties in its mathematical structure. Chow (2009) developed a control theoretic formulation of DSO assignments with departure time choice for non-ride-sharing systems. Future work can focus on expanding such a DSO assignment problem to ride-sharing systems.

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