

論文 / 著書情報
Article / Book Information

題目(和文)	
Title(English)	Popularity on Variations of Multidimensional Roommate Games
著者(和文)	GE Steven
Author(English)	Steven Ge
出典(和文)	学位:博士(学術), 学位授与機関:東京科学大学, 報告番号:甲第386号, 授与年月日:2025年3月26日, 学位の種別:課程博士, 審査員:伊東 利哉,田中 圭介,鹿島 亮,安永 憲司,横井 優
Citation(English)	Degree:Doctor (Academic), Conferring organization: Institute of Science Tokyo, Report number:甲第386号, Conferred date:2025/3/26, Degree Type:Course doctor, Examiner:,,,,,
学位種別(和文)	博士論文
Category(English)	Doctoral Thesis
種別(和文)	論文要旨
Type(English)	Summary

(博士課程)
Doctoral Program

論文要旨

THESIS SUMMARY

系・コース： Department of, Graduate major in	Mathematical and Computing Science	系 コース	申請学位 (専攻分野)： Academic Degree Requested	博士 Doctor of	(Philosophy)
学生氏名： Student's Name	Steven Ge		審査員主査： Chief Examiner	Toshiya Itoh	

要旨 (英文 800 語程度)

Thesis Summary (approx.800 English Words)

The formation of stable or optimal coalitions in multi-agent systems is a computational problem that has several variations with different conditions on the coalition size and preferences of the agents. The study of computational coalition formation problems began in 1962 with Gale and Shapley, who introduced the Stable Marriage Problem in which two disjoint sets are to be stably matched according to their preferences. A matching is stable if no unmatched pair prefers each other over their assigned partners in the matching. Gale and Shapley also provided an algorithm that computes a stable matching, which is guaranteed to exist, in polynomial time. Additionally, Gale and Shapley introduced the Stable Roommates Problem, which is essentially the Stable Marriage Problem but with only one set. Unlike the Stable Marriage Problem, a stable matching is not guaranteed to exist in the Stable Roommates Problem. Irving constructed a polynomial time algorithm that returns a stable matching, if one exists. Both the Stable Marriage Problem and the Stable Roommates Problem consider coalitions of size two. Knuth initiated the study of three- or multidimensional variants of these problems. We study two variants of the Multidimensional Stable Roommates problem.

There are various notions that define the stability or optimality of a partitioning of agents. The notion of popularity, which blends elements of both stability and optimality, was introduced by Gärdenfors in 1975. A partitioning of agents is popular if there does not exist another partitioning in which more agents are better off than worse off. Furthermore, a partitioning of agents is strictly popular if it is preferred by a majority of agents over any other partitioning. A probability distribution p over all possible partitionings of agents is mixed popular if the partitioning returned by p is expected to be at least as popular as the partitioning returned by any other probability distribution.

Computing a popular partition in a Stable Roommates game is NP-hard, even if the preferences are strict. One method of obtaining tractability results from intractable coalition formation problems is to put restrictions on the agents' preferences for which the associated computational problems become tractable. The two variants of the multidimensional Stable Roommates problem we study, called the Roommate Diversity Problem and the 3D-Euclidean Stable Roommates Problem, fall into the same type of approach.

Boehmer and Elkind introduced a restricted variant of the multidimensional Stable Roommates problem called the roommate diversity problem: each agent belongs to one of two types, and the agents' preferences over the rooms solely depend on the fraction of agents of their own type among their roommates. Ties are allowed in the preferences of the agents. The roommate diversity problem has been studied with the notions envy freeness, exchange stability, core stability, and Pareto optimality. For most of these notions, computing a stable or optimal partitioning in a roommate diversity game is NP-hard or co-NP-hard. On the other hand, for some of the stability notions the problem is FPT when using the room size as parameter. We show that popularity on the roommate diversity problem, with the room size fixed to 2, is tractable. Particularly, a popular partitioning of agents is guaranteed to exist and can be computed in polynomial time. Additionally, a mixed popular partitioning of agents is guaranteed to exist in any roommate diversity game. By contrast, when no restrictions are placed on the room size in a roommate diversity game, a popular partitioning may not exist and the problem becomes intractable. Our intractability results are summarized as follows:

- Determining the existence of a popular partitioning is co-NP-hard, even if the agents' preferences are trichotomous.
- Determining the existence of a strictly popular partitioning is co-NP hard, even if the agents' preferences are dichotomous.
- A mixed popular outcome for a roommate diversity game is not computable in polynomial time, even if the agents' preferences are dichotomous, unless $P=NP$.

Arkin et al. introduced a restricted variant of the Stable Roommates problem called the Geometric Stable Roommates: each agent is associated with a point in d -dimensional metric space \mathbb{R}^d and an agent prefers to be matched with an agent located close to it over an agent that is located far from it. Chen and Roy investigated a restricted variant of the Geometric Stable Roommates, called the Euclidean d -Dimensional Stable Roommates (Euclid- d -SR) problem, where every agent is associated with a point in 2-dimensional Euclidean space. The agents are partitioned into rooms of size d . Chen and Roy found that determining the existence of a core stable outcome for $d \geq 3$ is NP-complete for the Euclid- d -SR problem. We study a modified version of the Euclid- d -SR problem, called the 3D Euclidean Multidimensional Stable Roommates problem, where we consider 3-dimensional space instead of 2-dimensional, with the notion of popularity. We show that determining the existence of a strictly popular outcome in a 3D-Euclidean Multidimensional Stable Roommates game with room size 3 is co-NP-hard.

備考：論文要旨は、和文 2000 字と英文 300 語を 1 部ずつ提出するか、もしくは英文 800 語を 1 部提出してください。

Note：Thesis Summary should be submitted in either a copy of 2000 Japanese Characters and 300 Words (English) or 1 copy of 800 Words (English).

注意：論文要旨は、東京科学大学リサーチリポジトリ(T2R2)にてインターネット公表されますので、公表可能な範囲の内容で作成してください。

Attention: Thesis Summary will be published on Science Tokyo Research Repository Website (T2R2).