論文 / 著書情報
Article / Book Information

題目(和文)	フィードバックシステムにおける安定性と次元
Title(English)	Stability and Dimension in Feedback Systems: A Differential Lyapunov Framework
著者(和文)	加藤留偉
Author(English)	Rui Kato
出典(和文)	学位:博士(工学), 学位授与機関:東京工業大学, 報告番号:甲第12772号, 授与年月日:2024年3月26日, 学位の種別:課程博士, 審査員:石井 秀明,三宅 美博,小野 功,瀧ノ上 正浩,SLAVAKIS KONSTANTINO,畑中 健志
Citation(English)	Degree:Doctor (Engineering), Conferring organization: Tokyo Institute of Technology, Report number:甲第12772号, Conferred date:2024/3/26, Degree Type:Course doctor, Examiner:,,,,,
学位種別(和文)	博士論文
Category(English)	Doctoral Thesis
種別(和文)	論文要旨
Type(English)	Summary

（博士課程）
Doctoral Program

論 文 要 旨

THESIS SUMMARY

系・コース： Department of, Graduate major in	情報工学 系 ライフエンジニアリング コース	申請学位（専攻分野）： Academic Degree Requested	博士 Doctor of	（ 工学 ）
学生氏名： Student's Name	加藤 留偉	審査員主査： Chief Examiner	石井 秀明	

要旨（英文 800 語程度）
Thesis Summary（approx.800 English Words ）

Lyapunov's first and second methods, or the theories of Lyapunov exponents and Lyapunov functions, are powerful mathematical tools to study the asymptotic behaviors of dynamical systems. Historically, Lyapunov exponents have been studied in the dynamical systems community, whereas Lyapunov functions have been studied in the systems and control community. Motivated by the gap between theoretical approaches in the two different communities, we aim to integrate the feedback theory based on Lyapunov functions and the complexity theory based on Lyapunov exponents. This integration is important because dynamical systems cannot necessarily be controlled in the sense of Kalman. In this thesis, we are devoted to the analysis of global stability and Lyapunov dimension for feedback systems.

A feedback system is represented as an interconnection of two or more than two subsystems. There, the input/output relations of the subsystems and the interconnection structure determine the closed-loop dynamics. This perspective is important when a suitable feedback controller is designed or when a large-scale networked system is analyzed. Such a problem was first considered in absolute stability theory, and the so-called Aizerman and Kalman conjectures have been well studied in 1940s and 1950s. The solutions of the absolute stability problem by the Popov and circle criteria, which were reported around 1960, greatly influenced subsequent developments of comprehensive frameworks such as dissipativity theory and IQC theory. This thesis proposes a new approach to this problem by investigating infinitesimal changes in the variables of interest. A differential Lyapunov framework is suitable for estimating certain characteristic exponents, and hence, not only stability but also dimension can be addressed in a unified way.

In Chapter 2, we provide a short overview on a differential framework for dynamical systems. We start with introducing differential representations of autonomous systems (flows) and those of input/state/output systems (behaviors). After recalling the incremental notions of stability and dissipativity, we describe some relations between Lyapunov's first and second methods. In particular, it is shown that a Lyapunov exponent can be estimated via a differential Lyapunov function and that certain bounds of the largest Lyapunov exponent result in incremental stability of the flow. Based on differential dissipativity theory, we also consider incremental properties of interconnected systems.

In Chapter 3, we investigate global stability of a Lur'e system, which is a feedback system consisting of a linear time-invariant system and a memoryless nonlinearity. Although this system has long been studied in control theory, the differential approach in this thesis is different from previous studies. We first derive sufficient conditions for global stability in terms of input/output properties of the linear system and those of the derivative of the nonlinear map. In particular, we propose the use of what we call a "differential Lyapunov function of the Leonov form." This function is a product of two separated functions: a quadratic form and a Lyapunov-like function. The Lyapunov-like function in our framework originates in Leonov's work on dimension theory and provides novel flexibility in the global stability conditions. Indeed, we show that our stability conditions are necessary and sufficient conditions for dissipative systems in the sense of Levinson. A key difference from the existing methods such as IQC theory is that the linear block and the nonlinear block are allowed to have different input/output properties. This is because the nonlinear effects are absorbed by the Lyapunov-like function.

In Chapter 4, we continue to consider a Lur'e system. The main objective of this chapter is to estimate Lyapunov dimension, and hence, local stability is not assumed. To obtain an upper estimate of Lyapunov dimension, we extend Leonov's method for autonomous systems to feedback systems. In particular, the results in this chapter are generalizations of stability analysis in the previous chapter. Furthermore, we aim to obtain the exact Lyapunov dimension formula for the feedback system. This problem is inspired by the Eden conjecture, which states that the local and global Lyapunov dimensions coincide at an equilibrium. Because our approach is based on a differential Lyapunov function of the Leonov form, we need not consider the points at which the time derivative of a Lyapunov-like function does not vanish. This is similar to the basic idea in the LaSalle invariance principle. The main contribution of this chapter is the derivation of a suitable characterization of the class of nonlinearities for which the Eden conjecture is valid. Because the exact Lyapunov dimension is a good analytical estimate of the Hausdorff and box dimensions of an attractor, our result is important for reduction of complex dynamics.

Finally, in Chapter 5, we propose the notion of volume dissipative systems, which combines the concepts of volume contracting systems and differentially dissipative systems. The former originates in the classical Liouville theorem for Hamiltonian systems, and the latter originates in dissipativity theory by Willems. Based on this notion, we develop a general framework of dimension analysis for interconnected systems. In addition, we propose a computationally efficient method for estimating attractor dimension by generalizing Smith's method. The developed framework is meaningful in the sense that large-scale interconnected systems can be analyzed in a scalable way. We conclude this thesis with a discussion of future directions. Particularly, we emphasize that the concepts of dimension and entropy may be important in control theory for broader applications.