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TOKYO INSTITUTE OF TECHNOLOGY

DOCTORAL THESIS

A Study of Cayley Parametrizations and Their Applications to Optimizations with Orthogonality Constraints [Outline]

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in

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Outline

An optimization problem with the orthogonality constraints has been playing a crucial role in signal processing and machine learning applications, e.g., sparse principal component analysis. The main challenge in this problem is the severe nonlinearity of the orthogonality constraint set, called the Stiefel manifold, which hinders the development of effective optimization algorithms.

In this thesis, to resolve the nonlinearity of the Stiefel manifold, we present the generalized left-localized Cayley transform (for short, the generalized Cayley transform) for parametrization of an open dense subset of the Stiefel manifold in terms of a Euclidean space. Via the generalized Cayley transform, we propose a (Naive) Cayley Parametrization (NCP) strategy, which translates the original problem into an optimization problem over the Euclidean space. For the translated problem, we can directly apply any Euclidean optimization algorithm in the NCP strategy. Our numerical experiments demonstrate that the NCP strategy outperforms the standard retraction-based strategy for the optimization problem with the orthogonality constraints. However, the NCP strategy may suffer from a slow convergence in an exceptional case where a minimizer lies near singular points where the generalized Cayley transform can not be defined. We call this performance degradation a singular-point issue in this thesis.

To suppress the singular-point issue, we propose an Adaptive Localized Cayley Parametrization (ALCP) strategy, which adaptively adjusts the location of singular points of the generalized Cayley transform. For a smooth cost function, we also present convergence analyses of the ALCP strategy in terms of a stationary point in cases where broad classes of Euclidean optimization algorithms are incorporated into the strategy. Numerical experiments demonstrate that the ALCP strategy dramatically improves the convergence speed of the NCP strategy.

Furthermore, we address an optimization problem over a nonconvex constraint set with a nonsmooth but weakly convex cost function. For this problem, we propose an optimization algorithm with guaranteed global convergence in terms of a stationary point inspired by (i) a parametrization of the constraint set similar to the NCP strategy and (ii) a variable smoothing, which uses a smooth approximation of the cost function for updating estimates of a solution. As an application of the proposed algorithm, we also address a Sparse Spectral Clustering (SSC) through a new formulation with a weakly convex regularizer over the Stiefel manifold. Our numerical experiments demonstrate that the proposed SSC successfully outperforms the existing SSC.